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ADVANCED TEXTBOOK
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ELECTRICITY & MAGNETISM

VOLUME II.
ELECTRODYNAMICS

BY

ROBERT WILLIAM HUTCHINSON, M.Sc.

AUTHOR OF "INTERMEDIATE TEXTBOOK OF MAGNETISM AND ELECTRICITY"
"JUNIOR TECHNICAL ELECTRICITY," "WIRELESS: ITS PRINCIPLES AND PRACTICE"
JOINT AUTHOR OF "TECHNICAL ELECTRICITY"
LATE PRINCIPAL OF THE MUNICIPAL TECHNICAL COLLEGE, SMETHWICK

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PART II.

CHAPTER X.

PRIMARY CELLS AND GENERAL EFFECTS OF CURRENTS.

142. Introduction. The Simple Galvanic or Voltaic Cell.—It has been shown that in order to secure a continual flow of electricity from one point to another some conducting path must be provided between them, and means must be devised for maintaining the two points at different potentials, in which case electricity will continue to flow from the higher to the lower potential; this latter is the function performed by voltaic cells and batteries, dynamos, etc. In all cases the conductor acquires certain new properties attributed to this mysterious agent "flowing" along it; technically we refer to it as a "current" of electricity in the conductor, and the branch of our subject dealing with this is called *current electricity* or *electrodynamics.*

The existence of a current of electricity is known by various effects which it produces, *e.g.* (1) A compass needle suitably placed near the conductor (say a wire) is deflected, and if the wire be coiled round a bar of iron the latter is converted into a magnet; these are known as the *magnetic effects.* (2) If the wire be severed, the ends soldered to two suitable metal plates, and these placed some short distance apart in various liquids, many of the latter are decomposed, one of the products of decomposition appearing at one plate and another at the second plate; these are known as the *chemical effects.* (3) The conductor through which the current flows becomes heated; these are referred to as the *heating effects.*

In magnetism we are concerned with magnetic fields and tubes of magnetic force *at rest* and in electrostatics with electric fields and tubes of electric force *at rest*; the reader will see later that in current electricity we are really concerned with the *coexistence* of the above two fields *while in a condition of relative motion.*

One method of producing the difference of potential necessary for a continual flow of electricity is by chemical action.

Exp. (1) Place a plate of common zinc (Zn) in dilute sulphuric acid (H_2SO_4); a violent action ensues, the zinc is eaten away, zinc sulphate ($ZnSO_4$) is formed, hydrogen gas (H_2) is evolved, and *on the whole energy is liberated* appearing as heat in the solution. The chemical action is expressed by the equation

$$Zn + H_2SO_4 = ZnSO_4 + H_2.$$

(2) Amalgamate the zinc (*i.e.* coat its surface with mercury) and replace in the acid; no action is observed.

(3) Insert a plate of copper in the acid; again no action is observed. Place the amalgamated zinc and the copper side by side in the acid but without touching each other, and still nc action is observed.

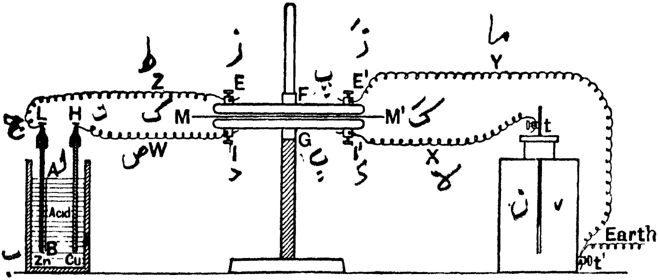


Fig. 273.

(4) Fit up the apparatus shown in Fig. 273, where *V* is an electroscope, *F* and *G* two brass plates, the former being provided with a handle and the latter fixed to an insulating support, *MM'* a sheet of paper between *F* and *G*, and *H* and *L* the copper and amalgamated zinc plates respectively, standing in the dilute sulphuric acid; the connections are as indicated, from which it will be seen that the zinc plate being earthed *is at zero potential*, whilst *H*, *G* and *V*, being connected, are at a common potential. Now remove *W* by insulating tongs and then take away the plate *F*; the leaves of the electroscope diverge, and by the method of Art. 69 it can be

proved that *their potential is positive.* Hence we may conclude that in (3), although no action is observed, *the potential of the copper plate is higher than that of the zinc plate.* Repeat this experiment with the copper plate *H* joined to *E*, *i.e.* earthed and *at zero potential,* and the zinc plate *L* joined to *I*. The leaves diverge as before, but on testing it is found that *their potential is negative*; this also supports the above, viz. that in (3) *the potential of the copper plate is higher than that of the zinc plate.* An explanation of this experiment is given below.

(5) Place the two plates in the acid and connect them outside by a wire (Fig. 274); it will be found that (*a*) the zinc is eaten away and zinc sulphate is formed, (*b*) hydrogen gas appears *at the copper plate*, (*c*) a current of electricity flows in the circuit, as can be readily proved by bringing a compass needle near the wire; the direction of the current is from copper to zinc in the connecting wire, zinc to copper in the liquid.

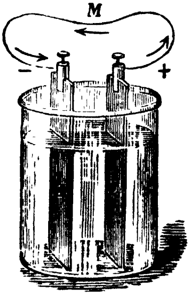


Fig. 274.

Such an arrangement is called a simple, galvanic, or voltaic cell; the copper plate is at a higher potential than the zinc and is called the *high potential plate*, the portion of it outside the liquid being called the *positive pole*; the zinc is known as the *low potential plate*, the portion of it outside being called the *negative pole.* In the outside circuit the current *naturally* flows from the high potential copper to the low potential zinc; inside the energy of the chemical action forces the electricity from the low to the high potential. The chemical action is, in fact, similar to that of a pump lifting water from a lower to a higher level, from which position the water would naturally run down again, doing work in virtue of the energy conferred upon it. Thus in the cell the consumption of the zinc really furnishes the energy which maintains the current in the circuit. The difference in potential between the zinc and the copper when they are merely immersed in the acid and not connected, *i.e.* when the cell is on "open circuit," is called the **electro-motive force** (E.M.F.) of the cell.

The explanation of the results noted in (4) above may now be given briefly as follows:—*F* and *G* form a condenser of fairly large capacity. When arranged as in the figure, H, G and V acquire a common potential equal in fact to the E.M.F. of the cell, and, of course, a positive charge has been gained by G; the potential is, however, not strong enough to cause the leaves to diverge. When F is removed we have no longer a condenser but merely a plate G of much smaller capacity, and the charge it has acquired raises its potential to such an extent that the leaves diverge.

On the electron theory the current from a cell is a flow of *negative* electrons from zinc to copper outside.

143. Preliminary Ideas on the "Chemical" Theory of the Simple Cell.

143. Preliminary Ideas on the "Chemical" Theory of the Simple Cell.—The theory of the simple cell is dealt with in Chapter XIV., but the following elementary treatment at this stage will considerably assist the reader to understand much that follows.

Investigations relating to the alteration of the freezing point, boiling point, and vapour pressure of water, produced by dissolving acids and salts therein, have led to the conclusion that dilute solutions are "dissociated," *i.e.* the molecules are broken up into atoms, or groups of atoms, and further, that there is a constant interchange of atoms between the molecules; thus at any instant a large number of atoms, in the act of passing from molecule to molecule, will be *free* or *dissociated*, and these free atoms are supposed to be electrically charged—metallic ones positively and non-metallic ones negatively. Such free, charged atoms are called **ions.** ✓

Consider the zinc and copper plates in the dilute acid—practically acidulated water—*but not connected.* The liquid contains a large number of oxygen and hydrogen ions, the former negatively, the latter positively, charged. Now zinc has great affinity for oxygen; it attracts the negative oxygen ions within a very narrow film round about it until its potential becomes so strongly negative that it begins to repel the oxygen ions electrically as intensely as it attracts them chemically; equilibrium is soon attained, the final result being, however, that the potential of the zinc is lowered by an amount e_1, say, below that of the outer surface of the film. The acidulated water, being a conductor, has the same potential throughout.

The copper also attracts the negative oxygen ions within a very narrow film, so that its potential is, say, e_2 below that of the outer surface of this film; the attraction is, however, less than in the case of the zinc, so that e_2 is less than e_1, and *the copper is, therefore, at a higher potential than the zinc.*

The P.D. between the zinc and the copper is clearly $e_1 - e_2$ and this measures the E.M.F. (E) of the cell. Fig. 275 (a) represents (not to scale) the potential slopes referred to, the vertical distances denoting potentials, and AB and CD the outer surfaces of the "films" at the zinc and copper respectively.

When the plates are connected by a wire the condition of equilibrium is upset. Electricity flows along the wire, from copper to zinc, lowering the potential of the former and raising that of the latter; the zinc again attracts negative oxygen ions and the copper now repels them, and this motion of negative ions in the direction copper to zinc in the liquid necessarily implies a motion of positive ions (*i.e.* hydrogen ions) in the opposite direction zinc to copper, and this is the direction recognised as that of the current inside, and, as far as the potential slopes are concerned, there is on the whole a perpetual process towards readjustment, the P.D. in the film at the zinc being maintained equal to e_1 and that at the copper equal to e_2 (neglecting the influence of the hydrogen there). Now, however, there is a fall of potential in the liquid from the outer surface of the zinc film to the outer surface of the copper film equal to V, say, and a potential fall in the wire equal, say, to e. These potential slopes are shown in Fig. 275 (b); clearly—

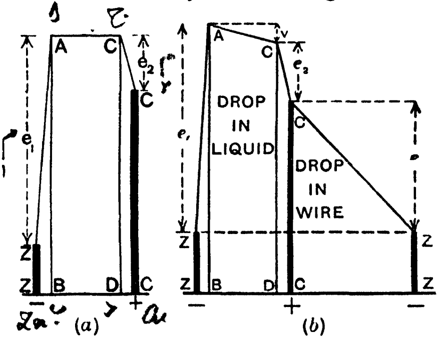


Fig. 275.

$$e_1 = V + e_2 + e, \quad i.e.\ e_1 - e_2 = e + V,$$
$$\therefore\ E = e + V$$
$$V = E - e$$
$$e = E - V.$$

The same general idea may be expressed in a slightly different form. When a piece of metal, say zinc, is placed in say sulphuric acid, *positive* zinc ions pass from the plate to the solution (thus making the potential of the zinc *negative with respect to the solution*) until the negative zinc and the positive solution give rise to such an electrostatic force that equilibrium is attained and no more zinc ions can leave the plate; thus the potential difference e_1, in Fig. 275 (*a*), is established, and a somewhat similar explanation may be given for e_2 (see Chapter XIV.).

When the plates are joined by a wire electricity flows, as explained, from the copper to the zinc, *lowering* the potential of the copper and *raising* the potential of the zinc. The latter result *lessens* the electrostatic force which prevents zinc ions going into solution, and in consequence more positive zinc ions leave the plate and the potential difference e_1 is maintained. The lowering of the potential of the copper results, similarly, in positive hydrogen ions being discharged towards it (and negative ions being repelled the other way). Thus the current is maintained in the circuit—in the wire from copper to zinc, in the cell by a procession of positive ions in the direction zinc to copper, and negative ions in the direction copper to zinc.

In the theory outlined above the sign of the P.D. between the plates and the acid is the same for both plates, and the E.M.F. (E) *is the difference of these two*, viz. $e_1 - e_2$. Experiments on the single potential differences at the surfaces of metals and solutions, made by means of "dropping electrodes" and capillary electrometers (Chapter XIV.), seem to indicate, however, that the potential differences at the surfaces of zinc and copper have opposite signs, *i.e.* that the acid is, say, v_1 *above* the zinc and that the copper is, say, v_2 *above* the acid, so that $E = v_1 + v_2$. It is uncertain which of these views strictly represents the case of the simple cell; certainly there is some doubt as to the accuracy of single P.D. determinations such as these by dropping electrodes and capillary electrometers.

According to the theory thus briefly outlined the E.M.F. of a cell is due to the different tendencies of the two plates to combine with oxygen, or to the different tendencies of the metals to project ions into solution, and to this should be added, as will be seen later, the tendency of ions to diffuse through the liquid owing to differences in concentration.

In the simple cell plates of zinc and copper are employed, but any two substances which differ in their affinities for oxygen may be used. The following list is arranged in decreasing affinity for oxygen (and decreasing solution pressures); any two of these may be chosen as the elements of the cell, the further the substances are apart on the list the greater will be the E.M.F. and the substance which is the higher on the list will constitute the negative pole of the cell:—Manganese, Zinc, Lead, Tin, Iron, Copper, Mercury, Silver, Platinum, Carbon.

144. Local Action and Polarisation.—Common zinc contains many impurities, such as iron, lead, arsenic, etc.; these, together with the zinc, being in contact with the acid give rise to a number of local currents all over the surface of the plate, the result being that the zinc is consumed without any advantage being gained therefrom. This, termed "*local action*," is prevented by amalgamating the zinc. The mercury dissolves the zinc, forming a uniformly soft amalgam which covers up the impurities; as the zinc is consumed in the cell the impurities fall to the bottom. Local currents between portions of the plate differing in hardness are also prevented by this device.

We have seen that when the cell is giving a current hydrogen bubbles appear *at the copper plate.* One explanation of their appearance there was given by Grotthüs, and is shown in Fig. 276.

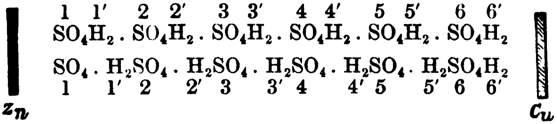

1 1′ 2 2′ 3 3′ 4 4′ 5 5′ 6 6′
$SO_4H_2 . SO_4H_2 . SO_4H_2 . SO_4H_2 . SO_4H_2 . SO_4H_2$
$SO_4 . H_2SO_4 . H_2SO_4 . H_2SO_4 . H_2SO_4 . H_2SO_4H_2$
1 1′ 2 2′ 3 3′ 4 4′ 5 5′ 6 6′
Zn
Cu

Fig. 276.

The upper row shows the arrangement of the molecules before the poles are connected, the lower row after the connection is made. The zinc (Zn) combines with the sulphion (SO_4), and alternate separations and recombinations take place until finally the hydrogen of the molecule on the right is liberated at the copper plate. The dissociation theory of Clausius gives a more modern explanation: the positive hydrogen ions travel towards the copper, and the negative ions towards the zinc. The deposition of the hydrogen on the copper is termed "*polarisation*"; it weakens the current and spoils the cell in two ways—

(*a*) The gas has a large resistance (Art. 154), *i.e.* it strongly opposes the flow of the current.

(*b*) A back pressure, or back E.M.F., greater than e_2 is set up, so that the E.M.F. (E) of the cell (viz. $e_1 - e_2$) is greatly diminished.

Modern primary cells are mainly devices for the elimination of polarisation.

145. Various Forms of Cells.—As it is essential for the student to see, handle, and work with these cells, detailed descriptions will not be given here.

(1) DANIELL'S CELL.—The Daniell's Cell (Fig. 277) consists of an outer cylindrical copper vessel forming the high potential element. This contains a concentrated solution of copper sulphate ($CuSO_4$), which acts as the "depolariser," *i.e.* the substance which prevents polarisation. In this stands a porous earthenware pot containing dilute sulphuric acid and an amalgamated zinc rod. To the upper portion of the copper cylinder a perforated shelf is attached (not shown), carrying crystals of copper sulphate; these are partially covered by the solution, and by gradually dissolving maintain the strength of the latter.

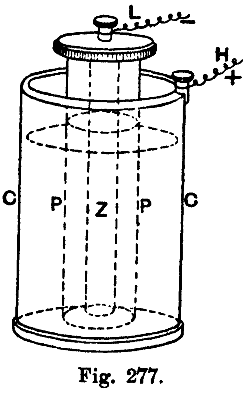


Fig. 277.

Briefly, the action is as follows:—The solutions ionise so that we have positive hydrogen and copper ions and negative sulphions (SO_4), and positive zinc ions pass from the zinc plate into solution. In the porous pot zinc sulphate is formed according to the equation

$$Zn + H_2SO_4 = ZnSO_4 + H_2,$$

and, at the same time, with repeated combinations and dissociations, the positive ions travel, under the influence of the electric field, towards the outer compartment. In the outer compartment the positive copper ions travel similarly towards the copper vessel, where they are deposited and give up their charges, whilst the hydrogen, which has entered from the inner compartment, joins with the SO_4, forming sulphuric acid according to the equation

$$H_2 + CuSO_4 = H_2SO_4 + \mathbf{Cu}.$$

Thus copper is deposited on the high potential plate, hydrogen does not appear there, and polarisation is prevented. Frequently zinc sulphate is used in place of sulphuric acid, in which case we may write—

$$\mathbf{Zn} + ZnSO_4 = \mathbf{Zn}SO_4 + Zn$$
$$Zn + CuSO_4 = ZnSO_4 + Cu.$$

The E.M.F. is about 1·1 volts (Art. 153), and the resistance (Art. 154) rather high; but both are fairly constant, so that the cell is useful when small, but constant currents are required.

In *Gravity Daniell's Cells* no porous pot is used, the denser copper sulphate solution being placed at the bottom of the cell, the lighter zinc sulphate resting on it. The *Minotto Cell* (Fig. 277*a*) is a modification; at the bottom is a copper plate and copper sulphate crystals, and above this sand or sawdust moistened with zinc sulphate solution; at the top is the zinc plate.

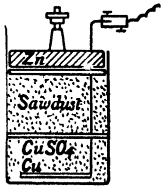


Fig. 277*a*.

(2) Grove's Cell and Bunsen's Cell.—In Grove's Cell the zinc, Z (Fig. 278), is cast in the form of a **U**, and is placed in dilute sulphuric acid. In the bend stands the porous pot containing strong nitric acid (HNO_3) and the high potential plate, viz. a sheet of platinum. The chemical reactions are—

$$Zn + H_2SO_4 = ZnSO_4 + H_2$$
$$H_2 + 2HNO_3 = 2H_2O + 2NO_2 \text{ (nitric peroxide).}$$

Thus hydrogen does not appear at the platinum and there is no polarisation; the nitric peroxide gas is soluble in strong nitric acid, and does not, therefore, gather on the platinum plate. The E.M.F. is about 1·9 volts, and the resistance fairly low; hence it is useful when strong and fairly constant currents are required. The fumes of nitric peroxide are disagreeable, and the use of platinum makes the cell expensive.

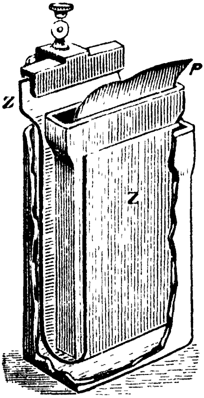


Fig. 278.

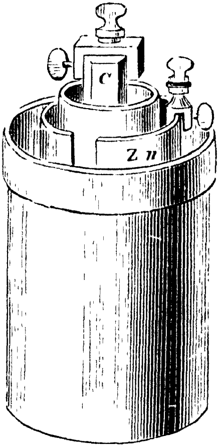


Fig. 279.

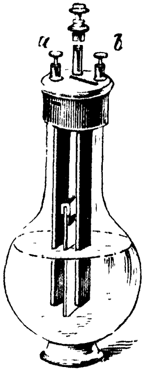


Fig. 280.

Bunsen's Cell (Fig. 279) is a modification of Grove's, in which the platinum is replaced by a rod of cheap gas coke. The liquids used and the action are the same as the preceding.

(3) Poggendorff's Bichromate and the Chromic Acid Cells.—These are single fluid cells, the former being made by mixing bichromate of potash (3 oz.), sulphuric acid (3 oz.), and water (1 pint), or corresponding quantities, according to the size of the cell. In this hang two plates of carbon (Fig. 280) connected to one terminal, and in between them a plate of zinc connected to another terminal. The reactions are—

$$(1)\quad \underset{\substack{\text{Potassium}\\\text{bichromate}}}{K_2Cr_2O_7} + \underset{\substack{\text{Sulphuric}\\\text{acid}}}{7H_2SO_4} + \underset{\text{Water}}{H_2O} = \underset{\substack{\text{Chromic}\\\text{acid}}}{2H_2CrO_4} + \underset{\substack{\text{Potassium}\\\text{sulphate}}}{K_2SO_4} + \underset{\substack{\text{Sulphuric}\\\text{acid}}}{6H_2SO_4}.$$

$$(2)\quad 3Zn + 3H_2SO_4 = 3ZnSO_4 + 3H_2.$$

$$(3)\quad \underset{\text{Hydrogen}}{3H_2} + \underset{\substack{\text{Chromic}\\\text{acid}}}{2H_2CrO_4} = \underset{\substack{\text{Chromic}\\\text{oxide}}}{Cr_2O_3} + \underset{\text{Water}}{5H_2O}.$$

$$(4)\quad \underset{\substack{\text{Chromic}\\\text{oxide}}}{Cr_2O_3} + \underset{\substack{\text{Sulphuric}\\\text{acid}}}{3H_2SO_4} = \underset{\substack{\text{Chromium}\\\text{sulphate}}}{Cr_2(SO_4)_3} + \underset{\text{Water}}{3H_2O}.$$

$$(5)\quad \underset{\substack{\text{Potassium}\\\text{sulphate}}}{K_2SO_4} + \underset{\substack{\text{Chromium}\\\text{sulphate}}}{Cr_2(SO_4)_3} = \underset{\substack{\text{Chrome}\\\text{alum}}}{K_2Cr_2(SO_4)_4}.$$

The waste product, chrome alum, crystallises on the plates and impairs the cell. To avoid this *chromic acid* is now invariably used instead of potassium bichromate, in which case equations (2), (3), and (4) represent the action.

The E.M.F. is about 2 volts and the resistance low, but the current soon begins to fall off; it is useful where a strong current is wanted for a short period.

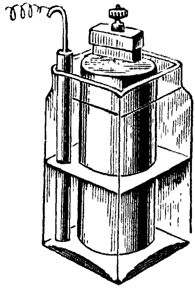


Fig. 281.

(4) Leclanché Cell. — In the outer vessel (Fig. 281) is a zinc rod immersed in a solution of ammonium chloride (sal ammoniac), and in the porous pot is a rod of carbon surrounded by a mixture of broken carbon and black oxide of manganese. The reactions are—

$$Zn + \underset{\substack{\text{Ammonium}\\\text{chloride}}}{2NH_4Cl} = \underset{\substack{\text{Zinc}\\\text{chloride}}}{ZnCl_2} + \underset{\text{Ammonia}}{2NH_3} + H_2 \ldots\ldots\ldots\ldots (1)$$

$$H_2 + \underset{\substack{\text{Black oxide of}\\\text{manganese}}}{2MnO_2} = \underset{\substack{\text{Brown oxide of}\\\text{manganese}}}{Mn_2O_3} + H_2O \ldots\ldots\ldots\ldots (2)$$

The hydrogen, however, is liberated quicker than the MnO_2 can use it up, so that after a time polarisation sets in and the current falls off. If allowed to rest for a few minutes the MnO_2 performs its work, as per equation (2), and the cell regains its strength; hence Leclanché Cells are adapted for intermittent work, *e.g.* electric bells and telephone calls. The E.M.F. is about 1·5 volts, but the resistance is frequently high.

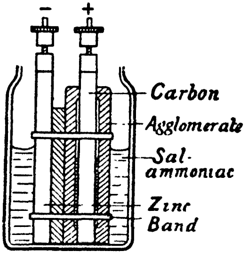


Fig. 282.

(5) Agglomerate Leclanché Cell.—This type (Fig. 282) dispenses with the porous pot. A mixture consisting of 40 parts of black oxide of manganese, 55 parts of gas coke, 3 parts of shellac, 2 parts of potassium sulphate, and a little sulphur is heated to a high temperature, and, by hydraulic pressure, formed into a compact mass. Two blocks of this agglomerate are fixed by indiarubber bands to a rod of gas coke forming the high potential element. Other details will be gathered from Fig. 282. The resistance is less than that of the ordinary type.

(6) Dry Cells.—The so-called dry cells are mainly modifications of the Leclanché. The E.C.C. type (Fig. 283) consists of a zinc cylinder, next to which is a paste, *W*, composed of plaster of Paris, flour, zinc chloride, sal ammoniac, and water. Adjoining this is a paste, *B*, of carbon, oxide of manganese, zinc chloride, sal ammoniac, and water. *C* is a rod of carbon. The whole is covered with a case of mill-board, is sealed with pitch, and is provided with a vent for the escape of gas. The E.M.F. is about the same as that of an ordinary Leclanché, but the internal resistance is much less.

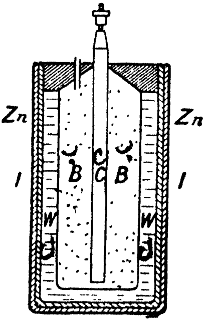


Fig. 283.

The *Hellesen Cell* consists of two cylinders of zinc, the inner one being perforated and lined with paper. Between these is a paste consisting of sal ammoniac, plaster of

Paris, and gum tragacanth. In the centre of the cell stands the carbon rod, around which is the depolariser, composed of oxide of manganese, plumbago, and sal ammoniac. The whole is sealed with pitch and provided with a gas vent.

(7) Edison-Lalande Cell.—This is a one-fluid cell, the plates being compressed copper oxide and zinc, both immersed in a solution of sodium hydrate. Its E.M.F. is about ·75 volt, but the resistance is low, so that large currents can be obtained from it, and it is free from local action and polarisation. The chemical reactions are—

$$\mathrm{Zn} + 2\mathrm{NaHO} = \mathrm{ZnNa_2O_2} + \mathrm{H_2}$$

$$\mathrm{H_2} + \mathrm{CuO} = \mathrm{H_2O} + \mathrm{Cu}.$$

The cell is largely used in America for railway work.

(8) Benkö Batteries.—This is made up of the latest type of primary cell. The elements consist of a rod of zinc placed inside a flattened porous carbon cylinder. The latter is closed at the bottom, and is surrounded by an outer closed chamber of lead. The liquid (some chromic acid solution) is contained in a vessel fixed some distance above the level of the cell. From this height it flows into the surrounding chamber, slowly passes through the porous carbon towards the zinc, and is finally pumped back to the containing vessel again or passed to waste. The constant renewal of the solution in contact with the carbon eliminates polarisation. The hope is entertained that batteries of these cells may, in time, to some extent replace accumulators (Art. 209). Single cells are, naturally, not in use; the standard article is a seven-cell battery (Art. 160). [illegible] ضروری

(9) Latimer Clark Standard Cell.—The Board of Trade pattern of the Latimer Clark Standard Cell is shown in Fig. 284. The containing vessel is a small test-tube about 2 cm. in diameter and 4 or 5 cm. deep. Mercury is placed in the bottom of the tube and forms the high-potential element of the cell. Above this is placed a mixture of mercurous sulphate and saturated

zinc sulphate solution in the form of a thick paste, and above the paste saturated zinc sulphate solution is added. The low-potential element is a rod of zinc supported as indicated in the figure. Contact with the mercury is made by means of a platinum wire protected by a glass tube. The whole is sealed with marine glue coated with sodium silicate. The E.M.F. of this cell is usually taken as 1·434 volts at 15° C., and it falls in value with rise of temperature; its E.M.F. at t° C. is

$$E_t = 1{\cdot}434 \{1 - {\cdot}00077\ (t - 15)\} \text{ volts};$$

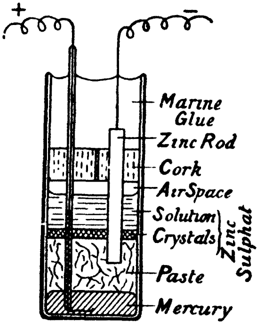


Fig. 284.

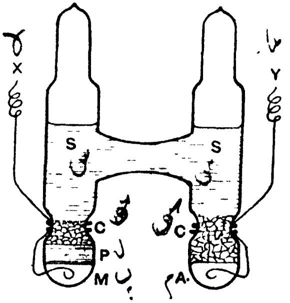


Fig. 285.

but the cell must have been at this temperature for some time for the relation to be true, for the variation of the E.M.F. lags behind the temperature change. At the 1908 International Conference it was decided to take the E.M.F. at 15° as 1·4326 volts.

(10) Weston Cadmium Standard Cell.—In this cell (Fig. 285) mercury (M) is the positive pole, an amalgam of mercury and cadmium (A) the negative pole, cadmium sulphate (C and S) the electrolyte, and mercurous sulphate paste (P) the depolariser. (C = crystals of, and S = a saturated solution of, cadmium sulphate.) Its E.M.F. may be taken as 1·0195 volts at 15° C. Certificates issued by the National Physical Laboratory take the

E.M.F. as 1·0184 volts (international) at 20° C. and at any temperature $t°$ C.—

$$E_t = E_{20} - \cdot 0000406\,(t-20) - \cdot 00000095\,(t-20)^2 + \cdot 00000001\,(t-20)^3.$$

Cells grouped together are called "batteries"; there are three groupings frequently adopted, viz. *series*, *parallel*, and *mixed* (Art. 160).

It should be mentioned in passing that the E.M.F. of a cell depends on the temperature and on the materials employed in its construction; it is independent of the size of the plates and their distance apart. On the other hand the internal resistance depends (1) on the size of the plates—the larger the area of the plates the less the resistance; (2) on the distance apart of the plates—the less the distance the less the resistance; (3) on the concentration, etc., of the liquid or liquids employed. *Thus a cell with large plates has the identical E.M.F. of a small one of the same kind, but the big one has the lesser resistance.* The student will understand these points better after reading Chapter XI.

A good voltaic cell should meet the following requirements:—

(1) Its electromotive force should be high and constant.

(2) Its resistance should be small.

(3) It should be free from polarisation.

(4) It should give a constant current for a considerable time, and should therefore have a good supply of suitable working materials, which should not be rapidly exhaustible.

(5) No chemical action should go on in it except when the current is passing.

(6) It should be convenient and economical in use.

Exercise. Criticise the cells dealt with from the point of view of these requirements.

146. Magnetic Effects of a Current.—It was soon discovered by experiment that *a conductor which is carrying a current has a magnetic field surrounding it.*

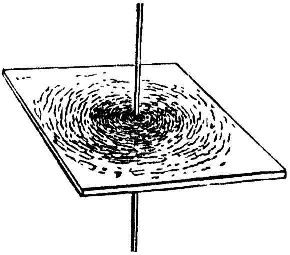


Fig. 286.

Exps. (1) Bore a hole in a piece of cardboard, fix it in a horizontal position, pass a wire vertically through it, and let a strong current flow through the wire. Sprinkle iron filings on the cardboard and gently tap the latter. The filings arrange themselves in concentric circles round the wire as their common centre; these indicate the lines of force in the magnetic field due to the current (Fig. 286).

(2) Arrange that the current is flowing *down* the wire. Move a small compass needle round the wire and note in which direction the north pole points. In this case it will be found that the north pole points as indicated by the arrows in Fig. 287 (*a*), *i.e.* the positive direction of the lines of force is *clockwise.* Repeat with the current flowing *up* the wire: the north pole will move in the opposite direction, *i.e.* the positive direction of the lines is *counter-clockwise* (Fig. 287 (*b*)).

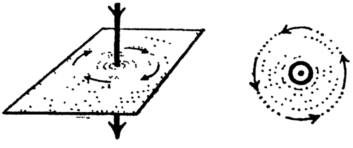

(*a*) (*b*)

Fig. 287.

(3) Place a compass needle on the table and hold a wire *above* and parallel to it, as shown in Fig. 288. Pass a current through the wire (*a*) from south to north, (*b*) from north to south, and note in which direction the north pole of the needle is deflected; it will be found that in (*a*) the north pole of the needle moves towards the *west*, and in (*b*) towards the *east.* Hold the wire *below* the needle and again pass a current (*c*) from south to north, (*d*) from north to south; it will be found that in (*c*) the north pole of the needle is deflected towards the *east* and in (*d*) towards the *west.*

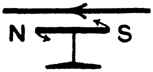


Fig. 288.

The first experiment above was originally due to **Arago** and the last to **Oersted**. The results need not be committed to memory; they may be obtained from the following rules which the reader should verify from the experiments:—

(1) Ampère's Rule.—*Imagine a man swimming in the circuit in the direction of the current and with his face towards the needle; the north pole of the needle will be deflected towards his left hand.*

(2) Right Hand Rule.—*Hold the thumb of the right hand at right angles to the fingers. Place the hand on the wire with the palm facing the needle and turn the fingers in the direction of the current; the thumb will point in the direction in which the north pole will be deflected.*

(3) Maxwell's Corkscrew Rule.—*Imagine an ordinary right-handed screw to be along the wire and to be*

twisted so as to move in the direction of the current; the direction in which the thumb rotates is the direction in which the north pole tends to move round the wire.

From (*a*) and (*d*), or (*b*) and (*c*), of experiment (3) above, it follows that if a compass needle be placed at the centre of a coil of several turns of wire, the plane of the coil being set in the meridian, and a current be passed, the currents in all the wires above and below the needle will tend to deflect it in the same direction and thus a weak current is enabled to produce a deflection; this is utilised in many **galvanometers** (Chapter XII.), which are instruments for the detection and measurement of electric currents.

Exps. (4) Bend a copper wire into a circle of about 10 inches diameter and fix to a horizontal sheet of cardboard as shown in Fig. 289. Pass a strong current and obtain the lines of force by filings as before. Set the coil with its plane in the magnetic meridian, place a compass needle at the centre of the coil and note the direction in which the north pole is deflected. This gives the direction of the magnetic field at the centre of the coil; in Fig. 289 the positive direction is away from the observer.

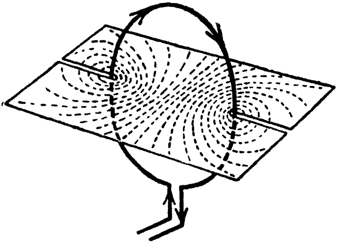


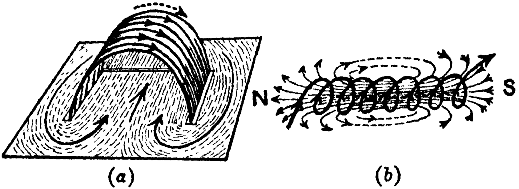
Fig. 289.

(5) Reverse the current and repeat the experiment. The lines of force will be as before, but the direction of the field will be reversed.

The direction of the field at the centre of a circular coil carrying a current may be obtained from the rules given above, but the following is also convenient:—*Looking at the face of the coil, if the current is clockwise, the positive direction of the lines inside the coil is away from the observer; if the current is counter-clockwise, the positive direction is towards the observer.*

Exp. (6) Make a "solenoid" by winding insulated copper wire on a glass or cardboard tube and fix in a horizontal sheet of cardboard as shown in Fig. 290. Pass a current and obtain the lines of force by filings and the direction of the field inside and outside by a compass Reverse the current and repeat.

An examination of Fig. 290 (*a*) will show that the magnetic field in the case of a solenoid carrying a current resembles the magnetic field of a bar magnet. The lines of force leave one end of the solenoid, pass through the outside field and enter the other end, completing their circuit through the solenoid itself. In fact, Fig. 290 (*a*), for example, corresponds to a bar magnet, the near end, *at which the current is circulating clockwise*, corresponding to the *south pole*, and the remote end, *where the current is circulating counter-clockwise*, corresponding to the *north pole*. Fig. 290 (*b*) will serve to emphasise these facts. The following experiments also show the magnetic properties of a current-carrying solenoid and circular coils.



(*a*) (*b*)

Fig. 290.

Exps. (7) Arrange the apparatus shown in Fig. 291, where *m*, *m* are small fixed cups containing mercury and *w*, *w* are wires communicating with a battery. The solenoid is therefore suspended and free to move in a horizontal plane. On passing a current the solenoid sets itself in the magnetic meridian just as a suspended magnet does. Find also its north and south ends by means of a magnet and note the directions of the current at these ends.

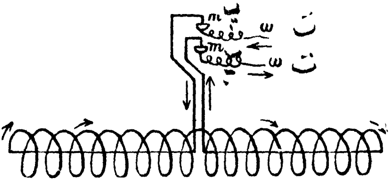


Fig. 291.

(8) Construct apparatus similar to Fig. 292 (De La Rive's floating battery), where Z and C are plates of zinc and copper, fitted in a cork floating in dilute sulphuric acid, the plates being joined by a coil of insulated copper wire. Bring the poles of a magnet near the coil, and note the resulting attraction or repulsion, and the direction of the current.

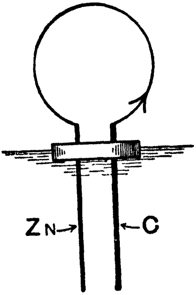


Fig. 292.

From the results of the above experiments the following rules for the polarity of a current-carrying solenoid are deduced :—

(1) Ampère's Rule.—*Imagine a man swimming in the circuit in the direction of the current and with his face towards the inside of the solenoid; his left hand will be towards the north end of the solenoid.*

(2) Right Hand Rule.—*Hold the thumb of the right hand at right angles to the fingers. Place the hand on the solenoid with the palm facing the inside and turn the fingers in the direction of the current; the thumb will be towards the north end of the solenoid.*

(3) End Rules.—*Look at the end of the solenoid; if the current is* **counter-clockwise** *that end is a* **north,** *if it is* **clockwise** *that end is a* **south.**

If a bar of iron be placed inside the solenoid and a current passed through the latter, the iron is converted into a magnet. The polarity of the iron is the same as that of the solenoid and is given by the above rules (see p. 21). Such magnets are called "*electromagnets.*"

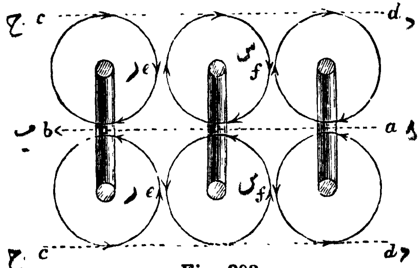


Fig. 293.

The reason for the "end to end" distribution of the lines in the case of a solenoid will be gathered from Fig. 293, which represents three turns. At *e* and *f* the circular lines, due to adjacent turns, are in opposite directions and neutralise each other. Inside, the general direction of the resultant force is evidently from *a* to *b*, and outside, from *c* to *d*.

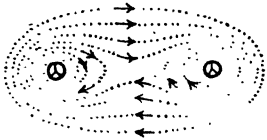


Fig. 294.

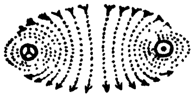


Fig. 295.

Fig. 294 gives the field in the case of two parallel wires carrying equal currents in the *same* direction (downwards) and Fig. 295 the field about two parallel wires carrying equal currents in *opposite* directions.

The student should note that the above rules are assuming the current to be from positive pole to negative pole outside, and should note the change in the wording if the electronic current be considered.

147. Chemical Effects of a Current.—In Art. 142 we have mentioned the fact that many liquids are decomposed by electricity. The process is termed *electrolysis*, the liquid is called the *electrolyte*, and the containing vessel the *voltameter*. The metal plates by which the current enters and leaves the liquid are termed the *electrodes*; that by which it enters is the *anode*, that by which it leaves is the *kathode*; the constituents of the liquid which are liberated and appear at the electrodes are called the *ions*, that travelling towards the kathode being the *kation*, and that travelling towards the anode the *anion*.

Thus, if the poles of a Bunsen's cell be connected to two pieces of platinum foil immersed in water acidulated with sulphuric acid, the passage of the current through the liquid decomposes it into oxygen and hydrogen. These gases are liberated separately at the surfaces of the pieces of platinum foil—the oxygen coming off from that connected with the positive pole of the cell (anode), and the hydrogen from the other (kathode). Remembering that the dissociated atoms (or ions) of oxygen and hydrogen in the liquid are supposed to be charged respectively negatively and positively, this result is just what should be expected; for the foil connected with the positive pole of the cell, being charged positively relative to that connected with the negative pole, at once attracts up to its surface the dissociated oxygen atoms in its neighbourhood, and the dissociated hydrogen atoms seek the other piece. Thus the gases are liberated separately at the two platinum terminals—oxygen at the one and hydrogen at the other—and the current is maintained through the liquid by the stream of dissociated atoms passing from one piece to the other.

The chemical action in this case is very similar to that which goes on in a voltaic cell, but it is essential to distinguish clearly between the two cases. In the voltaic cell the stream of dissociated atoms is set up by a difference between the chemical attractions of the two cell plates for oxygen; but in the case just considered the platinum plates used are exactly similar and have no chemical attraction for oxygen at all, so that it is only on electrifying them to different potentials, by putting them in contact with the poles of a cell, that the necessary difference of attraction for the dissociated atoms is established, and a current thereby set up. Further, since in the voltaic cell the motion of the dissociated oxygen atoms is due to the greater attraction of one of the plates, these atoms on reaching that plate combine with it, and the chemical action which maintains the current goes on. In the case here considered, however, neither the oxygen nor hydrogen atoms combine with the plates to which they are attracted, but their charges, which they give up before liberation, neutralise the charges on the plates at the same rate as these charges are supplied by the cell to which the plates are connected. In this way a current is maintained round the circuit, and the chemical decomposition produced is the result, and not the cause, of its existence.

Exp. (1) *Electrolysis of dilute sulphuric acid* (*or of water*).—Use the Hofmann's voltameter shown in Fig. 296. Two vertical glass tubes *T*, *T* are joined at the bottom by a horizontal tube and are fitted with stop-cocks *S*, *S* at the top. From the horizontal tube springs a central tube terminating in a reservoir *B*. Two platinum electrodes *E*, *E* are joined to terminals *t*, *t*, to which the poles of a Bunsen's battery of two or three cells may be attached. The central tube and *B* are first filled with the dilute acid and the cocks *S*, *S* are then opened. The acid rises in the tubes *T*, *T*, and when these are full the cocks are closed and the current passed. The result is that oxygen and hydrogen appear in the tubes *T*, *T*, oxygen at the one containing the anode and hydrogen at the one containing the kathode, the volume of hydrogen being about double that of the oxygen. The hydrogen may be identified by the property that it burns with a pale blue flame when a light is applied and the oxygen by the fact that it ignites a glowing splinter.

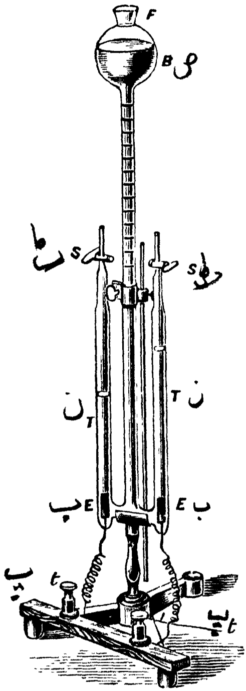


Fig. 296.

When the acid is mixed with water it ionises so that we have a large number of positively charged hydrogen ions and negatively charged sulphions; the result of the dissociation may be written

$$H_2SO_4 = H_2 + SO_4.$$

The hydrogen ions, under the influence of the electric field, "travel" towards the kathode, so that hydrogen is given off there. The sulphion "travels" in the opposite direction and at the anode we get the reaction

$$SO_4 + H_2O = H_2SO_4 + O,$$

so that oxygen is given off there. Combining these two equations we have

$$H_2SO_4 + H_2O = H_2SO_4 + H_2 + O,$$

and deducting H_2SO_4 from each side

$$H_2O = H_2 + O,$$

so that the final result is just the same as if water had been decomposed. The electrolysis of dilute sulphuric acid is thus often spoken of as the electrolysis of water. Pure water (*i.e.* without the acid) cannot, however, be decomposed.

Hydrochloric acid (HCl) may be decomposed into *hydrogen* and *chlorine*, the former appearing at the kathode, the latter at the anode. Since the chlorine attacks platinum a form of Hofmann's voltameter fitted with carbon electrodes may be used.

Exp. (2) *Electrolysis of sodium sulphate* (Na_2SO_4).—Place a porous pot inside a glass vessel and partially fill both with a solution of sodium sulphate. Add to the solution in the porous pot a little reddened litmus and to the solution in the outer vessel a little blue litmus. Use platinum electrodes, one in the porous pot, the other in the outer vessel, the latter being the anode and the former the kathode. Bubbles of gas will rise from each plate, the red solution in the porous pot will be turned blue and the blue solution in the glass vessel will be turned red.

The sodium sulphate contains positive sodium ions and negative sulphions, the former moving towards the kathode and the latter towards the anode. A secondary reaction occurs at the kathode, thus—

$$2Na + 2H_2O = 2NaHO + H_2,$$

so that *hydrogen is evolved at the kathode* and the caustic soda (NaHO) being an alkali turns the red solution blue. A secondary reaction takes place at the anode, thus—

$$SO_4 + H_2O = H_2SO_4 + O,$$

so that *oxygen is evolved at the anode* and the acid turns the blue solution red.

The electrolysis of *potassium sulphate* (K_2SO_4) gives results similar to the above. The potassium set free at the kathode gives a secondary reaction, thus—

$$2K + 2H_2O = 2KHO + H_2,$$

so that hydrogen is evolved there and potassium hydrate is formed. At the anode the secondary reaction mentioned above takes place, oxygen being evolved and sulphuric acid formed.

The electrolysis of a *solution of common salt* or *sodium chloride* (NaCl) gives more complex results. The sodium at the kathode gives the secondary reaction

$$2Na + 2H_2O = 2NaHO + H_2,$$

so that hydrogen is evolved there and caustic soda appears. At the anode some of the chlorine is evolved, some reacts with water forming hydrochloric acid and liberating oxygen, and some forms hypochlorous acid, etc.

The electrolysis of *potassium iodide* yields free iodine at the anode and hydrogen at the kathode, thus—

$$KI = K + I$$
$$2K + 2H_2O = 2KHO + H_2.$$

This furnishes a very delicate test for the existence of weak currents. The free iodine turns starch paste blue so that if the terminals of a circuit be placed a short distance apart on starch iodide paper the point at which the positive terminal rests is indicated at once by a blue dot.

Exp. (3) *Electrolysis of copper sulphate* (*$CuSO_4$*).—Place two pieces of platinum foil a short distance apart in a beaker containing a solution of copper sulphate. Pass a current through the liquid. It will be found that metallic *copper is deposited in a thin layer on the kathode* whilst oxygen is liberated at the anode, the actions being expressed by the equations $CuSO_4 = \mathbf{Cu} + SO_4$; $SO_4 + H_2O = H_2SO_4 + \mathbf{O}$. Reverse the current. Copper will be deposited on the new kathode, the copper previously deposited will gradually disappear from the new anode and then oxygen will be liberated there.

If copper electrodes be used in the above, then, as before, copper is deposited in a thin layer on the kathode. The SO_4 attacks the copper anode forming $CuSO_4$; some also joins with water forming sulphuric acid and liberating oxygen as previously shown. Again, the oxygen may combine with the copper forming copper oxide and this may again dissolve in sulphuric acid forming copper sulphate. In the majority of cases the action at the anode is of the complex character indicated, but if the copper anode be in such a condition that it is readily attacked by the SO_4 copper is merely taken from it to form $CuSO_4$, and *the loss in weight of the anode will be equal to the gain in weight of the kathode.* It looks, in fact, as if copper were merely carried through the liquid from one plate to the other, the average concentration of the liquid remaining the same.

Exp. (4) *Electrolysis of silver nitrate* (*$AgNO_3$*).—Repeat the experiment with a solution of silver nitrate (*a*) with platinum electrodes, (*b*) with silver electrodes. Results similar to those mentioned above will be obtained: in both cases silver will be deposited on the kathode.

The above experiments illustrate the method of electroplating; thus to silverplate an iron spoon it must be made to form the kathode in a silver solution, the anode being a silver plate. In practice special arrangements have to be made in order to get a coherent layer which will afterwards take a good polish. The following points, however, may be mentioned:—

SILVERPLATING.—*Anode* = Silver; *Electrolyte* = Solution of double cyanide of silver and potassium.

NICKELPLATING.—*Anode* = Nickel; *Electrolyte* = Solution of nickel-ammonium sulphate and ammonium sulphate.

COPPERPLATING.—*Anode* = Copper; *Electrolyte* = Solution of copper sulphate.

ELECTROGILDING.—*Anode* = Gold; *Electrolyte* = Solution of double cyanide of gold and potassium.

Most electrolytes, like those described above, are liquids. It is not, however, always necessary to make a solution of a salt in order to effect its electrolysis, for many salts, especially chlorides, when fused conduct electrolytically. Thus, when fused, silver, magnesium, and aluminium chlorides are readily decomposed by a current, the pure metal appearing at the kathode. This, in fact, is one important method of obtaining a number of metals in a pure state, and the metal potassium was first discovered by Sir Humphry Davy by subjecting the solid hydrate to electrolysis. The hydrate was allowed to deliquesce slightly, so as to become like a paste; then, on passing a strong current through it, it quickly liquefied, and small globules of potassium appeared round the wire where the current left the hydrate. The metal here appeared in a free state because there was not sufficient water present to combine with all of it as it formed. Incidentally it may be mentioned that many substances are now obtained *on a commercial scale* by electrolysis, *e.g.* metallic sodium (Castner process), caustic soda and chlorine (Castner-Kellner process), metallic calcium, aluminium, etc.

The preceding are the more elementary points in connection with the chemical effects of a current; the subject is more exhaustively dealt with in Chapter XIV.

148. Heating Effects of a Current.—The fact that a conductor is heated by the passage of an electric current is well known owing to its application in electric lighting, etc.; the following experiments are, however, instructive:—

Exps. (1) Join the poles of a Bunsen's cell by a short piece of iron wire about half a millimetre in diameter. Hold the wire between the fingers; it gradually becomes hot, and in time may become too hot to touch.

(2) Take short pieces of thick copper wire, thin copper wire, platinum wire, iron wire, and nickel wire of equal lengths, and fasten them end to end. Connect the extreme ends to a battery of three or four Bunsen's cells. The same current flows through each, but the heating effect in each is different. The platinum, iron, and nickel wires will, at different rates, probably become white hot, the thin copper wire may probably become red hot, but the thick copper wire will be only slightly heated.

Now, every substance offers a certain amount of opposition to the passage of electricity through it, and this is referred to as the *resistance* of the substance; the exact definition of resistance is given in Chapter XI. In decreasing order of resistance the substances given may be arranged as follows: Platinum, nickel, iron, thin copper, thick copper. Hence we may conclude, in a general way, that *the heat produced depends on the resistance, being greater the greater the resistance.*

Exp. (3) Connect a piece of platinum wire by copper wires to the terminals of a Bunsen's cell and note the heating effect; disconnect. Now join the positive and negative poles of another Bunsen's cell to the positive and negative poles of the first cell, and once more connect up the platinum. It will be noted that the rise in temperature of the platinum is much faster than before, showing the development of a greater amount of heat.

It will be seen later that joining the two cells as above results in a greater current in the wire; hence we may conclude that *the heat produced depends on the current, being greater the stronger the current.*

By taking the temperature of a cell *before* its poles are connected, and afterwards when the current is flowing, it is readily seen that *heat is produced in the inside as well as in the outside circuit.* Further, from the various experiments we may reasonably conclude that *the heat produced depends on the time, being greater the greater the time during which the current flows.*

The above experiments are merely of a general character; in Chapter XIII. it will be shown that the following are the exact laws relating to the heating effects of a current:—

(1) *The heat produced is proportional to the resistance;* thus, if one wire has twice the resistance of another, twice the amount of heat will be produced in it by the same current in the same time.

(2) *The heat produced is proportional to the square of the current;* thus, if the current flowing through a wire be doubled, four times the amount of heat will be produced in the same time.

(3) *The heat produced is proportional to the time the current flows.*

In symbols $H \propto I^2Rt$, where H denotes the heat produced, I the current, R the resistance, and t the time.

We have seen that when zinc is merely dissolved in sulphuric acid the energy of the chemical action *goes direct to heat.* When it is dissolved in a cell the poles of which are joined by a simple conductor, the energy is first utilised in forcing electricity from the low to the high potential, from which position it flows round the circuit, *the energy, however, finally going to heat,* partly in the conductor and partly in the cell itself. Should the external circuit contain, say, a motor, a certain amount of the energy is utilised in mechanical work, *but the balance again appears as heat* in the circuit.

The whole subject of the heating effects of currents is more exhaustively dealt with in Chapter XIII.

149. Path of Energy in the Circuit of a Voltaic Cell.—In speaking of the circuit of a voltaic cell and of the current in this circuit, it is convenient to speak as if the electrical actions going on were confined to the conducting circuit. It will, however, be understood from what has been said in discussing the energy in an electrical field, that the aether medium surrounding the circuit is really the vehicle of the electrical energy liberated by chemical action in the cell. If the circuit of the cell is not closed the terminals of the cell are oppositely charged, and tubes of force pass, as shown in Fig. 297, from one terminal to the other, and each tube of force is, as explained in Art. 97, the seat of a definite quantity of energy. When the circuit is closed these tubes of force travel outwards from the cell as indicated in Fig. 298. As the ends of each tube approach each other along the conducting circuit, the energy in the tube decreases by transformation into heat in the circuit. The energy from the cell thus passes to any point in the circuit *through the aether*, and the conducting circuit through which the current is usually said to pass *is the circuit along which the ends of the tubes move;* it determines the direction of transmission of the energy, and is the seat of the transformation of this energy into heat. The resistance of the circuit must from this point of view be associated with the rate of dissipation of energy in the circuit, and may be defined, as will be seen later, as the ratio of this rate of dissipation to the square of the current strength. In fact if the heat, H, be expressed in energy units, W, it will be seen later that the statement $H \propto I^2Rt$ (Art. 148) becomes $W = I^2Rt$, and therefore $R = W/I^2t$, *i.e.* $W/t \div I^2$ (Art. 255).

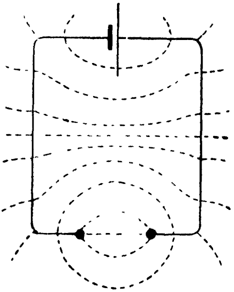


Fig. 297.

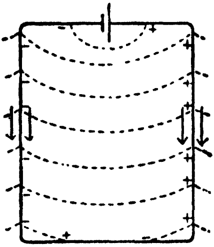


Fig. 298.

Poynting's Theorem treats in detail the principles outlined above, and shows that the paths along which the energy passes through the medium into the circuit are the intersections of the electrostatic and magnetic equipotential surfaces. In the case of a telegraph cable, for example, the magnetic lines are circles about the wire, and the magnetic equipotential surfaces are therefore planes passing through the wire: the electrostatic lines are radial, and (owing to the fall of potential along the wire) the electrostatic equipotential surfaces are frustra of cones: the lines of intersection of these two surfaces are the lines along which the electrical energy travels from the medium into the circuit.

The student should note that in practice a current is regarded as a flow of "electricity" (positive) from the positive to the negative pole in the outside circuit. On the above view the current consists of the passage of tubes of force across the field, the positive ends of the tubes moving in one direction along the wire and the negative ends in the other direction. In a later chapter a current will be viewed as a transference of electrons (negative) in the opposite direction to that usually regarded as the "direction of the current." The student will understand these views better, and be able to correlate them, later.

150. Other Effects of a Current.—The effect of an electric current in stimulating the nerves of a living body belongs rather to Physiology than to Physics.

When electricity under a high and intermittent pressure is passed through a vacuum tube a corpuscular discharge, known as *Kathode rays*, passes in the tube, and if the tube is suitably exhausted invisible rays, known as *Röntgen rays*, or *X rays*, emanate from the tube. The electrically caused *Gamma rays*, from radioactive bodies, are somewhat similar in their nature to X rays.

When electricity under a high pressure discharges between two spheres, say, in air, *Hertzian waves* are set up under certain conditions; these are the waves utilised in wireless telegraphy. *Light waves* are also of an electrical origin.

Light waves, Hertzian waves, X rays, and Gamma rays are all electrical in origin, and are really different rates of vibration of the *aether*; light waves are aether vibrations of a higher rate than Hertzian waves, but of a lower rate than X rays and Gamma rays. Kathode rays, on the other hand, are *not* aether waves, but negatively charged corpuscles or electrons.

These radiant effects will be discussed in detail in later chapters.

Exercises X.

Section A.

(1) Explain local action and polarisation, and show how these are avoided in Daniell's cell.

(2) Give a brief account of some theory put forward to explain the action of the simple cell.

(3) Give convenient rules (*a*) to determine the direction in which a compass is deflected by a current, (*b*) to ascertain the polarity of a solenoid carrying a current.

(4) Describe Leclanché's cell. Why does its E.M.F. diminish when it is short-circuited?

Section B.

(1) You have access to the terminal wires of a hidden battery. Explain how you would tell which wire was connected to the zinc and which to the platinum pole of the battery, by observing what happened when the wires were connected to the terminals of a voltameter containing a solution of silver nitrate. (B.E.)

(2) A vertical partition of porous earthenware is fitted into a tumbler, and dilute sulphuric acid is poured into each compartment. Rods of common zinc and copper are placed respectively in the two compartments and connected by a wire. State what will be observed with regard to the evolution of gas, and how the observed phenomena will be modified when copper sulphate solution is poured into the compartment containing the copper rod. (B.E.)

(3) An electric current (which is the same in all the parts of the trough) flows horizontally in a trough filled with copper sulphate. A rod of copper is then supported horizontally in the trough, with its length parallel to the direction in which the current is flowing. How will the rod be affected by the current? (B.E.)

(4) Two long wires are placed parallel to each other in the same horizontal plane and in the magnetic meridian. A magnetic needle capable of turning in any direction about its point of suspension is placed half-way between them. How will it behave if the same electric current flows through the easterly wire from south to north and through the westerly wire from north to south? [The action of the earth on the magnetic needle may be neglected.] (B.E.)

Section C.

(1) Explain the meaning of the term, and the cause of, polarisation of a voltaic cell. Give two instances of the use in cells of a depolariser; one being an instance of a cell with a single electrolyte, the other of a cell with two electrolytes. (Inter. B.Sc.)

(2) An electric current is flowing along a wire. You are given a pivoted compass-needle, and are required to find out by its aid which way the current is flowing. How would you proceed (*a*) if the wire in question lies horizontally, (*b*) if the wire runs vertically, (*c*) if the wire is coiled up in a circular coil or open hank? (Inter. B.Sc. Hons.)

CHAPTER XI.

FUNDAMENTAL DEFINITIONS, UNITS, AND THEORY.

151. Current Strength and Quantity.—Consider a pipe, AB, through which water is flowing steadily from A to B. Some idea of the strength of this water current may be obtained from a statement of the quantity of water entering A, leaving B, or passing any section of the pipe in a definite time, say one second; in short, *the strength of the current may conveniently be defined as the rate of flow of water through the pipe.* Accepting, then, this definition, *the total quantity flowing past any section in a given time will obviously be obtained by the product of the current strength and the time in seconds.* Further, when the pipe is quite full, the quantity entering A per second must be equal to the quantity leaving B or passing any section of the pipe in that time, however uneven the bore may be; that is, *the current strength is the same at all parts of the pipe.*

These elementary ideas have their electrical analogies. "**Current strength**" is defined as **the quantity of electricity passing any section of the conductor per second,** *i.e.* as **the rate of flow of electricity in the circuit,** and *the total quantity which passes in a stated period is given by the product of the current strength and the time in seconds;* stated algebraically, if I denotes the current strength and Q the quantity transferred in t seconds, $I = Q/t$ and $Q = It$. Further, *the current strength is the same at all parts of a simple conductor*, but, as we have seen, there is a fall of potential in the direction in which the current is flowing.

In Art. 80 the C.G.S., or absolute electrostatic unit quantity has been defined, and if the quantity Q be measured in these units, the ratio Q/t will be the current strength in electrostatic units; thus *the C.G.S. electrostatic unit current is that current in which an electrostatic unit quantity of electricity is conveyed across each section of the conductor in one second.* For reasons which will appear later, the electrostatic units are unsuitable for measurements in current electricity, and others, known as "*C.G.S. electromagnetic units*" (and "*practical units*") are employed.

We have referred to the magnetic effects of a current, and in the present branch of our subject a current is measured in terms of the intensity of the magnetic field produced at a given distance from the conductor carrying the current. Consider a wire bent into a circle of radius r arbitrary units, and carrying a current of strength I arbitrary units; investigation shows that the intensity, F, of the field at the centre of the circle (*i.e.* force in dynes on unit pole) is—

(*a*) directly proportional to the current I;

(*b*) directly proportional to the length, $2\pi r$, of the circular conducting path;

(*c*) inversely proportional to the square of the radius, r, or, stated algebraically—

$$F \propto \frac{2\pi r I}{r^2} \propto \frac{2\pi I}{r} = A \cdot \frac{2\pi I}{r},$$

where A is a constant depending only on the units adopted.

Now let the unit current be so chosen that with r in cm. and I in these units the constant A becomes unity in the above, so that $F = \frac{2\pi I}{r}$, in which case, if I be unit current and r be 1 centimetre, F will be 2π dynes; hence, **The C.G.S. electromagnetic unit current is that which, flowing in a single circular coil of 1 centimetre radius, exerts a force of 2π dynes on a unit pole at the centre.**

Again, since 2π cm. is the circumference of a circle of radius 1 cm., if we consider the force *due to unit length* of the coil, we find that it is 1 dyne; hence—

The C.G.S. electromagnetic unit current is that which, flowing in a wire one centimetre long bent into an arc of one centimetre radius, exerts a force of one dyne on unit pole at the centre.	**The practical unit current is the ampere,** which is $\frac{1}{10}$ of the electromagnetic unit; in the case opposite, the ampere would exert a force of ·1 dyne on the unit pole at the centre.

1 electromagnetic current unit $=3\times10^{10}$ electrostatic current units.
1 ampere $=\frac{1}{10}$ e.m. unit $=3\times10^{9}$ electrostatic units.

Since $Q=It$, unit current flowing for unit time will result in the transfer of unit quantity; hence—

The C.G.S. electromagnetic unit quantity is the quantity conveyed by the electromagnetic unit current in one second.	**The practical unit quantity is the coulomb; the coulomb is the quantity conveyed by one ampere in one second.**

1 electromagnetic quantity unit
$=3\times10^{10}$ electrostatic quantity units.
1 coulomb $=\frac{1}{10}$ e.m. unit $=3\times10^{9}$ electrostatic units.

The above are the exact definitions of the electromagnetic units of current strength and quantity and of the **true ampere** and **true coulomb,** but the chemical effects of a current may be utilised to provide convenient *working* definitions of the various units, and this is the method adopted by the Standards Committee of the Board of Trade in its legal definition of the ampere and coulomb. Thus consider a copper sulphate voltameter (Art. 147) with copper electrodes: on passing a current copper is deposited on the kathode, and careful experiment has established the facts :—

(*a*) That one ampere in one second deposits ·0003293 gramme of copper;

(*b*) That the chemical action is proportional to the current strength;

(*c*) That the chemical action is proportional to the time the current flows.

Thus from (*a*) above we have the following convenient definition of what is referred to as the **international ampere**, viz.:—**The international ampere is that steady current which, flowing through a solution of copper sulphate, deposits ·0003293 gramme of copper on the kathode in one second.**

The Standards Committee recommends the use of silver nitrate as the electrolyte, and the legal definition of the international ampere is:—**The international ampere is that unvarying current which, when passed through a solution of nitrate of silver in water, deposits silver at the rate of 001118 of a gramme per second.**

The international ampere was, of course, intended to be a practical realisation of the true ampere of 1/10 absolute unit, but to be exact it is *just a very little less* than the true ampere (about ·025 per cent.). For experimental and calculation purposes this slight difference may be ignored and the true ampere taken as depositing ·001118 gramme of silver per second, or ·0003293 gramme of copper per second, and the electromagnetic unit current as depositing ·01118 gramme of silver per second or ·003293 gramme of copper per second from the respective solutions.

Again, since unit current flowing for unit time gives the transfer of unit quantity, **the international coulomb is that quantity which liberates ·0003293 gramme of copper from a solution of copper sulphate and ·001118 gramme of silver from a solution of silver nitrate.** Further, neglecting the slight difference mentioned above, the true coulomb may, for calculation purposes, be taken as liberating ·001118 gramme of silver or ·0003293 gramme of copper, and the electromagnetic unit quantity as liberating ·01118 gramme of silver or ·003293 gramme of copper from the respective solutions.

Another quantity unit, the "ampere-hour," is employed. **An ampere-hour is the quantity conveyed by a current of one ampere flowing for one hour** (1 ampere-hour = 3600 coulombs).

It will be seen later that the existence of a **natural unit** of (negative) electricity—the **electron**—has been clearly established; quantities smaller than this cannot be obtained. This natural unit quantity or "atom" of electricity is equal to $4·65 \times 10^{-10}$ electrostatic quantity units, or $1{\cdot}55 \times 10^{-20}$ electromagnetic quantity units, or $1{\cdot}55 \times 10^{-19}$ coulombs.

From the preceding it follows that if a current of I amperes flows through a solution of copper sulphate for t seconds, and if w grammes of copper be deposited on the kathode,

$$w = {\cdot}0003293 \times I \times t,$$

$$\therefore \mathrm{I} = \frac{w}{{\cdot}0003293 \times t},$$

an expression from which, knowing w in grammes and t in seconds, an unknown current of I amperes may be determined.

The amount of an ion liberated in electrolysis by unit current in unit time, i.e. *by unit quantity, is called the "electro-chemical equivalent" of the ion;* thus ·003293 and 0003293 are the absolute (or C.G.S.) and ampere electro-chemical equivalents of copper respectively. Hence, if z denotes the electro-chemical equivalent of an ion, the formulae previously given may be put in the general forms

$$(1)\ w = zIt;\ (2)\ w = zQ;\ (3)\ I = \frac{w}{zt};\ (4)\ Q = \frac{w}{z}.$$

152. Preliminary Note on Units of Energy and Power.—We have frequently referred to the *dyne* as the C.G.S. or absolute unit of force, and to the *erg* as the C.G.S. or absolute unit of work or energy. The erg, being small, a larger unit of energy, known as the *joule*, is often employed; it is equal to 10,000,000, *i.e.* 10^7 ergs.

Power is defined as "rate of working," *i.e.* it is measured by the work done in unit time. The C.G.S. or absolute unit of power is *one erg per second,* but the larger unit, *one joule per second,* is often employed; it is known as the *watt.* It is shown in Chapter XXI. that the relation between the watt and the British gravitational power unit, viz. the *horse-power,* is that the horse-power is equal to 746 watts. Still larger units of energy, viz. the *watt-hour* (3600 joules or 3600×10^7 ergs) and the *kilowatt-hour* (1000 watt-hours), and a still larger unit of power, viz. the *kilowatt*

(1000 watts = 1000 joules per second), are also employed in electrical work.

These various units of energy and power are defined from an electrical standpoint in Art. 156.

153. Potential Difference (P.D.) and Electromotive Force (E.M.F.). Irreversible and Reversible Energy Transformations.—In Chapters V. and VI. potential has been defined as that electrical condition which determines the direction in which electricity will flow, and it has been shown that the P.D. in electrostatic units between two points is represented numerically by the work done in ergs in the transference of the electrostatic unit quantity from one point to the other. Similar reasoning applies to the present branch of the subject. As a simple illustration consider two points A and B in a simple wire through which a current is flowing in the direction A to B, and in which, therefore, A is at the higher potential. The conductor AB is heated, and by the law of conservation of energy this heat must have been produced at the expense of an equivalent amount of energy which has disappeared from the electric circuit; we have in short an *energy transformation* between the two points, electric energy being transformed into heat energy.

Imagine, now, that the electromagnetic unit quantity (*i.e.* 10 coulombs) passes from A to B and that w ergs is the energy which, in this case, is *subtracted* from the electric circuit and appears as heat; the P.D. between the two points is w electromagnetic units of potential. If the unit quantity is *forced* from a low to a high potential work must be done on it, and energy is *added* to the electric circuit; but in either case *the P.D. in electromagnetic units between the two points is represented numerically by the amount of energy transformed in ergs when the electromagnetic unit quantity passes between the two points*, and therefore if the energy transformed be one erg the P.D. is one C.G.S. electromagnetic unit of potential.

The practical unit is the **volt**, which is equal to 10^8 electromagnetic units. Clearly if the P.D. be one volt the energy transformed will be 10^8 ergs when the electromagnetic unit quantity passes, and therefore $\frac{1}{10}$ of 10^8, *i.e.* 10^7 ergs or *one joule*, when one coulomb passes. Hence—

The P.D. between two points is one C.G.S. electromagnetic unit if the energy transformed is one erg when the electromagnetic unit quantity passes.

The practical unit is the volt. The P.D. between two points is one volt if the energy transformed is one joule when one coulomb of electricity passes.

$$1 \text{ electromagnetic unit} = \frac{1}{3 \times 10^{10}} \text{ electrostatic unit.}$$

$$1 \text{ volt} = 10^8 \text{ e.m. units} = 1/300 \text{ electrostatic unit.}$$

The **international volt** (which is *just a little bigger* than the above **true volt** of 10^8 e m. units) is defined in Art. 154.

The P.D. between two points is, in practical work, sometimes called the " pressure " and sometimes the " voltage " between the two points.

Another term in frequent use is **" electromotive force,"** referred to in Art. 143. To illustrate its meaning further, consider a battery *on open circuit,* i.e. with the outside circuit disconnected ; imagine an electrostatic voltmeter joined to its terminals and that the reading is 50 volts ; this measures the total pressure it is capable of developing and this total pressure *given by the P.D. at its terminals on open circuit* is the electromotive force (E.M.F.) of the battery. If the outside circuit be now switched on, the voltmeter reading will fall by an amount depending on circumstances ; imagine the reading is now 47 volts ; this measures the terminal P.D. under present conditions, *i.e.* the volts used in driving the current through the *external* circuit, the other three volts being used in driving the current through the *internal* circuit, *i.e.* through the battery itself. Thus **the E.M.F. of a battery is the total pressure it is capable of developing and is measured by the P.D. at its terminals on open circuit.**

When the current flows the terminal P.D. may be anything according to circumstances, *but it is always less than* *the E.M.F.* To express the facts algebraically, if E volts be the E.M.F., e volts the terminal P.D. when the outside circuit is closed, and V the volts used in driving the current through the battery itself,

$$E = e + V, \quad \therefore e = E - V \text{ and } V = E - e,$$

as was stated in Art. 143.

All these points in connection with the working of a cell (or battery) may probably be more easily understood by considering the following hydrostatic analogy. Let $ABCD$ (Fig. 299) represent a circuit of pipes filled with water and having a stop-cock at T and a small paddle-wheel at W. By rotating the paddle-wheel the water can, when T is open, be driven round the circuit in the direction $CDAB$; but if T is closed, then no current can be produced, but the water is driven into the tube DA until the pressure due to the difference of level in the tubes DA and CB is sufficient to balance the force exerted by the driving wheel at W. The left-hand drawing in Fig. 299 indicates this case, and the difference of level (E) is a measure of the driving force of the paddle-wheel. When, however, the cock T is opened and a current established in the circuit, the levels in the tubes DA and CB change in the way shown in the right-hand drawing of the figure, and *the difference of level* in the vertical tubes is now considerably reduced. The work done by the paddle-wheel is now spent in driving the current of water round the circuit, and the difference of the levels at A and B (e) is a measure of that portion of the driving force of the wheel which is spent in driving the current through the tube AB.

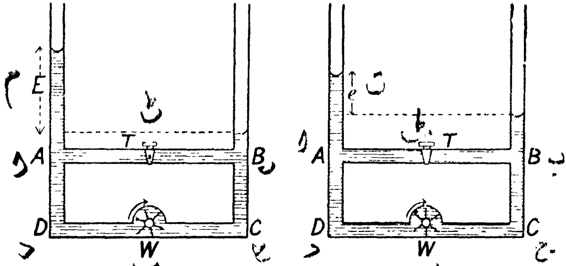


Fig. 299.

Now this hydrostatic arrangement roughly illustrates the action of a voltaic cell—the pressure exerted by the paddle-wheel at W corresponds to the electromotive force of the cell; and if AB represent the external portion of the circuit, the difference of levels at

A and B represents the difference of potential at the poles of the cell. Hence we see that when the circuit is open and no current flows, the potential difference at the poles is a measure of the electromotive force of the cell; but when the circuit is closed, it is only a measure of that portion of the electromotive force which is spent in driving the current through the external portion of the circuit.

The preceding explains what for *practical purposes* may be taken as the distinction between P.D. and E.M.F.; *the latter term should be used when referring to the total pressure developed, which total pressure is given by the P.D. between the terminals on open circuit, the former when referring to the pressure between two points of the circuit.* The *true* distinction, however, between P.D. and E.M.F. will be gathered from the following considerations relating to reversible and irreversible energy transformations.

We have seen that when a current flows along a simple conductor an energy transformation takes place, energy being *subtracted* from the electric circuit and appearing as heat. If the same current flows for the same time in the opposite direction the same energy transformation takes place and the same amount of heat is produced. Such an energy transformation is therefore said to be **irreversible.** It should be noted that in both cases energy is *removed* from the electric circuit and in both cases *there is a fall of potential in the direction in which the current is passing.*

Consider now a Daniell's cell giving a current. In the cell this current flows from zinc to copper, zinc passes from the zinc plate into solution and copper is deposited on the copper plate from the solution. The dissolving of the zinc liberates more energy than is required to deposit the copper, the surplus energy being *added to* the electric circuit, and *there is a rise of potential in the direction in which the current is passing,* i.e. the copper is at a higher potential than the zinc. Now imagine that another generator sends a current through this cell from copper to zinc. The actions will be reversed, copper being dissolved from the copper plate and zinc being deposited on the zinc plate. The dissolving of the copper liberates less energy than is required to deposit the zinc, the deficit being *subtracted* from the electric circuit and *there is a fall of potential in the direction* *in which the current is passing.* In this example the action of one current is exactly reversed by an equal opposite current and the energy transformation is said to be **reversible.** Now—

(*a*) Whenever a potential difference exists between two points in a circuit an energy transformation occurs between them.

(*b*) *If the energy transformation is completely reversible the potential difference is spoken of as an electromotive force.* The E.M.F. is *direct* if energy is *added to* the electric circuit, and it is inverse or *back* if energy is *subtracted from* the circuit. Thus the E.M.F. of a cell is really measured by the energy it provides *in reversible processes* when unit quantity passes, and a back E.M.F. is measured by the work done *reversibly* at the expense of the energy of the circuit when unit quantity passes. The algebraic sum of the direct E.M.F.'s and back E.M F.'s is the resultant pressure, more often spoken of as the **resultant E.M.F.**

(*c*) *If the energy transformation is completely irreversible we speak of the potential difference between the two points;* if partly reversible and partly irreversible we also speak of the potential difference between the two points.

Of course two points may be at the *same potential*, yet E.M.F.'s may exist in the paths between them. Thus, if A and B be the poles of a battery of, say, two similar cells in series, C the mid-point of the battery and D the mid-point of the connecting wire, then C and D are at the same potential but there is an E.M.F. in CAD and in CBD, and a P.D. between A and D and between B and D. Practically we may say in general that if a current is flowing from X to Y there is an energy transformation, the P.D. being equal to the energy so transformed per unit quantity. Any *back* E M.F. in XY equals energy *reversibly* transformed per unit quantity and if this be negative (*i.e.* other forms *reversibly* transformed to current energy) it equals *direct* E.M.F.

154. Resistance and Conductance.—The resistance of a body may be defined in a general way as *that property of it which opposes the flow of electricity,* but more precise definitions are as follows.

Consider first a simple conductor, say a wire, through which a current is passing, and in which, therefore, we have the *irreversible* process, the production of heat. Experiment (and theory) shows that the heat, in energy units, is proportional to the square of the current and the time, and it depends also on the material, dimensions, etc., of the conductor. In symbols—

$$W \propto I^2t = RI^2t,$$

where W is the heat energy, I the current, t the time in seconds, and R is a factor depending on the material, etc., and known as its *resistance.* If I and t be each unity, $W = R$; hence—

The resistance of a wire (temperature uniform) is represented numerically by the heat, in energy units, developed in one second when unit current passes; or, more general—*The resistance of any conductor is represented numerically by the heat, in energy units, developed in one second by* **irreversible processes** *when unit current passes.*

If W, I, and t be each unity, R is unity; hence—

The resistance of a conductor is one C.G.S. electromagnetic unit if the heat produced per second by irreversible processes when the electromagnetic unit current passes is one erg.

The practical unit is the ohm. The resistance of a conductor is one ohm if the heat produced per second by irreversible processes when one ampere passes is one joule.

Since a current of 1/10 absolute unit produces in the ohm 10^7 ergs of heat per second, if R be the value of the ohm in absolute units $10^7 = R(1/10)^2$ or $R = 10^9$, *i.e.*

$$1 \text{ ohm} = 10^9 \text{ electromagnetic units.}$$

Another definition of resistance is important. Again consider a simple wire with a steady P.D. between two points A and B; a certain steady current will be flowing. If the P.D. be altered in magnitude the strength of the current will also be changed, but experimentally it can be proved that *if the temperature of the wire be kept constant the ratio of the P.D. to the current is constant; this constant is called the "resistance" of the part AB of the wire*; stated algebraically—

$$\frac{\text{Potential difference}}{\text{Current}} = \text{a constant} = \text{resistance},$$

or
$$\frac{E}{I} = R,$$

where E is the P.D., I the current, and R the resistance. This formula is also true for a complete circuit, in which R is the resistance of the circuit, I the current, and E the **resultant E.M.F.,** *i.e.* the algebraic sum of direct E.M.F.'s and back E.M.F.'s.

If E and I be each unity, R is unity; hence—

A conductor has a resistance of one C.G.S. electromagnetic unit if a P.D. of one electromagnetic unit applied to its ends causes a current of one electromagnetic unit to flow through it.

A conductor has a resistance of one (true) ohm if a P.D. of one (true) volt applied to its ends causes a current of one (true) ampere to flow through it.

$$1 \text{ ohm} = \frac{1 \text{ volt}}{1 \text{ ampere}} = \frac{10^8 \text{ e.m. units}}{1/10 \text{ e.m. unit}} = 10^9 \text{ e.m. units.}$$

The **international ohm,** which was intended to be a practical realisation of the above **true ohm** of 10^9 e.m. units (but which is really *just a little bigger*) is defined by the Standards Committee as follows:—**The international ohm is the resistance of a column of mercury 106·3 cm. long, 1 sq. mm. in cross-section (mass 14·4521 grm.) at the temperature of melting ice.** (See Arts. 299, 300.)

Conductance is the reciprocal of resistance: thus a wire of resistance R has a conductance $1/R$. The practical unit is the **mho,** which is the conductance of a body of resistance one ohm.

From the relation $E/I = R$ it follows that if two of these be unity the third is unity; hence: (1) the **true volt** is the P.D. which must exist at the ends of (say) a wire of resistance one true ohm, in order that the current passing may be one true ampere; (2) **the international volt is the P.D. which must exist at the ends of (say) a wire of resistance one international ohm in order that the current may be one international ampere.** The true and international amperes and the true and international ohms may be similarly defined.

155. Ohm's Law.—This important law has already been given in dealing with the second definition of resistance (Art. 154); taking first the case of a simple conductor, it may be briefly stated as follows:—

If the temperature of a conductor be kept constant the ratio of the steady direct P.D. applied to its ends to the steady direct current flowing through it is constant; this constant measures the resistance of the conductor, hence

$$\frac{\text{Potential difference}}{\text{Current}} = \text{a constant} = \text{resistance},$$

i.e. $$\frac{E}{I} = R, \therefore E = IR \text{ and } I = \frac{E}{R}.$$

With regard to units, if E in the last expression be in volts and R in ohms, then I will be in amperes; whilst if E and R be in electromagnetic units I will be in electromagnetic units, and so with the other expressions.

To take the more general case, let E denote **the resultant E.M.F.** in a circuit (*i.e.* the algebraic sum of the direct E.M.F.'s and back E.M.F.'s), I the current, and R **the total resistance.** The net energy provided by the *reversible processes* is EQ, i.e. EIt, where Q is the quantity transferred in time t (Arts. 151, 153). The energy represented by the *irreversible heat production* is I^2Rt; hence

$$EIt = I^2Rt,$$

i.e. $$E = IR, \ \frac{E}{I} = R, \text{ and } I = \frac{E}{R}.$$

We can thus apply the above results to the whole or part of an electric circuit. Taking, for example, Fig. 299*a*, let E volts be the E.M.F., *i.e.* the driving influence for the whole circuit, e volts the terminal P.D. which is utilised in driving the current through the external circuit of

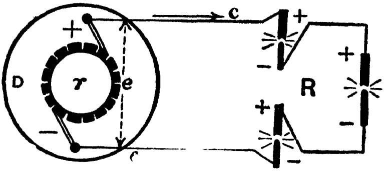


Fig. 299*a*.

resistance R ohms, and V the volts used in driving the current through the internal circuit of resistance r ohms,

$$I = \frac{\text{E.M.F.}}{\text{Total resistance}} = \frac{E}{r + R},$$

$$I = \frac{\text{Terminal P.D.}}{\text{External resistance}} = \frac{e}{R},$$

$$I = \frac{\text{Internal fall of potential}}{\text{Internal resistance}} = \frac{V}{r} = \frac{E - e}{r}.$$

156. Units of Electrical Energy and Power.—The absolute unit of energy is the **erg,** which (Art. 153) may be defined as *the work done* (*or energy transformed*) *between two points of a conductor when the P.D. between the points is one electromagnetic unit and the electromagnetic unit quantity passes.* If the P.D. be E e.m. units and one e.m. unit quantity passes the work done will be E ergs; if Q e.m. units pass the work will be EQ ergs, and if the Q e.m. units be transferred by a current I e.m. units in t seconds Q is equal to It and the energy transformation is EIt ergs; hence, **employing electromagnetic units**—

$$\text{Energy in ergs} = EIt = I^2Rt = \frac{E^2}{R}t \dots\dots\dots (1)$$

t being the time *in seconds.*

If the P.D. be one volt (10^8 e.m. units) and one coulomb (1/10 e.m. unit) passes the energy transformation is clearly 10^7 ergs; this is taken as the practical unit of electrical energy and is called a **joule**; hence *a joule is the work done* (*or energy transformed*) *between two points of a conductor when the P.D. between the points is one volt and one coulomb of electricity passes.* Clearly also, **if we employ practical units**—volts, amperes, ohms—

$$\text{Energy in joules} = EIt = I^2Rt = \frac{E^2}{R}t \dots\dots (2)$$

t being the time *in seconds.*

By (2), if the P.D. be one volt, the current one ampere, and the time one hour, the energy transformation will be 3,600 joules; this is another practical energy unit, called a

watt-hour; hence *a watt-hour is the work done (or energy transformed) between two points of a conductor when the P.D. between the points is one volt and one ampere flows for one hour.* Clearly **if we employ practical units**—

$$\text{Energy in watt-hours} = EIT = I^2RT = \frac{E^2}{R}T \;\ldots\; (3)$$

T being the time *in hours.*

A still larger practical energy unit is the **kilowatt-hour, kelvin,** or **Board of Trade unit**; it is equal to 1,000 watt-hours.

Power is "rate of doing work," and the absolute unit is **one erg per second**; it may be defined as *the power in a circuit when the P.D. is one e.m. unit and the e.m. unit current is passing.* Putting t equal to unity in (1) above and **using electromagnetic units,** we obtain the three expressions for the power in absolute units.

Referring to (2) above, if E and I be each unity the rate of work is **one joule per second**; this is adopted as the practical unit of electrical power, and is called a **watt**; hence *a watt is the power in a circuit when the P.D. is one volt and one ampere is passing; it is equal to* 10^7 *ergs per second or* 1/746 *horse-power.* Putting t equal to unity in (2) and **using practical units,**

$$\text{Power in watts} = EI = I^2R = \frac{E^2}{R} \;\ldots\ldots\ldots\; (4)$$

A still larger practical unit of power is the **kilowatt,** which is equal to 1,000 watts.

The reader should note that the watt and kilowatt are units of power, the watt-hour and kilowatt-hour units of work or energy.

157. Conductors in Series and in Parallel.—If several resistances, r_1 r_2 r_3 . . ., be arranged in *series* (Fig. 300) the total resistance R is clearly their *sum*, i.e. $R = r_1 + r_2 + r_3 + \ldots$

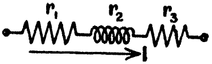


Fig. 300.

Consider now a number of resistances (say three) joined in *parallel* (Fig. 301), and let E denote the P.D. between A and B. Then

$$\text{Current in } r_1 = \frac{E}{r_1}, \text{ Current in } r_2 = \frac{E}{r_2}, \text{ Current in } r_3 = \frac{E}{r_3},$$

$$\therefore \text{ Total current} = \frac{E}{r_1} + \frac{E}{r_2} + \frac{E}{r_3}.$$

Let R denote the joint resistance of r_1, r_2, r_3; then

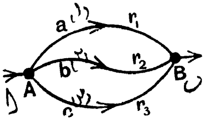


Fig. 301.

$$\text{Total current} = \frac{E}{R},$$

$$\textit{i.e.} \quad \frac{E}{R} = \frac{E}{r_1} + \frac{E}{r_2} + \frac{E}{r_3},$$

$$\therefore \frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \quad \ldots\ldots\ldots\ldots (1)$$

If the resistances are each equal to r_1 this becomes

$$\frac{1}{R} = \frac{3}{r_1}, \ \textit{i.e. } R = \frac{1}{3} r_1,$$

and, similarly, *the joint resistance of n equal resistances in parallel is* $\frac{1}{n}$ *of the resistance of one of them.*

In the case of two resistances in parallel

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 + r_2}{r_1 r_2}, \ \therefore R = \frac{r_1 r_2}{r_1 + r_2} \ \ldots\ldots (2)$$

i.e. *the joint resistance is the product of the resistances divided by the sum of the resistances.*

Whilst the current is the same at all parts of a simple (series) circuit, however the parts differ in resistance, in a parallel arrangement *the current divides directly as the conductances, and therefore inversely as the resistances;* thus, taking the wires a and b of Fig. 301—

$$\frac{\text{Current in } a}{\text{Current in } b} = \frac{\frac{E}{r_1}}{\frac{E}{r_2}} = \frac{r_2}{r_1} = \frac{\text{Resistance of } b}{\text{Resistance of } a}.$$

Example. *Three wires A, B, C of 2, 4, and 6 ohms resistance respectively are arranged in parallel, and the total current passing is 22 amperes. Find the joint resistance and the current in each wire.*

If R = joint resistance—

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{6+3+2}{12} = \frac{11}{12}, \quad \therefore R = \frac{12}{11} = \mathbf{1\tfrac{1}{11}\ ohms.}$$

Again—

Current in $A = \frac{6}{11}$ of 22 amperes = **12 amperes**
,, ,, $B = \frac{3}{11}$ of 22 ,, = **6** ,,
,, ,, $C = \frac{2}{11}$ of 22 ,, = **4** ,,

158. Laws of Resistance.—Experimental verifications of the principal laws which follow are dealt with in Chapter XVI.

(1) *The resistance of a conductor is directly proportional to its length and inversely proportional to its area of cross-section.* The latter part of this statement is important; thus, if one wire has twice the cross-section of another of the same material and length, the thick one will have half the resistance of the thin one.

(2) *The resistance of a conductor depends on the material.* Thus a piece of platinum of given dimensions has 6·022 times the resistance of a piece of silver of the same dimensions.

(3) *The resistance of a substance depends on its molecular condition, density, purity, hardness, etc.* A decrease in the density of copper has been shown to result in increased resistance. Wires subjected to mechanical strain have been found to increase in resistance. In general the resistance of an alloy is much greater than that of the substances forming it. Annealing diminishes the resistance of metals. The resistance of a rod of *bismuth* is considerably affected by a magnetic field, especially if the latter be transverse to the rod; thus, in an experiment due to Henderson the resistance of bismuth was increased 3·34 times under a field of 38,900 C.G.S. units. This effect on bismuth is utilised for the measurement of magnetic fields. *Selenium* decreases in resistance when exposed to light. The resistance of *tellurium* and *carbon* is also affected by light.

(4) *The resistance of a substance depends on its temperature.* The following are the main facts :—

(*a*) Metals.—These increase in resistance when heated, and decrease when the temperature is lowered. If R_t denote the resistance of a wire at t° C. and R_0 its resistance at 0° C.,

$$R_t = R_0 (1 + \alpha t + \beta t^2),$$

where α and β are constants for the same wire, but slightly different for different materials. For calculations the simpler relation

$$\mathbf{R}_t = \mathbf{R}_0 (1 + \alpha t)$$

may be employed ; α is known as the **temperature coefficient**, and may be defined as *the increase in unit resistance per unit rise in temperature* (*0° C. to 1° C.*). In the case of pure metals α may be taken as equal to ·0038 ; for mercury its value is ·00075 nearly.

It has long been surmised that at the absolute zero of temperature (– 273·7° C.) the resistance of all metals would be *nil*, this being based on the assumption that the laws of variation of resistance continued to hold at very low temperatures. Dewar, however, found that at – 250° C. the decrease in the resistance of platinum on cooling was *not so marked* as at – 200° C., and Dr. Harrison's experiments appeared to show that at – 253° C. the resistance of iron *was somewhat greater* than at – 191° C. On the other hand, in some recent experiments, Kamerlingh Onnes finds that at the temperature of liquid helium the resistance of certain pure metals is less than one-thousand-millionth of the values at 0° C., which is *considerably less* than if it fell in proportion to the absolute temperature.

(*b*) Alloys.—Most alloys increase in resistance with temperature, but not to the extent that pure metals do. *Manganin* increases in resistance from 0° C. to 35° C., after which its temperature coefficient becomes negative, but the variation is so small that it may be neglected. *German silver* has a temperature coefficient of about ·00044, *i.e.* $\frac{1}{8}$ that of pure metals, whilst for *Platinoid* the coefficient is only one half of this, viz. ·00022. *Platinum silver, Resista,* and *Eureka* are other alloys with low temperature coefficients.

(*c*) Carbon, Electrolytes, and Insulators.—These decrease in resistance when heated. The cold resistance of a carbon filament lamp is from 1·6 to 2·4 times the hot resistance under full voltage. The resistance of gutta-percha at 0° C. is about 24 times its resistance at 24° C. The decrease in the resistance of an electrolyte is of the order 2·4 per cent. for a rise in temperature of 1° C. when the temperature is 18° C. (Chapter XIV.).

The first two laws given above may be expressed by the equation

$$R = S \frac{l}{a} \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (1)$$

where R is the resistance of the conductor, l its length, a its cross-sectional area, and S a factor depending on the material and known as its **specific resistance** or **resistivity**; the reciprocal of S, viz. $1/S$, is referred to as the *specific conductivity*. If l be one centimetre and a one square centimetre, R is equal to S; hence *the specific resistance of any material is the resistance of a piece of it one centimetre in length and one square centimetre in cross-section*, i.e. *the resistance of a cube of one centimetre side;* frequently S is expressed in terms of the " inch cube."

Consider two wires of circular section. Let $R_1 S_1 l_1 a_1$ apply with their usual meaning to the first wire and $R_2 S_2 l_2 a_2$ to the second; let d_1 be the diameter of the first and d_2 the diameter of the second ($a = \pi r^2 = \cdot 7854 d^2$); then

$$\frac{R_1}{R_2} = \frac{S_1 \frac{l_1}{a_1}}{S_2 \frac{l_2}{a_2}} = \frac{S_1 \times l_1 \times a_2}{S_2 \times l_2 \times a_1} = \frac{S_1 \times l_1 \times (d_2)^2}{S_2 \times l_2 \times (d_1)^2} \quad \ldots\ldots\ldots\ldots (2)$$

The cross-section is sometimes given in terms of the mass and density (mass per unit volume) of the conductor. If w be the mass and ρ the density, the volume of the wire is w/ρ; but the volume is $a \times l$, hence

$$al = \frac{w}{\rho} \text{ and } a = \frac{w}{\rho l} \quad \ldots\ldots\ldots\ldots (3)$$

and substituting in (1)—

$$R = S\frac{\rho l^2}{w} \quad \ldots\ldots\ldots\ldots (4)$$

Again, extending (3) to (2),

$$\frac{R_1}{R_2} = \frac{S_1 \times l_1 \times \frac{w_2}{\rho_2 l_2}}{S_2 \times l_2 \times \frac{w_1}{\rho_1 l_1}} = \frac{S_1}{S_2} \times \frac{\rho_1}{\rho_2} \times \frac{l_1^2}{l_2^2} \times \frac{w_2}{w_1} \quad \ldots\ldots (5)$$

and if the wires be of the same material and density

$$\frac{R_1}{R_2} = \frac{l_1^2}{l_2^2} \times \frac{w_2}{w_1} \quad \ldots\ldots\ldots\ldots (6)$$

All these results are useful for calculation purposes.

159. Insulation Resistance of a Cable.—Let l be the length of the cable, S the specific resistance of the dielectric or insulating covering, and r_2 and r_1 the external and internal radii of the insulation (Fig. 302); let C be a layer of insulation of infinitely small thickness dr and radius r. For the resistance of this layer, say σ, we have

$$\sigma = S\,\frac{dr}{2\pi r l},$$

and for the total insulation resistance R

$$R = \frac{S}{2\pi l}\int_{r_1}^{r_2}\frac{1}{r}\,dr = \frac{S}{2\pi l}\Big[\log_e r\Big]_{r_1}^{r_2}$$

$$i.e.\ R = \frac{S}{2\pi l}\log_e\frac{r_2}{r_1}$$

$$= \cdot 366 \times \frac{S}{l} \times \log_{10}\frac{r_2}{r_1}.$$

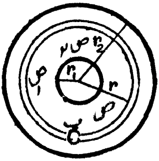


Fig. 302.

Taking two cables of lengths l_1 and l_2 insulated with the same material and having the same values of r_1 and r_2,

$$\frac{R_1}{R_2} = \frac{l_2}{l_1},$$

i.e. *the insulation resistances are inversely as their lengths;* thus, if the insulation resistance of a cable is 600 megohms per mile, half a mile will have an insulation resistance of 1,200 megohms. Further it should be noted that if two cables be joined end to end the conductor resistances are in series, *but the insulation resistances are in parallel.*

160. Grouping Similar Cells.

(1) SERIES GROUPING (Fig. 303).—In this grouping the negative pole of one cell is joined to the positive pole of the next. If there are n cells each of E.M.F. E and resistance r, the combined E.M.F. is nE and the total

internal resistance nr; hence, if I be the current and R the external resistance,

$$I = \frac{nE}{nr + R}.$$

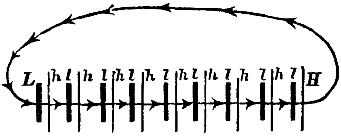


Fig. 303.

Extreme Cases.—(*a*) If the external resistance be very large compared with the internal, the latter may be neglected and $I = nE/R$; this is n times the current that one cell would give. (*b*) If the external resistance be very small compared with the internal, the former may be neglected and $I = E/r$; this is the current whichone cell alone would give. *Thus a series grouping lends itself to a large external resistance.*

(2) Parallel Grouping (Fig. 304).—In this grouping all the high potential plates are connected, forming, as it were, one large plate, and similarly all the low potential plates are connected. Since the E.M.F. does not depend on the size of the plates the combined E.M.F. is simply that of one cell, viz. E. The total internal resistance is, however, $1/n$ that of one cell, viz. r/n, and the external current I is given by

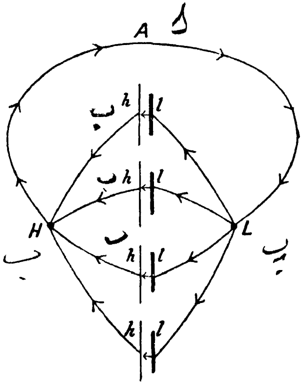


Fig. 304.

$$I = \frac{E}{\frac{r}{n} + R}.$$

Extreme Cases.—(*a*) If the external resistance be very large this becomes E/R, the current which one cell alone would give. (*b*) If the external resistance be very small I becomes nE/r; this is n times the current that one cell would give. *Thus a parallel grouping lends itself to a low external resistance.*

(3) Mixed Grouping (Fig. 305).—Using the facts of the preceding cases,

(*a*) E.M.F. of each row $= 4E$.
Resistance of each row $= 4r$.

(*b*) E.M.F. of 3 rows in parallel $= 4E$.

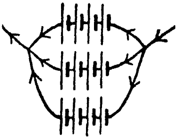


Fig. 305.

Resistance of 3 rows in parallel $= \frac{4r}{3}$.

Hence $$I = \frac{4E}{\frac{4r}{3} + R}.$$

If there are n cells in series per row and m rows in parallel, the external current is given by

$$I = \frac{nE}{\frac{nr}{m} + R}.$$

(4) GROUPING FOR MAXIMUM CURRENT.—Dividing numerator and denominator of (3) by n, we have

$$I = \frac{E}{\frac{r}{m} + \frac{R}{n}}.$$

The numerator is constant, hence I will be a maximum when the denominator is least. But the latter is the sum of two terms whose product is constant, and therefore will be least when the terms are equal; hence I will be a maximum if

$$\frac{R}{n} = \frac{r}{m}, \text{ i.e. if } R = \frac{nr}{m}.$$

Thus to secure the greatest current the resistance of the battery (nr/m) *must be made as near as possible equal to the external resistance* (R).

From the relation $\frac{nr}{m} = R$ we get

$$n^2r = nmR = pR, \text{ where } p = \text{total number of cells,}$$

i.e. $$n = \sqrt{\frac{pR}{r}}.$$

Hence the *maximum current* given by p cells (each of E.M.F. E and resistance r) to an external resistance R is

$$\text{Maximum current} = \frac{nE}{\frac{nr}{m} + R} = \frac{nE}{2R}$$

$$= \frac{E}{2}\sqrt{\frac{p}{Rr}}.$$

Examples. (1) *Find the arrangement for maximum current in the case of* 24 *cells each of resistance* 4 *ohms, the external resistance being* 6 *ohms.*

$$nm = 24, \quad \therefore m = 24/n.$$

Now $\frac{nr}{m} = R$ or $nr = mR$, i.e. $4n = 6m$,

$$\therefore n = \frac{6m}{4} = \frac{6 \times 24}{4n}, \quad \text{i.e. } n^2 = 36 \text{ or } n = 6.$$

The arrangement required is therefore 6 cells in series per row and 4 rows in parallel.

(2) *Find the minimum number of cells each of E.M.F.* $= E$ *and resistance* $= r$ *which will supply* w *watts to an external resistance* R.

The current must be the greatest which the required number (p) of cells can produce. If I be this maximum current,

$$I^2R = w, \quad \therefore I^2 = \frac{w}{R}.$$

But $I = \frac{E}{2}\sqrt{\frac{p}{Rr}}$, i.e. $I^2 = \frac{E^2}{4}\frac{p}{Rr}$.

Hence $\frac{E^2}{4}\frac{p}{Rr} = \frac{w}{R}$, $\quad \therefore p = \frac{4wr}{E^2}$.

161. Kirchhoff's Laws.—The joint resistance and the currents in the various branches of a divided circuit can readily be found by the methods of Art. 157, provided there are no cross connections such as are indicated in Fig. 307. In this and other more complex arrangements applications of **Kirchhoff's Laws** enable the solutions to be obtained.

Kirchhoff's two laws are as follows:—(1) *In any network of wires carrying currents the algebraic sum of the currents meeting at any point is zero.* (2) *In any closed path (or mesh) in a network the sum of the E.M.F.'s acting* *in that path is equal to the sum of the products of the resistances of, and currents in, the various parts of the path.*

The first law presents no difficulty; thus, to take a simple case (Fig. 306), it is clear that $I = I_1 + I_2 + I_3$, and therefore

$$I - I_1 - I_2 - I_3 = 0.$$

Current flowing *to* the point A is given the positive sign, and current flowing *from* A the negative sign in writing down the algebraic sum.

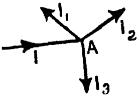


Fig. 306.

The second law will be understood from the details on the divided circuit shown in Fig. 307, where the letters P, Q, R, S, G, B denote resistances. Taking the mesh (a) the second law states that

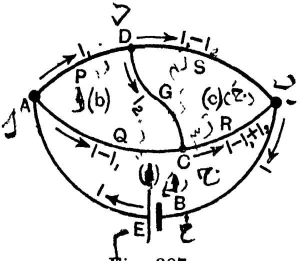


Fig. 307.

$$E = IB + (I - I_1)Q$$
$$+ (I - I_1 + I_2)R \dots (1)$$

for the mesh (b)—

$$0 = I_1P + I_2G - (I - I_1)Q \dots\dots\dots\dots\dots\dots (2)$$

and for the mesh (c)—

$$0 = (I_1 - I_2)S - (I - I_1 + I_2)R - I_2G \dots\dots (3)$$

In each mesh clockwise currents are taken as positive, and counter-clockwise ones as negative; thus in mesh (b) $I - I_1$ is counter-clockwise, and the same applies to I_2 and $I - I_1 + I_2$ for the mesh (c).

The truth of the above is obvious; thus, taking the mesh (b), P.D. between A and $D = I_1P$, P.D. between D and $C = I_2G$, so that the P.D. between A and C is $I_1P + I_2G$; but the P.D. between A and C is $(I - I_1)Q$, so that

$$I_1P + I_2G = (I - I_1)Q, \text{ i.e. } I_1P + I_2G - (I - I_1)Q = 0.$$

By solving equations (1), (2), and (3) the currents in the various branches are determined. The general application of the method to complicated networks is, however, troublesome, and Maxwell suggested the "cyclic current" device to simplify the solution. In each mesh a cyclic current of specified value is imagined to flow, all the cyclic currents being in the same direction; the current in any branch will thus be the difference between the cyclic currents of the meshes it separates. The method will be best understood from the following:—

Example. *Four points, A, B, C, D, are connected together as follows:—A to B, B to C, C to D, D to A, each by a wire of* 1 *ohm resistance; A to C, B to D, each by a cell of* 1 *volt E.M.F. and* 2 *ohms resistance. Determine the current flowing through each of the cells.*

The clockwise cyclic currents are x, y, and z (Fig. 308).

Applying Kirchhoff's Law 2 to the bottom compartment—

$$2x + (x - z) + (x - y) = 1$$

i.e. $$4x - y - z = 1 \quad \ldots\ldots (1)$$

Similarly for the compartment on the right—

$$y + (y - x) + 2(y - z) = 1$$

i.e. $$-x + 4y - 2z = 1 \quad \ldots\ldots\ldots\ldots (2)$$

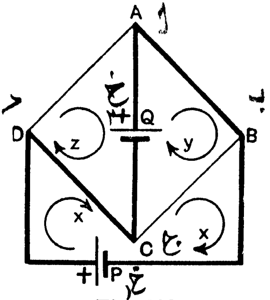


Fig. 308.

and for the remaining compartment—

$$z + 2(z - y) + (z - x) = -1$$

i.e. $$-x - 2y + 4z = -1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

In (3) the E.M.F. is given the negative sign, since its direction is *counter-clockwise* in that compartment.

Eliminating x from (2) and (3) $\quad y - z = \frac{1}{3}$ .... (4)

,, $\quad x$ from (2) and (1) $\quad 5y - 3z = \frac{5}{3}$ .... (5)

,, $\quad y$ from (4) and (5) $\quad z = 0$.

Hence from (4) $\quad y = \frac{1}{3}$, and from (1) $\quad x = \frac{1}{3}$.

$\therefore$ Current in cell $P = x = \frac{1}{3}$,

,, ,, $Q = y - z = \frac{1}{3}$.

Current in $AB = y = \frac{1}{3}$; current in $BC = y - x = 0$.

Current in $DC = x - z = \frac{1}{3}$; current in $DA = z = 0$.

The firm lines denote the path of the current.

162. Application to the Currents in the various Branches of a Wheatstone Net.—The Wheatstone bridge network consists of four resistances P, Q, R, S, a battery B (E.M.F. $= E$), and a galvanometer G arranged as indicated in Fig. 309; it is extensively employed in electrical measurements (Chapter XVI.). Using the letters to denote the resistances and cyclic currents as shown we get on writing down the equations as before and collecting—

$$\begin{aligned}
(P+Q+G)x \qquad & -Gy \qquad & -Qz=0\ldots(1)\\
-Gx+{} & (R+S+G)y \qquad & -Rz=0\ldots(2)\\
-Qx \qquad & -Ry+{} & (B+Q+R)z=E..(3)
\end{aligned}$$

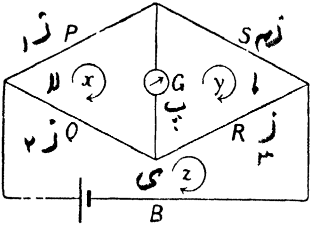


Fig. 309.

By solving these equations the values of x, y, and z can be determined, and the current in any branch deduced from these values; thus the current in the galvanometer is evidently $x-y$ or $y-x$, and the condition for no current in the galvanometer is that $x=y$. *This case is important in practice.* Multiplying (1) by R and (2) by Q we get—

$$\begin{aligned}
R(P+Q+G)x \qquad & -RGy-RQz=0\\
-GQx+{} & Q(R+S+G)y-RQz=0.
\end{aligned}$$

Subtracting—

$$(RP+RQ+RG+QG)x-(QS+RQ+RG+QG)y=0$$

i.e. $(RP+RQ+RG+QG)x=(QS+RQ+RG+QG)y.$

Hence if $RP=QS$, *i.e.* if $\dfrac{P}{S}=\dfrac{Q}{R}$, $x=y$ and the current in the galvanometer is zero. (See Chapter XVI.)

The ready solution of the general equations (1), (2), and (3) involves a knowledge of determinants, but any particular solution is simple. Thus in the case shown in Fig. 309, if $P=2$, $S=4$, $R=5$, $Q=6$, $G=8$, $B=10$, we get—

$$\begin{aligned}
8x-4y-3z&=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (4)\\
-8x+17y-5z&=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (5)\\
-6x-5y+21z&=E\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (6)
\end{aligned}$$

Eliminating x from (4) and (5)—

$$13y - 8z = 0 \qquad (7)$$

Multiplying (4) by 3 and (6) by 4—

$$24x - 12y - 9z = 0$$
$$-24x - 20y + 84z = 4E.$$

Adding $\qquad -32y + 75z = 4E \qquad (8)$

Eliminating y from (7) and (8) $\qquad z = \dfrac{52E}{719}.$

Substituting in (7) $\qquad y = \dfrac{32E}{719}.$

Hence from (4) $\qquad x = \dfrac{35{\cdot}5E}{719}.$

Thus the total current is $52E/719$, the current in the galvanometer is $x - y$, viz. $3{\cdot}5E/719$, and so on.

The *joint resistance* of the network formed by P, Q, R, S, and G is found thus: if R_1 = joint resistance, $B + R_1$ = total resistance, so that $E = (B + R_1)z$, i.e. $R_1 = \dfrac{E}{z} - B$; thus in the above example

$$R_1 = \frac{E}{\dfrac{52E}{719}} - 10 = \frac{719}{52} - 10 = 3{\cdot}82.$$

163. Application to Cells of Unequal E.M.F.'s in Parallel.—Let e_1, e_2, e_3 be the E.M.F.'s of three cells in parallel (Fig. 310), r_1, r_2, r_3 the three internal resistances, I_1, I_2, I_3 the three internal currents, r the external resistance, and I the external current. Applying Kirchhoff's Laws—

$$I_1 + I_2 + I_3 = I \qquad (1)$$
$$Ir + I_1r_1 = e_1 \qquad (2)$$
$$Ir + I_2r_2 = e_2 \qquad (3)$$
$$Ir + I_3r_3 = e_3 \qquad (4)$$

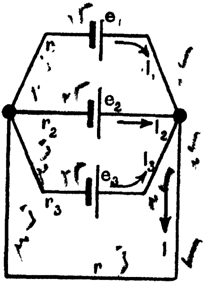


Fig. 310.

Dividing (2) by r_1, (3) by r_2, and (4) by r_3—

$$I\frac{r}{r_1}+I_1=\frac{e_1}{r_1}$$

$$I\frac{r}{r_2}+I_2=\frac{e_2}{r_2}$$

$$I\frac{r}{r_3}+I_3=\frac{e_3}{r_3}.$$

Adding, and substituting (1)—

$$I\left(\frac{r}{r_1}+\frac{r}{r_2}+\frac{r}{r_3}+1\right)=\frac{e_1}{r_1}+\frac{e_2}{r_2}+\frac{e_3}{r_3}$$

$$I\left(\frac{rr_2r_3+rr_1r_3+rr_1r_2+r_1r_2r_3}{r_1r_2r_3}\right)=\frac{e_1r_2r_3+e_2r_1r_3+e_3r_1r_2}{r_1r_2r_3}$$

$$\therefore I=\frac{e_1r_2r_3+e_2r_1r_3+e_3r_1r_2}{rr_2r_3+rr_1r_3+rr_1r_2+r_1r_2r_3}\ \ldots\ldots\ldots\ldots\ (5)$$

To obtain the current in each cell substitute (5) for I in equations (2), (3), and (4), and solve for I_1, I_2, and I_3.

If E be the total E.M.F.,

$$E=\text{total current}\times\text{total resistance}$$

$$=I\left(r+\frac{1}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}}\right)$$

$$=\frac{e_1r_2r_3+e_2r_1r_3+e_3r_1r_2}{r_2r_3+r_1r_3+r_1r_2}\ \ldots\ldots\ldots\ldots\ldots\ldots\ (6)$$

Note.—If the three E.M.F.'s be equal, say each e_1, and the three internal resistances be equal, say each r_1, (5) becomes

$$I=\frac{3e_1r_1^2}{r_1^3+3rr_1^2}=\frac{e_1}{\frac{r_1}{3}+r},$$

and (6) becomes

$$E=\frac{3e_1r_1^2}{3r_1^2}=e_1=\text{same as one cell,}$$

as was stated in Art. 160.

164. Current Sheets or Conduction in Two Dimensions.—Imagine a large flat sheet of copper and that electricity enters it at a certain point; clearly the current will flow away from this point by spreading out on all sides and we have evidently a case of conduction in two dimensions. The point at which the current enters the sheet is referred to as the *source*, and if the current be withdrawn from the sheet at another point, this latter point is referred to as the *sink*. Two cases only will be dealt with :—

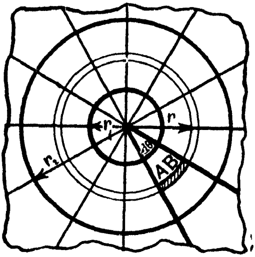


Fig. 311.

CASE 1. ONE SOURCE OR SINK.—Let the sheet be of unlimited extent; the lines of flow will be straight lines radiating from the point (Fig. 311), and the equipotential lines will be concentric circles with this point as centre. Consider two equipotentials of radii r and $r + dr$ and two flow lines enclosing an angle $d\theta$; let S be the specific resistance of the sheet and t its thickness. The resistance of the element bounded by these two flow lines and the two equipotentials is clearly $S\dfrac{dr}{t.r\,d\theta}$, since the breadth AB of the element is $rd\theta$. The whole circular strip consists of $\dfrac{2\pi}{d\theta}$ of these elements in parallel and the resistance of the circular strip is therefore

$$\frac{d\theta}{2\pi} \text{ of } S\frac{dr}{trd\theta} = S\frac{dr}{2\pi tr}.$$

Consider now a circular belt of radii r_2 and r_1; its resistance is clearly

$$\textit{Res.} = \frac{S}{2\pi t}\int_{r_1}^{r_2}\frac{1}{r}\,dr = \frac{S}{2\pi t}\log_e\frac{r_2}{r_1}.$$

Further, if V_2 be the potential at distance r_2, V_1 the potential at distance r_1, and I the total current, then

$$V_2 - V_1 = \text{Curr} \times \text{Res} = \mp\frac{IS}{2\pi t}\log_e\frac{r_2}{r_1},$$

the *minus* being used if the point is a *source* (in which case V_2 is less than V_1) and the *plus* if the point is a *sink*. If v be the potential at unit distance and V the potential at distance R,

$$V = v \mp \frac{IS}{2\pi t}\log_e R,$$

the *minus* being used for a *source* and the *plus* for a *sink*.

CASE 2. ONE SOURCE AND ONE SINK.—A little consideration will convince the student that the lines of flow and equipotential lines are similar to the lines depicted in Fig. 312.

Let R = distance of a point X from the source
R' = distance of the point X from the sink
V = potential at X due to the source alone
V' = potential at X due to the sink alone
P = the resultant potential at X
v = the potential at unit distance.

Now—

$$V = v - \frac{IS}{2\pi t}\log R, \quad V' = -v + \frac{IS}{2\pi t}\log R',$$

$$\therefore P = V + V' = \frac{IS}{2\pi t}\log_e \frac{R'}{R}$$

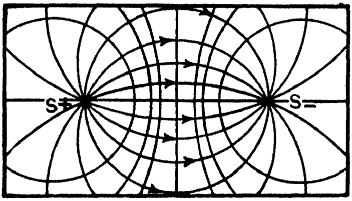


Fig. 312.

and is constant as long as the ratio of R' to R is constant.

Now consider a portion of the sheet bounded by two lines of flow which meet at an angle α and two equipotential lines of values P_1 and P_2; let R_1 and R'_1 denote the distances from the source and sink respectively of any point on P_1 and let R_2 and R'_2 have the same meaning for P_2, then

$$P_1 = \frac{IS}{2\pi t}\log_e \frac{R'_1}{R_1}, \quad P_2 = \frac{IS}{2\pi t}\log_e \frac{R'_2}{R_2},$$

$$\therefore P_1 - P_2 = \frac{IS}{2\pi t}\log_e \frac{R'_1 R_2}{R'_2 R_1}.$$

If I_1 be the current through the sheet bounded by the two flow lines in question, $I_1 : I = \alpha : 2\pi$, i.e. $I_1 = \frac{\alpha}{2\pi} I$; hence, if r denote the resistance of this portion of the sheet bounded by P_1 and P_2 and these two flow lines,

$$r = \frac{\text{Pot. Diff.}}{\text{Current}} = \frac{S}{\alpha t}\log_e \frac{R'_1 R_2}{R'_2 R_1},$$

and for the resistance of the whole space bounded by these two equipotentials

$$\text{Res.} = \frac{S}{2\pi t}\log_e \frac{R'_1 R_2}{R'_2 R_1}.$$

Examples.

(1) *Twelve cells are arranged in series, the total internal resistance being* 27 *ohms. The outside resistance is* 40 *ohms and the terminal* P.D. 6 *volts. Find* (i) *the current,* (ii) *the fall of potential in each cell,* (iii) *the E.M.F. of the whole battery,* (iv) *the E.M.F. of each cell,* (v) *the terminal P.D. for each cell.*

$$\text{(i) Current} = \frac{\text{Terminal P.D.}}{\text{External resistance}} = \frac{6}{40} = \textbf{·15 ampere.}$$

$$\begin{aligned}\text{(ii) Fall of potential in battery} &= \text{Current in battery} \times \text{Res. of battery}\\ &= {\cdot}15 \times 27\\ &= 4{\cdot}05 \text{ volts}\end{aligned}$$

$$\therefore \text{ Fall of potential per cell} = \frac{4{\cdot}05}{12} = \textbf{·3375 volt.}$$

$$\begin{aligned}\text{(iii) E.M.F. of battery} &= \text{Terminal P.D.} + \text{Fall of potential in battery}\\ &= 6 + 4{\cdot}05 = \textbf{10·05 volts.}\end{aligned}$$

$$\text{(iv) E.M.F. of each cell} = \frac{10{\cdot}05}{12} = \textbf{·8375 volt.}$$

$$\begin{aligned}\text{(v) Terminal P.D. for each cell} &= \text{E.M.F.} - \text{Fall of potential in cell}\\ &= {\cdot}8375 - {\cdot}3375 = \textbf{·5 volt.}\end{aligned}$$

Note also.—Fall of potential per cell $= \text{Current in cell} \times \text{Res. of cell}$

$$= {\cdot}15 \times \tfrac{27}{12} = {\cdot}3375 \text{ volt.}$$

$$\begin{aligned}\text{E.M.F. of battery} &= \text{Total current} \times \text{Total resistance}\\ &= {\cdot}15 \times (27 + 40) = 10{\cdot}05 \text{ volts.}\end{aligned}$$

$$\text{Terminal P.D. for each cell} = 6/12 = {\cdot}5 \text{ volt.}$$

(2) *A dynamo produces a fixed P.D. at its terminals of* 120 *volts and is* 300 *yards away from a house where there are two hundred* 100 *volt* 35 *watt glow lamps in parallel. What size leads should be employed between the dynamo and the house if the resistance of an inch cube of copper is* ·66 *microhm? Find also* (i) *the electrical horse-power supplied at the house,* (ii) *the watt-hours and Board of Trade units supplied in* 10 *hours,* (iii) *the cost of the energy at* 4*d. per unit.*

$$\text{Watts} = EI, \quad \therefore I = \frac{\text{Watts}}{E}.$$

Hence—

$$\begin{aligned}\text{Current in each lamp} &= \frac{\text{Watts taken by each lamp}}{\text{P.D. for each lamp}}\\ &= \tfrac{35}{100} = {\cdot}35 \text{ ampere}\end{aligned}$$

$\therefore$ Current for 200 lamps = ·35 × 200 = 70 amperes = current in the leads. Volts used in leads = 120 − 100 = 20

$\therefore$ Resistance of leads = $\frac{20}{70} = \frac{2}{7}$ ohm.

Again—

$$R = S\frac{l}{a}, \quad \therefore a = \frac{Sl}{R}.$$

S = ·66 microhm = $\cdot 66/10^6$ ohm per inch cube

l = 600 yards (go and return) = (600 × 36) inches

$R = \frac{2}{7}$ ohm

$$\therefore a = \frac{\cdot 66 \times 600 \times 36 \times 7}{10^6 \times 2} = \mathbf{\cdot 0499\ square\ inch}$$

Further—

Total watts supplied = 200 × 35 = 7000,

$\therefore$ **H.P. supplied** = 7000/746 = **9·4** approx.

Watt-hours supplied in 10 hours = Watts × hours,

$\therefore$ **Watt-hours** = 7000 × 10 = **70000**;

$$\text{and Board of Trade units} = \frac{\text{Watt-hours}}{1000},$$

$$\therefore \textbf{B.O.T. units} = \frac{70000}{1000} = \mathbf{70}.$$

Finally—

Cost of 70 units at 4d. per unit = £1 3s. 4d.

(3) *A battery of* 10 *volts and internal resistance* ·5 *ohm is connected in parallel with one of* 12 *volts and internal resistance* ·8 *ohm. The poles are connected by an external resistance of* 20 *ohms. Find the current in each branch.* (B.Sc.)

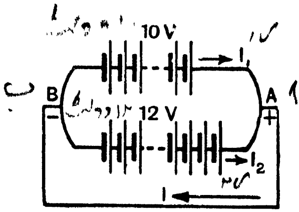


Fig. 313.

Let Fig. 313 represent the details. Applying Kirchhoff's Laws as in Art. 163,

$$20I + \cdot 5I_1 = 10 \quad \ldots\ldots\ldots\ldots (1)$$

$$20I + \cdot 8I_2 = 12 \quad \ldots\ldots\ldots\ldots (2)$$

Dividing (1) by ·5, (2) by ·8, adding, and remembering that $I_1 + I_2 = I$, we get

$$66I = 35, \quad \therefore \boldsymbol{I} = \frac{\mathbf{35}}{\mathbf{66}}\ \textbf{ampere}.$$

Substituting this in (1)—

$$\cdot 5I_1 = 10 - \frac{700}{66} = -\frac{40}{66}, \quad \therefore \boldsymbol{I_1} = -\frac{\mathbf{80}}{\mathbf{66}}\ \textbf{ampere},$$

and substituting in (2)—

$$\cdot 8I_2 = 12 - \frac{700}{66} = \frac{92}{66}, \quad \therefore \boldsymbol{I_2 = \frac{115}{66}} \textbf{ amperes.}$$

Thus the current through the stronger battery is 115/66 amperes, of which 35/66 ampere flows through the external resistance and 80/66 amperes flow *back through the weaker battery.*

Note that going along the wire from B to A the rise in potential is ($e = IR$) $35/66 \times 20 = 700/66$ volts; going through the stronger battery from B to A the rise is $E - I_2r = 12 - \frac{115}{66} \times \cdot 8 = 700/66$ volts; going through the weaker battery from B to A the rise is $E + I_1r_1 = 10 + \frac{80}{66} \times \cdot 5 = 700/66$ volts.

(4) *Twelve equal wires each of resistance r are joined up to form a skeleton cube and a current enters at one corner and leaves at the diagonally opposite corner. Find the joint resistance between these corners.* (Inter. B.Sc.)

An examination of Fig. 314 will show that the total current (say $6x$ for convenience) divides at A into three equal parts each equal to $2x$. At B, E, and D each of these again divides into two equal parts each equal to x. At H, C, and F these latter unite in pairs, giving three currents each equal to $2x$ uniting at G.

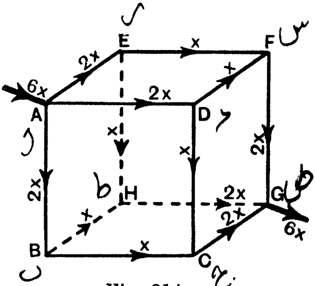


Fig. 314.

If E denote the P.D. between A and G, then, taking any one path, say $AEFG$,

$$E = 2xr + xr + 2xr = 5xr.$$

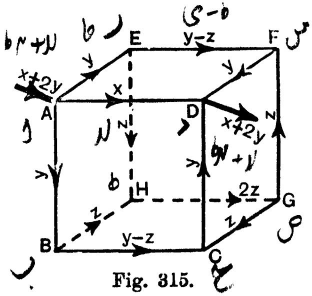


Fig. 315.

But if R denote the joint resistance,

$$E = 6xR, \quad \text{i.e. } 6xR = 5xr,$$

$$\therefore \boldsymbol{R = \frac{5}{6}r.}$$

(5) *Twelve wires each of* 1 *ohm resistance are joined up to form a cube. Show that the total resistance taken between two corners on the same edge is* $\frac{7}{12}$ *of an ohm.* (City and Guilds of London.)

An examination of Fig. 315 will show that the total current I ($= x + 2y$) divides at A into three parts, viz. x along AD and two equal parts y along AE and AB. At E and B these latter divide into z along EH and BH, and $y - z$

along EF and BC. A current $2z$ flows along HG, breaking up into z along GF and z along GC. The currents in FD and CD are therefore y.

If E = P.D. between A and D and r = resistance of each wire we have

$$E = xr \quad\ldots\ldots\ldots\ldots(1)$$

$$E = yr + (y - z)\,r + yr = 3yr - zr \quad\ldots\ldots\ldots\ldots(2)$$

$$E = yr + zr + 2zr + zr + yr = 2yr + 4zr \quad\ldots\ldots\ldots\ldots(3)$$

Eliminating z from (2) and (3),

$$y = \frac{5E}{14r} = \frac{5xr}{14r} = \frac{5}{14}x,$$

$$\therefore\ I = x + 2y = \frac{12}{7}x.$$

If R = joint resistance, $E = IR = \frac{12}{7}xR$; but $E = xr$,

$$\therefore\ \frac{12}{7}xR = xr,\ \textit{i.e.}\ R = \frac{7}{12}r = \frac{7}{12}\ \text{if}\ r = 1.$$

Exercises XI.

Section A.

(1) Explain "current strength," "resistance," and "electromotive force." Is there any real distinction between "potential difference" and "electromotive force"? If so, explain fully.

(2) Define the absolute electromagnetic and the practical units of current strength, quantity, potential difference, and resistance. Define also erg, joule, watt, kilowatt, Board of Trade unit of electrical energy, temperature coefficient of resistance, and electrochemical equivalent.

(3) State and explain Ohm's Law, and develop the formula for the joint resistance of conductors in parallel.

(4) State Kirchhoff's Laws, and show their application (i) to the determination of the currents in the various branches of the Wheatstone net, (ii) to the case of cells of unequal E.M.F.'s in parallel.

Section B.

(1) A circuit is formed of six similar cells in series and a wire of 10 ohms resistance. The E.M.F. of each cell is one volt, and its internal resistance 5 ohms. Determine the difference of potential between the positive and negative poles of any one of the cells.

(B.E.

(2) The terminals of a battery formed of seven Daniell's cells in series are joined by a wire 35 feet long. One binding screw of a galvanometer is joined by a wire to the copper of the third cell (reckoned from the copper end). With what point of the 35 ft. wire must the other screw of the galvanometer be connected so that the needle shall not be deflected? (B.E.)

(3) Two cells, *A* and *B* (E.M.F. and internal resistance of each are 1 volt and 1 ohm respectively), are arranged in series. The positive and negative poles of this battery are connected with the positive and negative poles respectively of a third cell, *C*, exactly like *A* and *B*, the connecting wires having negligible resistance. What is the current in the circuit, and what is the potential difference between the positive and negative poles of the cell *C*? (B.E.)

(4) The electrodes of a quadrant electrometer are joined to the terminals of a battery of five cells in series. In what ratio will the deflection of the needle be altered if the electrodes are also joined to the terminals of a battery of three cells in series similarly arranged, all the cells being alike and the connecting wires thick? (B.E.)

(5) A battery of twelve equal cells in series, screwed up in a box, being suspected of having some of the cells wrongly connected, is put into circuit with a galvanometer and two cells similar to the others. Currents in the ratio of 3 to 2 are obtained according as the introduced cells are arranged so as to work with or against the battery. What is the state of the battery? Give reasons for your answer. (B.E.)

(6) A circuit is made up of (i) a battery with terminals *A*, *B*, its resistance being 3 ohms and its E.M.F. 2·7 volts; (ii) a wire *BC*, of resistance 1·5 ohms; (iii) two wires in parallel circuit, *CDF*, *CEF*, with respective resistances 3 and 7 ohms; (iv) a wire *FA*, of resistance 1·5 ohms. The middle point of the last wire is put to earth. Find the potential at the points *A*, *B*, *C*, *F*. (B.E.)

Section C.

(1) State the Law of Ohm, and apply it to calculate how many Grove cells, each having an electromotive force of 1·8 volts and an internal resistance of 0·07 ohm, will be required to send a current of 10 amperes through a resistance of 2·2 ohms. (Inter. B.Sc.)

(2) State the law of subdivision of a current in a divided circuit. Explain how you would arrange 36 cells of a battery, each having an internal resistance of 1·6 ohms, so as to send the strongest possible current through an external resistance of 5·6 ohms. (Inter. B.Sc.)

(3) If a cell has an E.M.F. of 1·08 volts and ·5 ohm internal resistance, and if the terminals are connected by two wires in parallel of 1 ohm and 2 ohms resistance respectively, what is the current in each, and what is the ratio of the heats developed in each? (Inter. B.Sc.)

(4) An electric light installation consists of a group of lamps in parallel arc between the ends of leads. The leads have total resistance ·4 ohm, and bring current from sixty accumulators each with E.M.F. 2 volts and resistance ·01 ohm. When twenty-five lamps are switched on each takes ·4 ampere. Find the resistance of a lamp, and the watts used in each part of the circuit.
(Inter. B.Sc. Hons.)

(5) State the laws governing the distribution of current in a network of wires. A battery of 6 volts E.M.F. and 0·5 ohm internal resistance is joined in parallel with another of 10 volts E.M.F. and 1 ohm internal resistance, and the combination is used to send current through an external resistance of 12 ohms. Calculate the current through each battery. (B.Sc.)

(6) A framework is made out of six pieces of the same wire forming a square and its two diagonals. It is fixed together at the angles of the square and at the crossing point of the diagonals. What is the equivalent resistance of the frame between the two ends of the same diagonal? (B.Sc.)

(7) Show how to calculate the current through the galvanometer in the Wheatstone bridge arrangement of conductors. (B.Sc. Hons.)

(8) Let AB, BC, CD, DA be four uniform wires, each of unit resistance, joined in the form of a square $ABCD$; let E be a point in the side CD such that CE is to ED in the ratio of $\sqrt{2}$ to 1; and let A be joined to E by a wire AE of unit resistance. Show that, if the points A and C be maintained at different potentials, then the potential of B is equal to that of E, so that no current will flow along a wire joining BE. (Tripos.)

(9) A tetrahedral framework is made of wires cut from the same coil: if pairs of opposite edges be equal and of lengths a, b, c respectively, and a current enters and leaves the framework at the ends of an edge of length a, then the strengths of the currents in the pair of edges of length a are in the ratio

$$b(a+c) + c(a+b) : b(a+c) - c(a+b).$$

(Jesus College.)

(10) In the previous question show that the resistance of the whole framework is that of a length of wire equal to

$$\frac{1}{2}\left(\frac{ab}{a+c} + \frac{ac}{a+b}\right).$$

(St. John's College.)

CHAPTER XII.

MAGNETIC EFFECTS OF CURRENTS.

165. Field due to a Linear Current.—We must now consider what determines the *intensity* of the field at any point in the neighbourhood of a conductor carrying a current.

Exp. Fix a *long* wire vertically and pass a current *up* the wire. Using the oscillation magnetometer of Art. 40, find the number of oscillations (n_1) per minute at distance d_1 due magnetic east of the wire. Repeat at distance d_2 and let n_2 be the number per minute. Switch off the current and let n_0 be the number per minute under the influence of the earth alone. On the east of the wire the earth's field and the field due to the current are in the same direction; hence (Art. 40)—

$$n_1^2 - n_0^2 \propto \text{field due to the current at distance } d_1,$$
$$n_2^2 - n_0^2 \propto \text{field due to the current at distance } d_2,$$

and it will be found that $(n_1^2 - n_0^2) : (n_2^2 - n_0^2) = d_2 : d_1$, i.e. *the intensity of the field varies inversely as the distance.* This is known as Biot and Savart's Experiment.

Include an ammeter (Art. 187) in the circuit, and also an adjustable resistance, so that the current may be indicated and its strength varied. With current I_1 passing, let n_1 be the number of oscillations per minute at a certain point, and with current I_2 let n_2 be the number per minute *at the same point*; then

$$n_1^2 - n_0^2 \propto \text{field at a certain distance when the current is } I_1,$$
$$n_2^2 - n_0^2 \propto \text{field at the same distance when the current is } I_2,$$

and it will be found that $(n_1^2 - n_0^2) : (n_2^2 - n_0^2) = I_1 : I_2$ (approx.), i.e. *the intensity of the field varies directly as the current strength.*

In discussing the law established by Biot and Savart, Laplace showed that the result may be deduced mathematically from the following assumptions:—

Let AB (Fig. 316) represent a wire carrying a current; then, considering the field due to a very small element, ab, it is assumed that the intensity of the field at c due to the current in that element is—

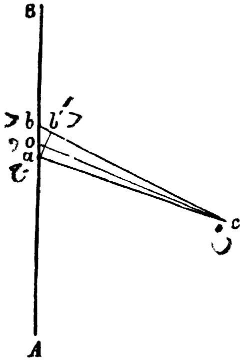


Fig. 316.

1. *Directly* proportional to the strength of the current in AB.

2. *Directly* proportional to the distance ab', that is to the apparent length of the element ab as seen from c. (The line ab' is drawn at right angles to the line co, which joins c to o, the middle point of ab.)

3. *Inversely* proportional *to the square of the distance* of the element ab from c, that is to co^2.

Hence, if H denote the intensity of the field at c due to the element ab, and I the strength of the current in AB, we have

$$H \propto \frac{I \,.\, ab'}{(oc)^2} \text{ or } H \propto \frac{I \,.\, ab \sin a}{(oc)^2} \text{ or } H \propto \frac{I \,.\, ab \cos \theta}{(oc)^2},$$

where a denotes the angle aoc, and θ the angle $\underline{Pco}$. ?

The direction of H is perpendicular to the plane through c and AB. To determine the intensity, at any point, due to a current in a conductor of given form and position, it is necessary to sum up the effects due to each element: the result of the summation gives the total intensity due to the current in the conductor taken as a whole.

In the case of an infinitely long straight conductor, by applying Laplace's assumptions to the two cases shown in Fig. 317, where the distances of the point c from the wire are d and d', we get

$$H \propto \frac{I \,.\, ab \sin a}{oc^2} \quad \text{and} \quad H_1 \propto \frac{I \,.\, a'b' \sin a}{o'c^2},$$

$$\therefore \frac{H}{H_1} = \frac{ab}{a'b'} \cdot \left(\frac{o'c}{oc}\right)^2.$$

But from the figure

$$\frac{o'c}{oc} = \frac{a'b'}{ab}, \qquad \therefore \frac{H}{H_1} = \frac{ab}{a'b'} \cdot \left(\frac{a'b'}{ab}\right)^2 = \frac{a'b'}{ab}.$$

And geometrically

$$\frac{ab}{a'b'} = \frac{d}{d'},$$

$$\therefore \frac{H}{H_1} = \frac{d'}{d}.$$

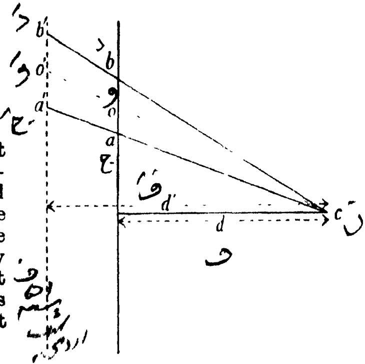


Fig. 317.

This indicates that the effect of each element varies inversely as the distance, and therefore by summation the total effect of the whole wire at any point varies inversely as the distance of the point from the wire. Thus Laplace's mathematics agrees with Biot and Savart's Experiment. ✓

As indicated in Art. 151, the units are so selected that the expressions H ∞ $Iab \sin a/(oc)^2$, etc., above *become equalities*; hence **the intensity of the field at any point due to a current in a straight wire of finite length** is calculated as follows:—

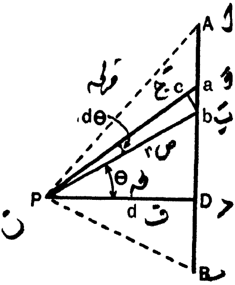


Fig. 318.

In Fig. 318 let *BA* be the conductor carrying a current *I* e.m. units and *P* the point, *d* cm. from the conductor, at which the intensity is required; let $Pb = r$, the angle $DPb = \theta$, and the small increment $bPa = d\theta$; let the angle $DPA = \theta_2$ and the angle $DPB = \theta_1$.

$$\text{Field at } P \text{ due to } ba = \frac{I\,.\,bc}{r^2} = \frac{Ird\theta}{r^2} = \frac{I\,.\,d\theta}{r}.$$

But $$\frac{d}{r} = \cos\theta, \textit{ i.e. } \frac{1}{r} = \frac{\cos\theta}{d},$$

$$\therefore \text{Field at } P \text{ due to } ba = \frac{I}{d}\cos\theta\,.\,d\theta,$$

and the total field (H) at P due to the whole conductor BA is the integral of the above between the limits $\theta = -\theta_1$ and $\theta = \theta_2$; that is—

$$H = \frac{I}{d}\int_{-\theta_1}^{\theta_2} \cos\theta \, . \, d\theta = \frac{I}{d}\Big[\sin\theta\Big]_{-\theta_1}^{\theta_2}$$

$$= \frac{I}{d}\Big[\sin\theta_2 - \sin(-\theta_1)\Big],$$

i.e. $$H = \frac{I}{d}\left(\sin\theta_2 + \sin\theta_1\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

In the case of an *infinitely long conductor* $\theta_1 = \theta_2 = \frac{\pi}{2}$, and therefore $\sin\theta_1 = \sin\theta_2 = 1$; hence

$$H = \frac{2I}{d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

In Art. 173 it is shown that the work done in moving unit pole once round an infinitely long conductor carrying a current I e.m. units is $4\pi I$. If H be the intensity of the field at distance d, H measures the force on unit pole at this distance, and the work done in moving the unit pole once round the conductor in a circle of radius d is given by the product of the force and the distance, *i.e.* by $2\pi dH$; hence

$$2\pi dH = 4\pi I, \text{ i.e. } H = 2I/d.$$

If I be in *amperes* the right-hand expressions of (1) and (2) must, of course, be divided by 10.

166. Field at the Centre of a Circular Current.—Consider the conductor AB to be looped into a circle as shown in Fig. 319, and let us determine the intensity of the magnetic field at c, the centre of the circle.

Considering the action of the element ab, the intensity of the field is $\frac{I \times ab}{r^2}$, where I denotes the current in AB and r the radius of the circle. Now, since each element of the conductor is similarly placed relatively to c, the intensity of the field at c due to the conductor AB taken as a whole is $\frac{I \times AB}{r^2}$, that is $\frac{I \times 2\pi r}{r^2}$ or $\frac{2\pi I}{r}$. Hence, if H denote the total intensity of the field at c, then

$$H = \frac{2\pi I}{r},$$

I being in e.m. units and r in centimetres.

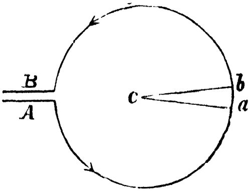


Fig. 319.

If the coil consists of n turns sufficiently thin to be regarded as coincident,

$$H = \frac{2\pi n I}{r} \quad \text{.........}(3)$$

The direction of the field at c is at right angles to the plane of the coil, and may be determined by Ampère's Rule or the Right Hand Rule (Art. 146); in Fig. 319 it is upwards towards the reader.

Fig. 320 shows roughly the lines of force; within a small space *abcd* at the centre the field is practically uniform, *i.e.* the lines of force are approximately parallel equidistant straight lines; this is utilised in the *tangent galvanometer* (Art. 177).

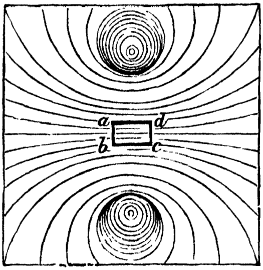


Fig. 320.

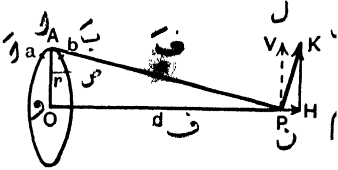


Fig. 321.

167. Field at any Point on the Axis of a Circular Current.—Consider first the field at P (Fig. 321) due to the small element ab of the coil. The intensity is clearly $\frac{I \cdot \overline{ab}}{a^2}$ in a direction at right angles to AP; let PK represent this, and resolve it into two components, viz. PH

along the axis, and PV perpendicular thereto. Only the component along the axis need be considered, for when the whole ring is taken into account the vertical components cancel each other. From the similar triangles HPK and OAP—

$$\frac{PH}{PK} = \frac{r}{a}, \quad \therefore \; PH = \frac{r}{a} PK,$$

or, denoting the horizontal component PH by h—

$$h = \frac{r}{a} \times \frac{I \, . \, \overline{ab}}{a^2} = \frac{rI\overline{ab}}{a^3}.$$

Clearly, for the whole ring, the intensity H at P will be obtained by summing the above for all the elements into which the ring is divided, *i.e.*

$$H = \frac{rI}{a^3} \Sigma \, \overline{ab} = \frac{r \, . \, I \, . \, 2\pi r}{a^3} = \frac{2\pi r^2 I}{a^3},$$

and if there are n turns (as in (3) above)—

$$H = \frac{2\pi n r^2 I}{a^3} = \frac{2\pi n r^2 I}{(d^2+r^2)^{\frac{3}{2}}} \quad \ldots\ldots\ldots\ldots\ldots \; (4)$$

When $d = 0$ this reduces to $H = 2\pi n I/r$ the result obtained above for the field at the centre of the coil.

168. Field midway between Two Similar Coaxial Circular Coils, the Distance apart being equal to the Radius.—From (4) of the preceding article it follows that the field due to unit current in the coil is $2\pi n r^2/(d^2 + r^2)^{\frac{3}{2}}$, and from this the variation of the field along the axis may be graphically represented by working out the value of the field for different values of d. The curve in Fig. 322 depicts this variation, the abscissae denoting distances along the axis and the ordinates values of the field. The curve is at first concave towards O, but the curvature becomes less and less and quickly changes sign, the curve becoming convex towards O.

The point of inflection or change of curvature is at the point where $d = \frac{1}{2}r$, and at this point the curve is, for a short length, practically a straight line. If, therefore, we have two equal circular coils placed with their axes coincident and *at a distance apart equal to the radius of either*, then, for the same direction along the common axis, the rate of increase of the field due to one coil at a point midway between the two coils is equal to the rate of decrease of the field due to the other coil at the same point, and the field for a fair distance on each side of this point will be practically uniform. This is shown graphically in Fig. 323. The dotted curves show the fields for the separate coils, and the full line curve, the resultant of the two dotted curves, represents the resultant field due to the two coils; the horizontal part indicates uniformity of field. Fig. 324 shows the field of force between the coils and clearly indicates that, in the middle of the field, there is a region

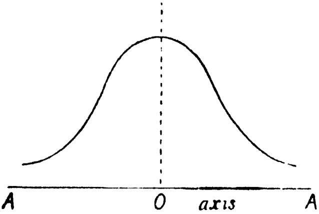


Fig. 322.

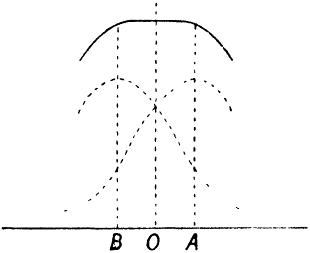


Fig. 323.

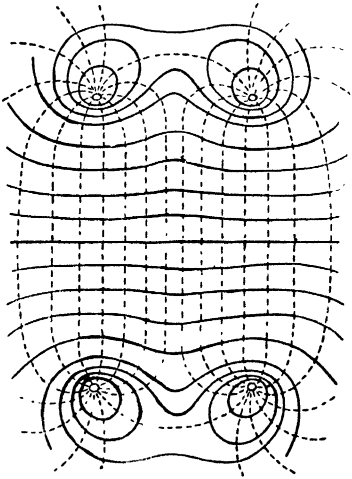


Fig. 324.

of comparatively fair extent where the field is *practically* uniform; this is utilised in the *Helmholtz tangent galvanometer* (Art. 178).

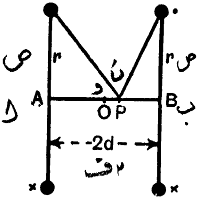


Fig. 325.

The truth of the above may be shown mathematically as follows:—Let $2d$ (Fig. 325) be the distance apart of the coils and O the mid point. Take a point P near to O and let $OP = x$; $AP = d + x$ and $BP = (d - x)$. Assume unit current to flow in the same direction in each coil, and let the radius of each coil be r; then

$$\text{Field at } P = H = 2\pi n r^2 \left[\frac{1}{\{(d+x)^2 + r^2\}^{\frac{3}{2}}} + \frac{1}{\{(d-x)^2 + r^2\}^{\frac{3}{2}}}\right].$$

Differentiating with respect to x—

$$\frac{dH}{dx} = -2\pi n r^2 \left[\frac{3(d+x)}{\{(d+x)^2 + r^2\}^{\frac{5}{2}}} - \frac{3(d-x)}{\{(d-x)^2 + r^2\}^{\frac{5}{2}}}\right].$$

To find the condition for uniform field $\frac{dH}{dx}$ must be equated to zero; hence $$\frac{d+x}{\{(d+x)^2 + r^2\}^{\frac{5}{2}}} = \frac{d-x}{\{(d-x)^2 + r^2\}^{\frac{5}{2}}},$$

i.e. $$(d+x)\{(d-x)^2 + r^2\}^{\frac{5}{2}} = (d-x)\{(d+x)^2 + r^2\}^{\frac{5}{2}} \dots\dots\dots(a)$$

Now $\{(d+x)^2 + r^2\}^{\frac{5}{2}} = (d^2 + r^2 + 2xd)^{\frac{5}{2}}$, neglecting x^2

$$= (d^2 + r^2)^{\frac{5}{2}}\left\{1 + \frac{2xd}{d^2 + r^2}\right\}^{\frac{5}{2}}$$

$$= (d^2 + r^2)^{\frac{5}{2}}\left(1 + \frac{5xd}{d^2 + r^2} + \quad\right).$$

Similarly

$$\{(d-x)^2 + r^2\}^{\frac{5}{2}} = (d^2 + r^2)^{\frac{5}{2}}\left(1 - \frac{5xd}{d^2 + r^2} + \quad\right)$$

and substituting in (a)

$$(d+x)\,(d^2 + r^2)^{\frac{5}{2}}\left(1 - \frac{5xd}{d^2 + r^2}\right)$$
$$= (d-x)\,(d^2 + r^2)^{\frac{5}{2}}\left(1 + \frac{5xd}{d^2 + r^2}\right),$$

$$\therefore\ d + x - \frac{5xd^2}{d^2 + r^2} - \frac{5x^2d}{d^2 + r^2} = d - x + \frac{5xd^2}{d^2 + r^2} - \frac{5x^2d}{d^2 + r^2}$$

i.e. $$\frac{5d^2}{d^2 + r^2} = 1, \quad \text{hence} \quad d = \frac{r}{2} \text{ or } 2d = r.$$

Thus, to secure a very uniform field in the central region between the coils, the distance apart must be equal to the radius of either.

The field H at the centre due to current I in the coils is clearly

$$H = 2\left\{\frac{2\pi n r^2 I}{\left(\frac{r^2}{4} + r^2\right)^{\frac{3}{2}}}\right\} = \frac{32\pi n I}{5\sqrt{5}\,.\,r} \quad\ldots\ldots\ldots\ldots (5)$$

169. Field due to a Solenoidal Current.—A further extension of the result of Art. 167 shows that the intensity of the field in the interior of a long closely wound solenoid is given by

$$H = \frac{4\pi S I}{l} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

where I is the current in e.m. units, S the total number of turns of the solenoid, and l its length. The field is practically uniform except near the ends, and is *half the above value at the ends.* If n be the number of turns *per unit length*, $n = S/l$ and

$$H = 4\pi n I \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

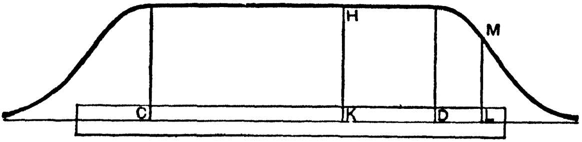


Fig. 326.

Fig. 326 shows approximately the variation of the field in the case of a solenoid.

The above relations may be established as follows:—Let i be the current per unit length of the solenoid so that $i\,.\,dx$ is the current in the thin slice dx (Fig. 327). The field at P due to this is $\frac{2\pi r^2 i dx}{(x^2 + r^2)^{\frac{3}{2}}}$. Now $\frac{a\,.\,d\theta}{dx} = \sin\theta$, i.e. $dx = \frac{a\,.\,d\theta}{\sin\theta}$; hence the field at P due to the slice dx is $\frac{2\pi r^2 i a d\theta}{(x^2 + r^2)^{3} \sin\theta}$.

Again, $\sin\theta = \dfrac{r}{a}$, $r^2 + x^2 = a^2$; hence

$$\text{Field at } P \text{ due to slice } dx = \frac{2\pi i r^2 d\theta}{a^2 \sin\theta}$$
$$= 2\pi i \sin\theta \,.\, d\theta.$$

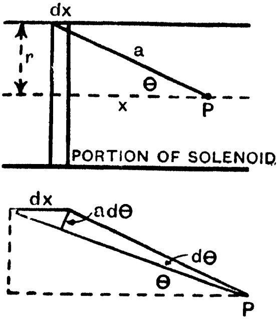


Fig. 327.

Thus if θ_1 and θ_2 be the values of θ for the ends of the solenoid, the total field H at P is clearly

$$H = 2\pi i \int_{\theta_1}^{\theta_2} \sin\theta \,.\, d\theta$$
$$= 2\pi i \Big[\cos\theta\Big]_{\theta_2}^{\theta_1}$$
$$= 2\pi i\,(\cos\theta_1 - \cos\theta_2).$$

If I be the current in the solenoid, S the total turns, and l the length, $i = SI/l$; hence

$$\boldsymbol{H} = \frac{\mathbf{2\pi SI}}{\boldsymbol{l}}\,(\mathbf{\cos\theta_1 - \cos\theta_2}) \ldots (8)$$

If the solenoid be *very* long and P be well removed from either end, practically $\theta_1 = 0$ and $\theta_2 = \pi$, so that $\cos\theta_1 - \cos\theta_2 = 2$ and

$$H = \frac{4\pi SI}{l}.$$

At the extreme end of a very long solenoid, practically $\cos\theta_2 = 0$ and $\cos\theta_1 = 1$, so that $H = \dfrac{2\pi SI}{l}$, i.e. *half the intensity at the central parts* of the solenoid.

A simple proof is based on the results of Art. 173, viz. that the work done in moving unit pole round a current-carrying wire is $4\pi I$. Imagine unit pole moved along the path $ABCDA$ (Fig. 328): the work done is $4\pi I$ for *each turn* of wire. If there are n turns per unit length, the total turns in the part considered are $n\overline{AB}$ and the work done is $4\pi I \times n\overline{AB}$.

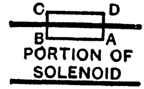


Fig. 328.

The work in going along BC and DA is *nil*, since these paths are at right angles to the lines of force. The work in going along CD is negligibly small compared with that in going along AB, for the lines of force which are crowded into a small space inside are spread out throughout the whole field outside, so that the force along CD, and therefore the work, is negligible. The work in going along AB is $H \times \overline{AB}$, where H is the intensity inside; hence

$$H \times \overline{AB} = 4\pi I \times n\overline{AB}, \quad \therefore H = 4\pi n I.$$

170. Force exerted on a Conductor carrying a Current in a Magnetic Field.—In Fig. 329*a* A is the cross-section of a straight wire placed at right angles to the magnetic field due to the poles N and S, *i.e.* at right angles to the plane of the paper; the lines at B and C indicate the general direction of the lines of force of the field. Imagine now that a current passes *upwards* in the wire; the circle about the wire indicates the general direction of the lines due to the current. On the side C the two sets of lines are in the same direction and repel each other; on the side B the two sets are in opposite directions and cancel each other. The result is that the wire is acted on by a force towards the right; the tendency is, therefore, for the conductor to move towards the right. The *actual* field between N and S is the resultant of the two fields above (see Fig. 329*b*).

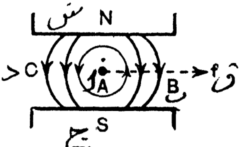


Fig. 329*a*.

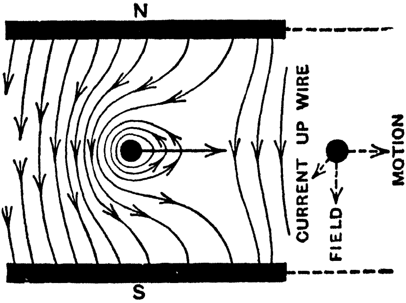


Fig. 329*b*.

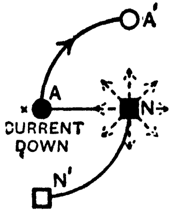


Fig. 330.

Fig. 330 shows the direction in which a current-carrying wire A tends to move round a north pole N, and the direction in which the north pole tends to move round the current. The latter is given by, say, the right hand rule of Art. 146; the former—the direction in which a conductor carrying a current tends to move in a magnetic field—is perhaps best given *in most practical cases* by the following **left hand rule** due to Fleming:—*Hold the thumb and the first two fingers of the left hand mutually at right angles; place the forefinger in the direction of the lines of force of the field in which the conductor is situated (N to S), and turn the hand so that the middle finger points in the direction of the current; the thumb will indicate the direction of motion of the conductor.* Clearly, if the direction of the current *or* the direction of the field be reversed the direction of the force will be reversed, but if *both* field and current be reversed the direction of the force will be unaltered.

Numerous experiments may be devised to illustrate these "rotations" and to verify the rules given. Thus in Fig. 331 the vertical magnet is surrounded by a ring trough containing mercury up to about the level of the mid point of the magnet, and a conducting circuit is pivoted on the end of the magnet as shown. On passing a current up the magnet and down the circuit continuous rotation of the latter ensues, and the left hand rule may be verified. If a solenoid carrying a strong current be substituted for the magnet, and the apparatus modified accordingly, the same effect takes place. Further, if a bar magnet be suitably weighted, so that it floats vertically in mercury, it can, by suitable arrangement, be caused to rotate round an insulated vertical conductor dipping into the mercury when a current is passed through the conductor.

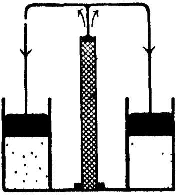


Fig. 331.

An expression for the *magnitude* of the force acting on the conductor may be obtained as follows:—The intensity h of the magnetic field at c (Fig. 316) due to the element ab is given by

$$h = \frac{I \times ab'}{oc^2},$$

the direction of the field at c being at right angles to the plane abc.

If we denote ab by l, oc by r, and the angle aoc by α, then

$$h = \frac{Il \sin \alpha}{r^2}.$$

This expression for the intensity of the magnetic field at c gives the force which a unit north pole would experience if placed at that point. It therefore gives the force which a unit north pole at c would exert on the element ab. But the strength of the magnetic field at o, due to a unit north pole at c, is $1/r^2$. Hence the force exerted on the element ab in a magnetic field of intensity $1/r^2$ is $I.l \sin \alpha . 1/r^2$; that is, in a field of intensity H the force exerted on the element ab of the conductor AB carrying a current I is

$$\text{Force} = IHl \sin \alpha.$$

If the conductor is straight and the field uniform, then this result may be applied to a conductor of finite length. Further, in the case of a conductor of length l, placed *at right angles* to a uniform field of strength H, we have (since $\sin \alpha = 1$)—

$$\text{Force} = I.H.l.$$

Example. If a straight wire 20 cm. long, carrying a current of 15 amperes, be placed at right angles to the earth's horizontal field ·18 unit, the force tending to move the wire at right angles to the current and to the field (left hand rule) is $\frac{15}{10} \times {\cdot}18 \times 20 = 5{\cdot}4$ dynes.

171. Work Done in Displacing in a Magnetic Field a Circuit carrying a Current.—

Consider first a straight conductor of length l cm. placed at right angles to a uniform field of strength H units and carrying a current of I e.m. units; the force on the conductor is IHl dynes. If the conductor moves as indicated in Fig. 332 through a distance x cm., the work done is $IHlx$ ergs; but $l \times x$ is the area swept out, and Hlx is, therefore, the total number of unit

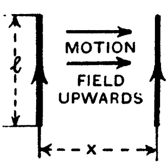


Fig. 332.

tubes of force (F) cut by the conductor during the displacement; hence

$$\text{Work} = I.F = \text{current} \times \text{tubes cut.}$$

Example. If a wire carrying 30 amperes moves in a field so as to cut 10,000 unit tubes, the work done is $\frac{30}{10} \times 10{,}000 = 30{,}000$ ergs.

A more general proof of the above is as follows. Let ab (Fig. 333) represent an element of a conductor carrying a current in the direction of the arrow, and let aH denote the direction and strength of the magnetic field acting on the element. Also let aF, drawn in a direction at right angles to the plane of ab and aH, represent the direction of the force f acting on the element ab.

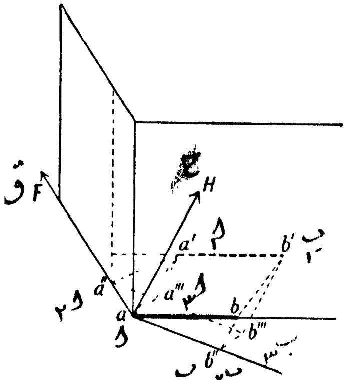


Fig. 333.

If now the element remaining parallel to its first position suffer a very small displacement to the position $a'b'$, the work done in effecting the displacement by the force acting on the element is measured by $f\,.\,aa''$, where aa'' is the projection of aa' on the direction of f. But if H denote the intensity of the field, then by Art. 170 we have $f = IHab \sin \alpha$, where α denotes the angle baH and I the current in the element, that is, the work done in the displacement is equal to $IHab \sin \alpha\,.\,aa''$. But $ab \sin \alpha\,.\,aa''$ is $ab''\,.\,aa''$, the area of $aa'''b'''b''$ which is the projection of the area $aa'b'b$ on a plane through a at right angles to the direction of the magnetic field. The quantity $H\,.\,ab \sin \alpha\,.\,aa''$ is, therefore, the flow of force through $aa'b'b$ or *the number of unit tubes of force cut by the element during its displacement* into the position $a'b'$. Thus we obtain, as before, the result that the work done = current × tubes cut

If a closed circuit be displaced in any way in a field it is evident that the work done may be determined by imagining the circuit divided up into very small elements and summing up the work done in the displacement of each element. Thus, if I denote the current in the circuit and n_1, n_2, n_3, . . . the number of tubes of force cut by the elements of the circuit, then the total work for the given displacement is ΣIn or $I\Sigma n$. In determining Σn, however, the *sign* of n for each element has to be considered. Assuming the tubes of force at any point to pass through the circuit from the negative side to the positive side (that is, in the direction which the tubes of force due to current in the circuit would pass), then n is positive if the tubes are so cut as to pass from without to within the area of the circuit, and negative if the tubes are so cut as to pass from within to without this area. It follows from this that Σn is really equivalent to the increase of the flow of force through the circuit in the positive direction. If this *increase* be denoted by F, then the work done *by* the electromagnetic force acting on the circuit in the magnetic field is measured by FI.

If the displacement is a very small linear displacement l, then FI/l is the work done *by* the electromagnetic forces per unit length, that is FI/l is the force causing the displacement. Similarly, if the circuit be rotated round an axis through a very small angle θ, then FI/θ gives the moment of the couple causing the displacement.

If the circuit be taken from an infinite distance up to any point in the field, then the work done against electromagnetic forces in bringing the circuit to that point is given by $-FI$, where F denotes the total flow of force through the circuit in the position it occupies in the field. The quantity $-FI$, therefore, measures the potential of the circuit in the field. The potential here defined evidently varies with the position of the circuit in the field and may be zero if F is zero. Its maximum value obtains when F is negative and has the greatest arithmetical value possible, and for its minimum value F is positive and as great as possible. The two positions of the circuit corresponding to the maximum and minimum values evidently make an angle of 180° with each other.

For a circuit free to move in the field the position of rest is that corresponding to the minimum potential energy, that is, to the position in which the tubes of force pass through it in the greatest number and in the same direction as the tubes of force due to the current in the circuit. A plane circuit, for example, freely suspended in a uniform field tends to set itself at right angles to the field in such a position that the positive direction of its axis is the same as that of the field. It is evident also that if the circuit is flexible or made up of movable parts it will, when a current is passing through it, tend to make the area enclosed by the circuit a maximum. A flexible circuit, therefore, tends to become circular in form, and in a circuit with a movable part the movable part moves in such a way as to increase the area of the circuit.

172. Current Circuits and Equivalent Magnets and Magnetic Shells.—Let a circular coil of radius r carrying a current I e.m. units be fixed with its plane in the meridian; the force on unit pole at P (Fig. 321) is $2\pi r^2 I/(d^2 + r^2)^{\frac{3}{2}}$, *i.e.* $2\pi r^2 I/d^3$, say, if r be small compared with d. Hence if a small magnet of pole strength m and length l be placed at P, the couple on it due to the coil is

$$\text{Couple} = \frac{2\pi r^2 I}{d^3} ml = \frac{2AIM}{d^3},$$

where A is the face area of the coil and M is the moment of the magnet at P.

If the coil be now replaced by a small "end on" magnet of moment M_1 the couple on the magnet at P is (Art. 34)

$$\text{Couple} = \frac{2M_1 M}{d^3}.$$

If these are identical $M_1 = AI$; thus in the case considered *the circular current is equivalent to a magnet the moment of which is numerically equal to the current in e.m. units multiplied by the area of the coil face.*

Now let the circular current be replaced by a magnetic shell of moment M_1 whose boundary coincides with the wire; this shell will be equivalent to the current if $M_1 = AI$, *i.e.* if $M_1/A = I$. But M_1/A is the moment per unit area, *i.e.* the *strength* of the shell (Art. 36); thus *the circular current is equivalent to a magnetic shell the strength of which is numerically equal to the current in e.m. units.*

The truth of the above can also be *directly* deduced from formulae previously established. Thus in Art. 36 it is shown that the field at P due to a shell of circular contour is $2\pi r^2 \phi/a^3$, where r is the radius of the face of the shell, ϕ its strength, and a the distance from the edge of the shell to the point P. If a circular current of the same boundary replaces the shell it is shown in Art. 167 that the field at P is $2\pi r^2 I/a^3$; hence the two are equivalent if, numerically, $\phi = I$.

So far we have dealt with the special case of a circular current for convenience, but the theorem of the *equivalent* *magnetic shell* is applicable to any closed current circuit. Thus it has been shown in Art. 36 that the potential of a magnetic shell in a magnetic field is $-\phi F$, where ϕ is the strength of the shell and F the flow of force through the contour of the shell from its negative side to its positive side. It has also been shown in Art. 171 that the potential of a closed circuit carrying a current I in a magnetic field is $-IF$, where I is the strength of the current and F the flow of force through the circuit from its negative to its positive side. It is evident that if $\phi = I$ these two quantities become equal; hence the general statement that *a closed circuit gives rise to the same field, and is subject to the same forces in a magnetic field, as a magnetic shell of the same contour as the circuit, if the strength of the shell is equal to the strength of the current in e.m. units;* this is referred to as Ampère's theorem.

If the air medium be entirely replaced by a medium of permeability μ the strength of the equivalent shell will be μI.

173. Work done in carrying a Unit Pole round a Current.—From the preceding it follows that all the theorems relating to magnetic shells can be applied to a closed circuit by supposing it replaced by the equivalent magnetic shell. Thus, for a magnetic shell of strength ϕ the potential at any point (Art. 36) is given by $\phi\omega$, where ω is the angle which the contour of the shell subtends at the point. Hence, the potential due to a closed circuit at any point is $I\omega$, where I is the electromagnetic measure of the current and ω the angle which the circuit subtends at the point. It should be noted, however, that this is the potential at the point, assuming that unit pole is brought from an infinite distance up to that point *without passing through the circuit.* If the pole passes through the circuit and back to the given point by a path outside the circuit, then the angle subtended by the circuit ~~from~~ at points along the path of the pole changes from ω to $4\pi + \omega$, and the potential changes from $I\omega$ to $I(4\pi + \omega)$, *i.e.* by an amount $4\pi I$; thus *the work done on a unit pole in threading the circuit from any point back to the same point is* $4\pi I$, and in threading the circuit n times the work is $4\pi In$.

The above will be clear from an examination of Fig. 334, which represents a current circuit with its plane at right angles to the plane of the paper. At A indefinitely near the north face of the circuit the solid angle is 2π and the potential $+2\pi I$. The potential falls in going along the path $ABC\ldots$, being πI at B, $\frac{1}{2}\pi I$ at C, zero at D, $\ldots -2\pi I$ at G, the angles subtended at $B, C, D, \ldots$ being indicated in the figure. The work done in moving unit pole from A to G along the path $ABCDEFG$ is therefore $4\pi I$ (the change in potential). The work required to complete the path GA is *nil*, since the distance is indefinitely small. Hence *the work done in moving unit pole from A along the path $ABCDEFG$ back to A*, i.e. *along a path linked once with the current, is* $4\pi I$. If the closed path is not linked with the current, *e.g.* the path $A'B'C' \ldots G'A'$, the work is *nil*.

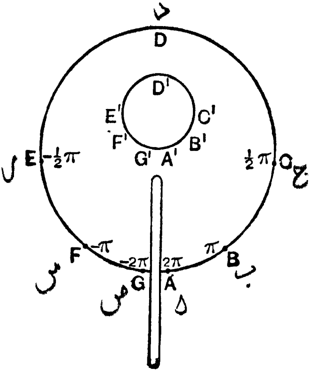


Fig. 334.

The work done in carrying unit pole along any path from one point to another is called the *line integral of the field* between the two points, and the line integral of a field round a closed path is called the *curl of the field*; hence for a path linked with a current

$$\text{Curl } H = 4\pi I.$$

It will be remembered that this relation was used in the alternative proofs of the formulae in Arts. 165 and 169.

174. Galvanoscopes and Galvanometers.—Instruments in which the magnetic effects are utilised for the detection of currents are termed galvanoscopes, more accurate forms designed for current measurement being known as galvanometers.

The simplest type of galvanoscope consists of a magnetic needle with a wire placed above it and in the meridian. To increase the sensitiveness of this arrangement the wire may be formed into a coil with the needle at its centre; an application of the *right hand rule* (Art. 146) will reveal the fact that in such a case all the currents both above and below the needle are urging it in the same direction, and thus a weak current will be better able to produce a deflection.

In both these cases it will be observed that the action of the earth on the needle is opposing that of the current, for the former is striving to set it in the meridian, whilst the latter is endeavouring to set it at right angles thereto, from which it follows that to secure greater sensibility the action of the earth must be partially eliminated. This is more or less accomplished by employing instead of a single needle an **"astatic" pair**; that is, a combination of two needles of equal length and strength (or equal "moment"), fixed parallel with unlike poles adjacent, and with their magnetic axes in the same vertical plane, and in which therefore the turning effect of the earth on one is cancelled by the equal turning effect in the opposite direction on the other. The two methods of winding such an arrangement so as to form an **"astatic galvanometer"** are shown in Fig. 335; by applying the right hand rule it will be seen that both needles are urged in the same direction.

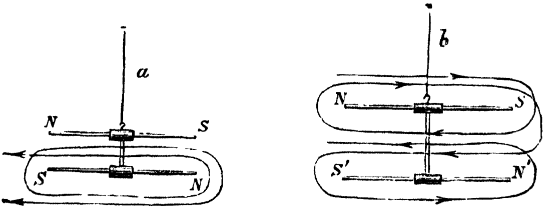


Fig. 335.

The construction of a *perfectly* astatic pair is difficult, and in fact undesirable here; but if partially astatic, in which case the controlling influence is mainly the torsion of the suspending fibre, a very sensitive galvanometer is obtained. The deflection may be read either by means of a pointer attached to the moving system or by the "lamp and scale" method explained in Art. 41.

The figure of merit of any ordinary galvanometer in which the deflection is given by a pointer moving over a scale of degrees is usually defined as the current in amperes necessary to produce a deflection of 1°; in the case of a reflecting galvanometer it is taken to be the current necessary to produce one millimetre deflection on a scale placed one metre from the mirror of the galvanometer.

The sensibility of a reflecting galvanometer is, however, in order to take various factors into account, more exactly defined as the number of millimetres deflection produced on a scale one metre from the mirror by a current of one-millionth of an ampere, reduced to the corresponding deflection for the same rate of expenditure of energy if the resistance of the galvanometer were one ohm and the period of vibration one second. Mathematically, if t be the period of vibration, r ohms the galvanometer resistance, and d mm. the deflection when the current is 10^{-6} ampere and the distance of the scale one metre, it can be shown that

$$\text{Sensibility} = \frac{d}{t^2\sqrt{r}}.$$

Galvanometers are often wound with two coils, one consisting of few turns, the other of many turns of wire. A moderately strong current is passed through the former, but a weak current through the latter, for the greater the number of times it is carried round the greater will be its effect on the needle. Of course the coil of many turns must necessarily be of fine wire to avoid the instrument approaching unwieldy dimensions.

The moving part of a galvanometer or other measuring instrument (ammeter, voltmeter, etc.) tends to oscillate about a mean position when the current is started, stopped, or varied. This is often undesirable and a "**damping**" device is frequently introduced to prevent this undue oscillation; the instrument is then said to be "**dead beat.**" Damping is secured by utilising (1) *the viscosity of liquids*—the moving part carries a light vane which moves in a liquid (usually oil); (2) *air friction*—the vane moves in air, often, in commercial instruments, taking the form of a piston moving in a tube; (3) *induced currents*—the moving part has currents developed in it which oppose the motion.

175. The Kelvin Mirror Galvanometers.—One type is shown in Fig. 336. The coil is wound on a comparatively small circular reel, *R*, enclosed in *B*, a cylindrical brass box with a glass front. The needle of the instrument is a short, carefully magnetised strip of steel, attached by shellac or cement to the back of a small concave mirror, *m*, suspended by a single silk fibre at the centre of the coil. The deflection of the magnet is measured by the mirror and scale method.

The large "controlling" magnet, *NS*, supported above the coil, has several important uses. If this magnet be removed, the needle of the galvanometer sets in the magnetic meridian and the magnetic field in which it lies is that due to the horizontal component of the earth's field. If now the magnet be replaced with its length in the magnetic meridian and its *south* pole pointing northwards, the magnetic field which it produces at the centre of the coil will be added to that due to the earth. The needle will now be more difficult to deflect, and, consequently, the galvanometer will be less sensitive. The nearer the magnet is to the coil, the greater will this effect be; hence, by lowering the magnet on its support, the sensitiveness may be very considerably diminished.

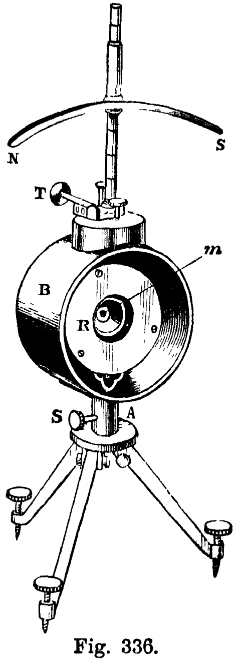


Fig. 336.

If, however, the magnet be replaced with its *north* pole pointing northwards, the field it produces at the centre of the coil tends to neutralise that due to the earth; and hence, by lowering the magnet, the field in the coil may be decreased until, at a certain point, the field of the magnet exactly balances that of the earth. For this position of the magnet the needle is unstable, and remains at rest in any position; and if the magnet be lowered still more, the direction of the field is reversed and the needle tends to turn round through 180°. Hence, in this case, the galvanometer cannot be used with the controlling magnet lower than the position of instability of the needle, but with the magnet slightly above this position the instrument is extremely sensitive. Currents of the order of one-millionth of an ampere can be detected by this galvanometer.

An even more delicate type is the **Kelvin High-resistance Astatic Galvanometer** shown in Fig. 337. It consists of four coils enclosed in four ebonite coil boxes arranged two above and two below as indicated. The coils are hollowed out at their centres and in the two cavities thus formed hang the two sets of magnetic needles. Each set consists of three or four needles, the two sets being fixed to an aluminium wire and arranged in astatic order, *i.e.* the poles of the upper set point in the opposite direction to the like poles of the lower set. A light vane of mica carrying a mirror is fixed to the wire in a position midway between the coils as shown; as it rotates in air it acts somewhat as a "damping" device. With this instrument currents of the order $\frac{1}{100,000,000}$ ampere can be detected.

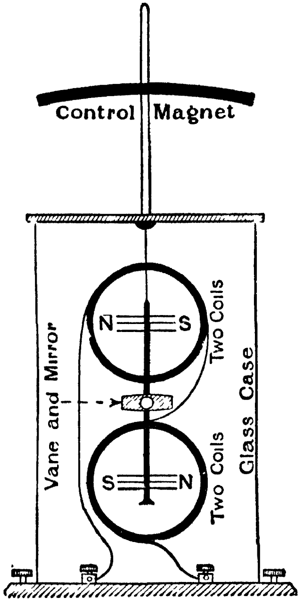


Fig. 337.

176. Moving Coil Galvanometers.—An early form invented by MM. Despretz and D'Arsonval is shown in Fig. 338. The rectangular movable coil consists of a number of fine wires well soaked in insulating varnish and suspended between the poles of a permanent horse-shoe magnet by wires or phosphor bronze strips, which also serve to conduct the current to and from the coil. The lower wire is attached to a small spring, and the upper suspension to the torsion head at the top of the instrument; in this way the coil is maintained firmly in its normal position, *viz.* with its plane along the lines of force of the field between the magnet poles. A cylindrical piece of soft iron *C* is supported within the coil, and by concentrating the lines results in an intense field in the space at each side in which the vertical wires of the coil move.

When a current passes in the coil, the latter tends to set itself so as to enclose as many lines of force as possible, *i.e.* it tends to set with its plane at right angles to the field between the poles, and this motion is resisted by the controlling couple furnished by the torsion of the suspension, the coil in consequence taking up an intermediate position in which the deflecting and controlling couples balance each other. The induced pressures developed in the coil when it moves in the magnetic field oppose the motion and render the instrument "dead beat." In some types the coil is wound upon a conducting frame of aluminium or silver ; in such cases the induced currents (eddy currents) developed therein still further oppose the motion and increase the dead-beat action of the galvano-

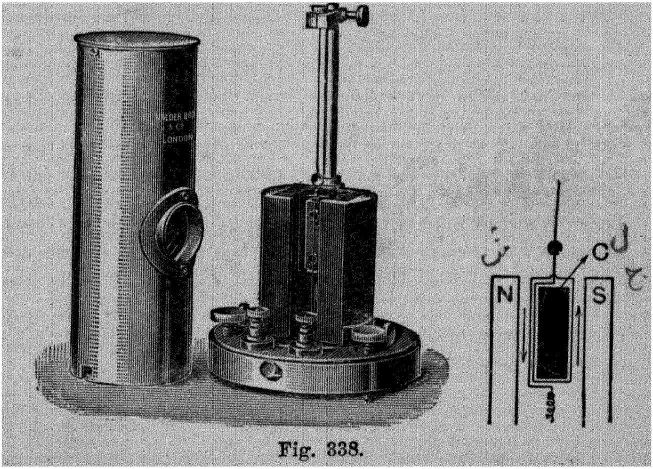


Fig. 338.

meter. Further, when the current ceases any oscillation of the coil may be prevented by merely connecting for a moment the galvanometer terminals by a wire of low resistance, the damping being as before produced by induced currents.

The direction of deflection is determined by Fleming's left hand rule; thus with the current passing as in Fig. 338 the left-hand side of the coil moves "out of the paper" and the right-hand side into it.

The law of this type of galvanometer may be developed as follows. Let $2d$ be the mean width of the coil, l its mean length, n the number of turns, and H the strength of the field in which the vertical branches of the coil hang. Let a, supposed small, be the twist of the wire. Then for a small additional angular deflection θ the number of tubes of force cut by each branch is approximately $ld\theta H$, and the work done is, as in Art. 171, given by $2nI \,.\, ld\theta H$. This gives the moment of the couple causing deflection as equal to $2nI \,.\, ldH$. If T denote the moment of the torsion couple for unit angular twist of the wire, we therefore get

$$2nIldH = Ta.$$

But $2ld$ is the mean area of the coil. If this be denoted by A we have

$$nIAH = Ta$$

or

$$I = \frac{T}{nAH}a.$$

This indicates that I is directly proportional to the deflection, provided H is constant throughout the space in which the coil moves.

The more recent type of moving coil galvanometer due to Ayrton and Mather is shown in Fig. 339. The permanent magnet is of the shape of a nearly complete cylinder—a very narrow air gap only existing between the poles. In this gap hangs the long narrow coil mounted in a thin silver tube; thus the "damping" is very efficient. The coil has no iron core, its shape and the narrow air gap rendering such unnecessary.

Fig. 340 depicts the Crompton type, in which the suspension is a bifilar one which leads the current to and from the coil.

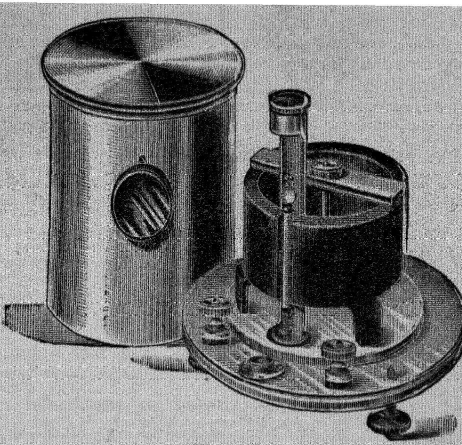


Fig. 339.

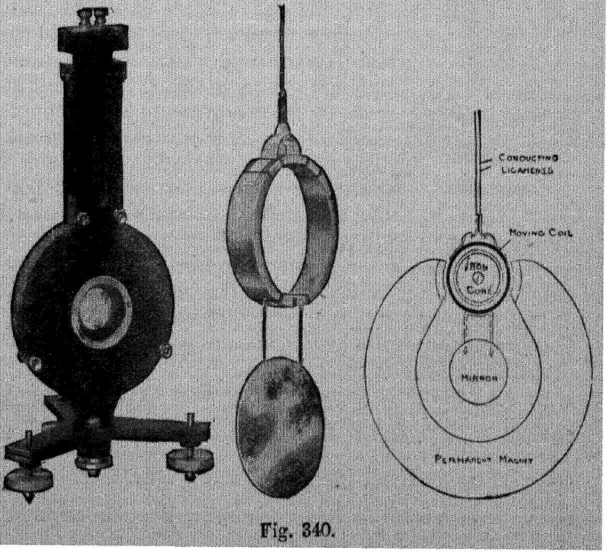


Fig. 340.

The advantages of these galvanometers may be said to be:—(1) They are practically independent of the earth's magnetic field and may be set up in any convenient position. (2) The field in which the coils hang is so strong that magnetic fields due to dynamos, etc., do not affect the readings *to any appreciable extent.* (3) They are remarkably dead beat and are not so sensitive to vibration in their vicinity as those of Art. 175.

If these galvanometers are intended for ballistic work (Art. 188) it is important that the oscillations should be unchecked, and therefore a *non-conducting* frame or tube for the coil must be used.

177. The Tangent Galvanometer.—A simple form of tangent galvanometer is shown in Fig. 341. It consists essentially of a circular coil of a few turns of insulated wire, with a *small* magnetic needle pivoted or suspended at the centre. The needle is small, so that the magnetic field due to the current in the coil may be assumed uniform over the entire space in which the needle moves and equal to the field at the centre of the coil. A light aluminium pointer attached to the needle enables the deflections to be read on a horizontal scale graduated in degrees. The needle, pointer, and scale are enclosed in the shallow cylindrical box *B*, which is fitted with a glass cover. Errors due to parallax are avoided by fixing a sheet of mirror glass below the pointer. In working with the instrument the coil is first set in the magnetic meridian, so that the needle and coil are in the same vertical plane; the current to be measured is then passed, and the angle of deflection read off from the scale.

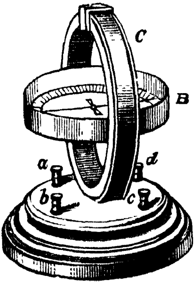


Fig. 341

In Art. 166 it is shown that the field at the centre of the coil is $\frac{2\pi nI}{r}$, and if the field is uniform in the region occupied by the small needle the force F on each pole due to the current is $\frac{2\pi nI}{r}m$ dynes, m being the pole strength of the needle. Further, the force on each pole due to the earth is mH dynes, H being the horizontal component of the earth's field.

Referring to Fig. 341a, it will be seen that the needle is at rest (deflection α) under the influence of two couples—a deflecting couple due to the current and a controlling couple due to the earth; equating these couples we have

$$F \times ad = mH \times bd,$$

$$\therefore\ F = mH \times \frac{bd}{ad} = mH \tan \alpha,$$

$$\text{i.e.}\quad \frac{2\pi n I}{r} m = mH \tan \alpha,$$

$$\therefore\ I = \frac{r}{2\pi n} \mathbf{H} \ \mathbf{tan}\ \alpha.$$

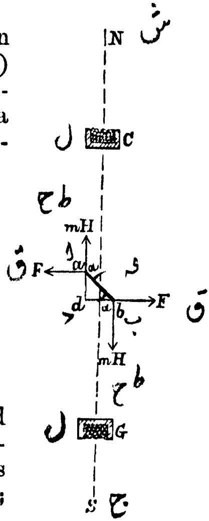


Fig 341a.

The factor $2\pi n/r$ (which gives the field at the centre of the coil due to unit current) is called the **coil constant**, and is the same wherever the instrument is used; denoting it by G, we have

$$I = \frac{H}{G} \tan \alpha.$$

The factor H/G is called the **reduction factor**, and it depends on the value of H, *i.e.* it varies with the place where the instrument is used; denoting it by K—

$$I = K \ \mathbf{tan}\ \alpha.$$

If I be in amperes we have

$$I = 10 \frac{r}{2\pi n} H \tan \alpha = 10 \frac{H}{G} \tan \alpha = 10K \tan \alpha.$$

To eliminate the errors mentioned in Art. 41 it is customary to read *both* ends of the pointer and then to *reverse the current* and again read both ends; the mean of the four readings gives the value of α.

When possible it is advisable to obtain deflections in the vicinity of 45°, as in practice the accuracy in reading is then at a maximum, a given variation in the current producing its greatest effect in this region. Thus, if da be a small increase in the deflection produced by a small increase dI in the current, we have

$$I = K \tan a, \quad \therefore \frac{dI}{da} = K \sec^2 a,$$

i.e.
$$\frac{dI}{I} = \frac{\sec^2 a}{\tan a} da = \frac{2}{\sin 2a} da.$$

Now dI/I is the relative change in the current, and for this to be *as small as possible for a given value of* da the factor $2/\sin 2a$ must be as small as possible, *i.e.* $\sin 2a$ must be as large as possible; this is so when $2a = 90°$, *i.e.* when $a = 45°$.

The reduction factor (K) is readily found in laboratory practice as follows:—

Exp. Arrange the tangent galvanometer, a small copper sulphate voltameter fitted with copper electrodes, an adjustable resistance, a convenient battery—say two Daniell's cells—and a reversing key as in Fig. 341*b*. Adjust the resistance until the deflection is about 45°. Switch off the current, dry, clean, and weigh the kathode and replace it. Pass the current for an exact time—say one minute—noting the galvanometer deflection and keeping it constant. Quickly reverse the current in G and run for another minute. In each case read both ends of the pointer, and take the mean of the four angles as the deflection a. Break circuit, wash, dry (by warm air), and re-weigh the kathode; let w grm. be the increase in weight. Now

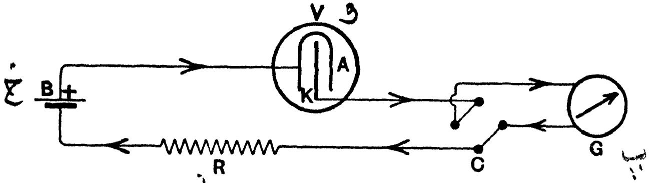


Fig. 341*b*.

$$I = \frac{w}{zt} \text{ (amperes)} \quad \textit{and} \quad I = 10K \tan a \text{ (amperes)},$$

where $z = \cdot0003281$ (Art. 151) and t is the time in seconds the current has been flowing; thus

$$K = \frac{I}{10 \tan a} = \frac{w}{z \,.\, t \,.\, 10 \,.\, \tan a}.$$

Examples. (1) *The coil of a tangent galvanometer consists of* 10 *turns of fine wire on a narrow ring of* 22 *cm. radius. Find the intensity of the magnetic field at the centre of the coil when a current of one ampere passes through it.*

The intensity of the magnetic field at the centre of the coil is given by

$$h = \frac{2\pi n I}{r}.$$

Here $n = 10$; $I = 1$ ampere $= \frac{1}{10}$ C.G.S. unit; and $r = 22$ cm. Therefore

$$h = \frac{2 \times 22 \times 10 \times 1}{7 \times 22 \times 10} = \frac{2}{7} \text{ unit.}$$

(2) *Calculate the reduction factor of the tangent galvanometer referred to in the preceding question, and find the deflection which a current of* 0·21 *ampere would produce when passed through the instrument.*

The reduction factor of the galvanometer is given by

$$K = \frac{rH}{2\pi n}.$$

Hence, taking $H = 0{\cdot}18$ unit, we have

$$K = \frac{22 \times {\cdot}18 \times 7}{2 \times 22 \times 10} = {\cdot}063 \text{ for C.G.S. units.}$$

Also if a denote the deflection produced by a current of 0·21 ampere, that is, 0·021 C.G.S. unit, then from

$$I = K \tan a$$

we have

$${\cdot}021 = {\cdot}063 \tan a,$$

or

$$\tan a = \frac{{\cdot}021}{{\cdot}063} = \frac{1}{3}.$$

That is, a is an angle whose tangent is $\frac{1}{3}$, and is therefore an angle of about 18° 26′.

178. The Helmholtz Tangent Galvanometer.—This consists of two equal coils arranged as described in Art. 168 at a distance apart equal to the radius of the coils (Fig. 342). The needle is suspended midway between them where the field is uniform.

As in the preceding case, we have

$$F = mH \tan a,$$

but the field in the region occupied by the needle is (Art. 168) $32\pi nI/5\sqrt{5r}$; hence

$$\frac{32\pi nI}{5\sqrt{5r}}m = mH \tan a$$

$$\therefore I = \frac{5\sqrt{5r}}{32\pi n}H \tan a.$$

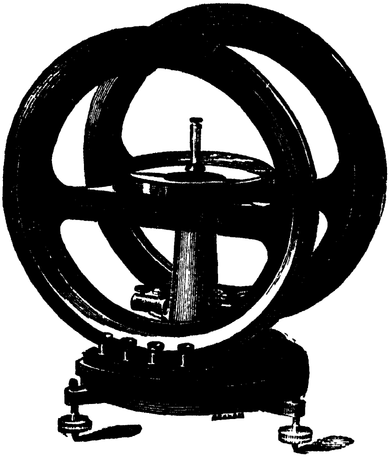


Fig. 342.

179. The Sine Galvanometer, one form of which is shown in Fig. 343, is an instrument exactly similar in principle and construction to the tangent galvanometer. It differs from it only in the fact that the coil and needle box can be rotated round a central vertical axis, and a horizontal circular scale is

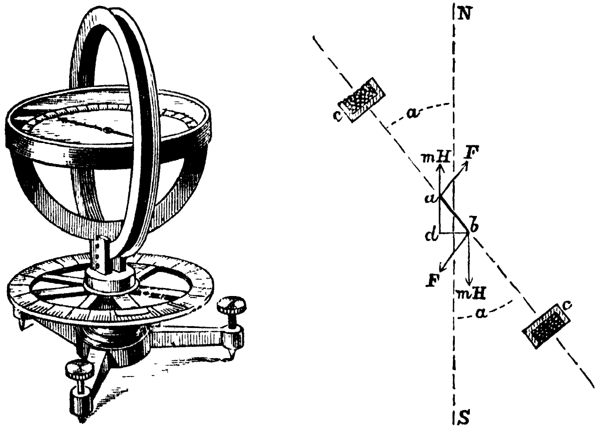


Fig. 343.

provided on which the amount of this rotation can be accurately read.

For use the instrument is adjusted in the same way as the tangent galvanometer, but when the needle is deflected the coil is rotated after it until the needle is overtaken by it and *in its deflected position* lies in the plane of the coil. The diagram of Fig. 343 shows the conditions of equilibrium of the needle, and as before we must have

$$F \, . \, ab = mH \, . \, bd,$$

i.e.

$$\frac{2\pi mnI}{r} \, . \, ab = mH \, . \, bd,$$

$$\therefore \quad I = \frac{r}{2\pi n} \, . \, H \, . \, \frac{bd}{ab} = \frac{r}{2\pi n} H \sin a,$$

where a denotes the deflection of the needle or the rotation of the coil; hence

$$\boldsymbol{I = K \, . \, \sin a},$$

where K denotes the reduction factor of the instrument.

Example. *A sine galvanometer with a short needle is used as a tangent galvanometer, and when a given current is passed through it a deflection of* 30° *is produced. Find the deflection which the same current should produce if the instrument were used as a sine galvanometer.*

Here, when the galvanometer is used as a tangent galvanometer, we have

$$I = K \tan a_1,$$

and when used as a sine galvanometer we have

$$I = K \sin a_2.$$

And from the conditions of the question we have

$$K \tan a_1 = K \sin a_2$$

or

$$\tan a_1 = \sin a_2.$$

But

$$a_1 = 30^\circ;$$

therefore

$$\tan 30^\circ = \sin a_2,$$

or

$$\frac{1}{\sqrt{3}} = \sin a_2.$$

That is, a_2 is an angle whose sine is $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$ (= ·577), and is therefore an angle of about 35° 15′.

180. The Duddell Thermo-Galvanometer.—This instrument is primarily intended for the detection and measurement of very small alternating or varying currents, *e.g.* such as are met with in telephone circuits and in the receiving aerials of wireless telegraphy. A loop of silver wire L hangs by a quartz fibre in between the poles N, S of a permanent magnet (Fig. 344). The lower ends of the loop are attached to pieces of bismuth and antimony respectively, the bismuth and antimony being in contact at the bottom. Below this junction is situated the heater, which is a filament of wire or a quartz fibre platinised; these heaters are of various resistances from 1 to 1,000 ohms. When the current passes through the heater, part of the heat developed is radiated to the bismuth-antimony junction; the result is (see Chapter XV.) that another current is set up in the suspended loop in the direction bismuth to antimony through the junction of these two, and the loop is deflected just as in the case of the moving coil galvanometer. The deflection is read by means of the mirror M, and a lamp and scale; it is proportional to the square of the current when the heater is central under the junction.

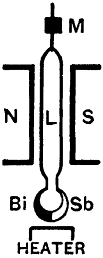


Fig. 344.

181. Shunts and Shunting.—To reduce the sensitiveness of a galvanometer or to prevent damage to the instrument it is frequently necessary to send only a portion of the current through it; this is effected by connecting the galvanometer terminals by a wire termed a "shunt" (Fig. 345).

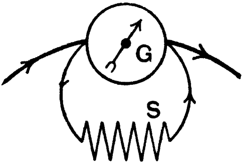


Fig. 345.

If I be the total current, I_1 the portion in the galvanometer, I_2 the portion in the shunt, G the galvanometer resistance, and S the resistance of the shunt, then (Art. 157)

$$\frac{I_1}{I_2} = \frac{S}{G}, \quad \therefore \frac{I_1}{I_1 + I_2} = \frac{S}{G + S},$$

i.e.
$$I_1 = \frac{S}{G+S}(I_1 + I_2) = \frac{S}{G+S}I \quad \ldots\ldots\ldots \quad (1)$$

and
$$I = \frac{G+S}{S}I_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Thus from (1) we learn that the current in the galvanometer is obtained by multiplying the total current by the factor $\frac{S}{G+S}$, and from (2) that the total current is obtained by multiplying the galvanometer current by the factor $\frac{G+S}{S}$; this latter factor is often denoted by m and is called the *multiplying power* of the shunt; thus

$$\frac{G+S}{S} = m,$$

$$\therefore S = \frac{G}{m-1} \quad \ldots\ldots\ldots\ldots \quad (3)$$

To send 1/10 only of a current through a galvanometer the multiplying power of the necessary shunt must be 10, and substituting this for m in (3) we find that $S = \frac{1}{9}G$, *i.e.* the shunt resistance must be $\frac{1}{9}$ of the galvanometer resistance. Similarly $\frac{1}{100}$ of a current or $\frac{1}{1000}$ of a current may be caused to pass through a galvanometer by using shunts having resistances $\frac{1}{99}$ and $\frac{1}{999}$ respectively of the galvanometer resistance. A shunt box designed to meet these requirements is shown in Fig. 346; by inserting a plug in any one of the three holes any desired shunt is put across the galvanometer terminals. Such a shunt box can only be used with the particular galvanometer for which it is constructed.

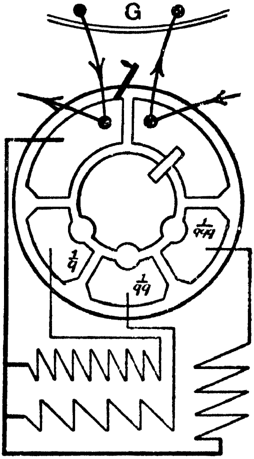


Fig. 346.

The insertion of a shunt evidently reduces the total resistance in the circuit by an amount

$$G - \frac{GS}{G+S} = G - \frac{G}{\frac{G+S}{S}} = G - \frac{G}{m}.$$

This measures the additional resistance which must be put in the main circuit to keep the total resistance and total current as before. In *constant total current shunts* the insertion of a shunt coil throws automatically this requisite resistance into circuit.

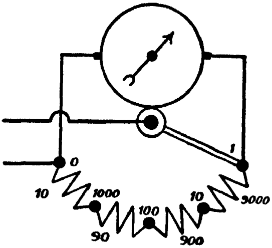


Fig. 347.

The *principle* of Ayrton's "Universal" shunt box, which may be employed with *any* galvanometer, will be understood from Fig. 347. With the lever on stud 1 the whole of the shunt coils are employed, and the current in the galvanometer is, by (1) above,

$$\frac{10000}{G+10000} \text{ of the total current.}$$

When the lever is moved to stud 10 the 9,000-ohm coil is inserted in the galvanometer branch and the shunt employed is of resistance 1,000 ohms ; the galvanometer current is now

$$\frac{1000}{(G+9000)+1000}, \textit{ i.e. } \frac{1000}{G+10000} \text{ of the total current,}$$

or $\frac{1}{10}$ of the galvanometer current in the first case.

Turning the switch to stud 100 will result in a galvanometer current $\frac{1}{100}$ of the first and so on, and clearly this is true whatever may be the value of G.

Exercise. Show that if a battery of very low resistance be connected to a high resistance galvanometer and the galvanometer be then shunted the deflection will remain practically unchanged.

182. Mutual Action of Currents.—If we place two flat spirals of wire a short distance apart with their axes in the same straight line and pass currents round them, then (provided they have suitable freedom of movement) they are found to approach or recede from one another, thus indicating respectively attraction or repulsion, according as the currents circulate round them in the same or in opposite directions. Bearing in mind that the current renders each spiral practically a magnet, and also the fundamental law of electromagnetic polarity, it is easy to see that these actions are in accordance with the laws of attraction and repulsion between two magnets.

Ampère demonstrated in the case of straight circuits the following laws :—

(1) *Two* PARALLEL *currents attract or repel one another according as they flow in the same or in opposite directions.*

(2) *Two* NON-PARALLEL *currents attract one another if both approach or both recede from the point of meeting of their directions, while they repel one another if one approaches and the other recedes from that point.*

These laws are illustrated in Fig. 348, which the student should carefully study. In the lower right-hand figure the vertical current attracts the portion *AM* of the horizontal one and repels the portion *MB*, but since *AM* is

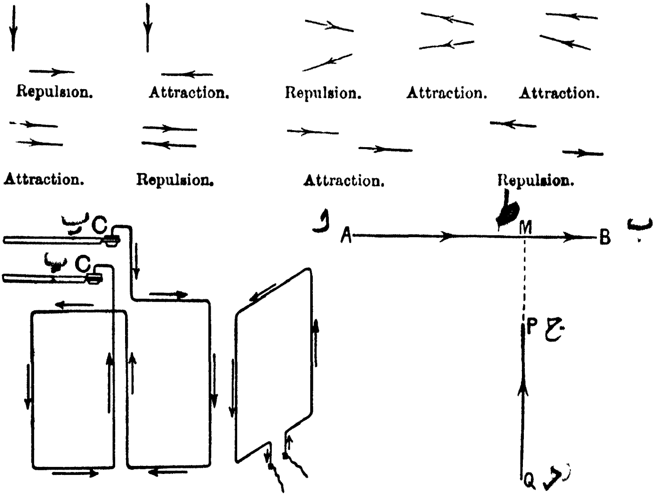


Fig. 348.

longer than *MB* there is, on the whole, attraction; if *M* were the middle point of *AB* the total force would be *nil*.

The experiments whereby Ampère's laws are established merely consist in having two circuits, one fixed and the other movable, placing them in various relative positions, and passing currents in sundry directions. One form is indicated on the left of Fig. 348; the movable coil is bent in the form of two rectangles in such a way that the current flows through them in opposite directions (thus nullifying the effect of the earth on the apparatus) and the coil is suspended from the mercury cups *C*, *C*; on bringing the second current-carrying circuit into various positions the laws may be verified. The truth of the first law is also evident from the distribution of the lines of force shown in Figs. 294 and 295.

183. Force between two infinitely long Parallel Conductors and between two Coaxial Coils.—Let I and I_1 be the currents in the two conductors M and N (Fig. 349). The field at any point in N due to the current in M is equal to $2I/d$, where d is the distance between the conductors and the direction of the field is at right angles to the plane of the conductors. The force *per unit length* exerted on N in this uniform field of intensity $2I/d$ is (Art. 170) given by the product of the current, the field, and the length considered; hence

$$\text{Force per unit length} = \frac{2II_1}{d} \quad \ldots\ldots \quad (1)$$

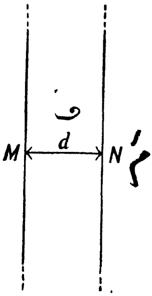


Fig. 349.

Similarly for the force on M due to the current in N, and by applying the hand rules previously given it will be found that the force is one of attraction if the currents are in the same direction and one of repulsion if they are in opposite directions.

Consider now two **equal** coaxial coils, A and B, at distance d apart, carrying currents I and I_1. The force on unit length of B due to A is $2II_1/d$ and the total force on B is the product of this and the circumference; thus, if r be the radius,

$$\text{Force} = \frac{2II_1}{d} \times 2\pi r = \frac{4\pi r II_1}{d} \quad \ldots\ldots\ldots\ldots \quad (2)$$

Next let B (Fig. 350) be a **small** coil of radius r_1 and let r be the radius of A. Let H be the field at the near side of B due to A. Let the equivalent magnetic shell for B have an amount of magnetism m per unit area and a thickness dx; then

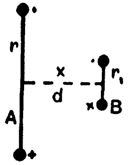


Fig. 350.

Force on the near side of $B = m\pi r_1{}^2 H$ ...... (a)

Field at the far side of $B = H - \frac{dH}{dx}dx$

$\therefore$ Force on the far side of B

$$= m\pi r_1{}^2\left(H - \frac{dH}{dx}dx\right) \text{ .... } (b)$$

Hence

$$\text{Resultant force on } B = (a) - (b) = m\pi r_1{}^2 \frac{dH}{dx} dx.$$

Now mdx is the moment per unit area or the *strength* of the equivalent shell and is therefore equal to the current I_1 in B. Again, the field H at B due to A is (Art. 167) given by

$$H = \frac{2\pi r^2 I}{(r^2 + x^2)^{\frac{3}{2}}}, \quad \therefore \frac{dH}{dx} = -\frac{6\pi r^2 Ix}{(r^2 + x^2)^{\frac{5}{2}}},$$

i.e.

$$\text{Force on } B = -\frac{6\pi^2 r^2 r_1{}^2 II_1 x}{(r^2 + x^2)^{\frac{5}{2}}} = -\frac{6\pi^2 r^2 r_1{}^2 II_1 d}{(r^2 + d^2)^{\frac{5}{2}}} \text{ .... } (3)$$

This is zero if $d = 0$ and numerically greatest if $d = r/2$.

Again, if d be great compared with r, so that r^2 in the denominator may be neglected, (3) becomes

$$\text{Force on } B = \frac{6\pi^2 r^2 r_1{}^2 II_1}{d^4} \text{ .... } (4)$$

In Art. 35 it is shown that the force between two small "end on" magnets is $6MM_1/d^4$; in the case of two equivalent current circuits $M = \pi r^2 I$ and $M_1 = \pi r_1{}^2 I_1$, and the expression for the force becomes $6\pi^2 r^2 r_1{}^2 II_1/d^4$; this is identical with (4) above.

In all the cases considered the force is proportional to the *product of the current strengths.* Measuring instruments depending on the mutual action between currents are known as *electrodynamometers.*

184. Siemens' Electrodynamometer.—This instrument (Fig. 351) consists of two coils, one of which, $ABCD$, is fixed, the other, $FGHL$, being movable and suspended by means of a silk thread with its plane perpendicular to that of the other. The spiral S has one end attached to the movable coil and the other to the torsion head T; the latter carries a pointer P, which moves over a graduated scale. M, M are mercury cups arranged one above the other, into which the lower ends of the moving coil dip, and through the medium of the upper cup the two coils are put in series. A pointer P_1 is also attached to the movable coil, its range of deflection being limited by the two stops h, h. The instrument is set up so that the plane of the moving coil is perpendicular to the meridian, in which position the earth will have no tendency to deflect it.

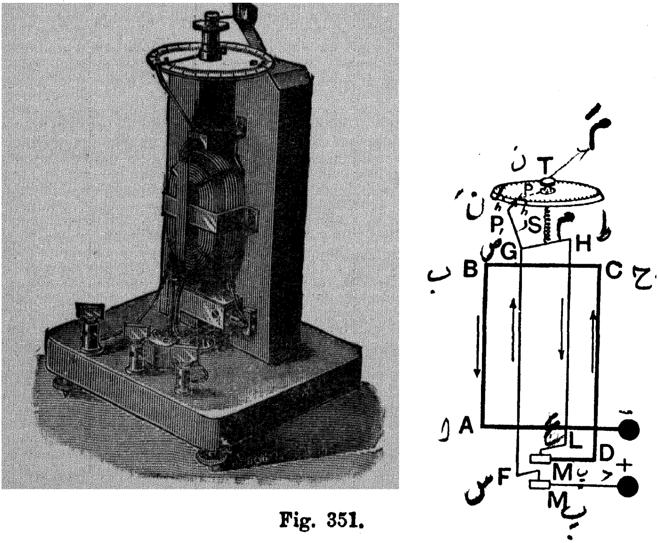


Fig. 351.

When a current passes, the mutual attractions and repulsions tend to set the moving coil parallel to the fixed one, and the torsion head and pointer P are turned in the opposite direction to the deflection until the pointer P_1 is again at zero. If I be the current and θ the angle through which P is turned,

$$\text{Couple between the coils} \propto I^2,$$
$$\text{Couple due to torsion} \propto \theta,$$

and since these balance each other—

$$I^2 \propto \theta, \quad i.e.\ I \propto \sqrt{\theta},$$

$$\therefore I = K\sqrt{\theta},$$

in which K is a constant which may be determined by passing a known current and noting the value of θ.

Reversing the current will not alter the direction of deflection, for it will be reversed in *both* coils; the instrument can, therefore, be used for alternating as well as for direct currents.

185. Weber's Electrodynamometer.—In this instrument a small coil is connected in series with a larger fixed coil and suspended at the centre of the latter by means of a bifilar suspension. The current is led to and from the small coil by means of the suspension. When no current flows the axis of the small coil is in the meridian and at right angles to the axis of the large coil. When a current flows the small coil tends to set itself coaxially with the large one, and this tendency is opposed by the action of the earth (if the moving coil has its north face northwards) and the bifilar suspension, the small coil finally taking up a position in which the opposing couples balance. The coils are frequently arranged after the manner of the Helmholtz galvanometer as shown in Fig. 352.

Let G be the constant of the large fixed coil, I the current, A the face area of the small movable coil (total area of *all* the turns), K the constant of the bifilar suspension, H the earth's horizontal component, and θ_1 the deflection of the small coil.

The deflecting couple is $GI^2A \cos \theta_1$ (compare Art. 177, viz. $\frac{2\pi nI}{r} m \times \overline{ad} = GIm \times \overline{ab} \cos \alpha = GI^2A \cos \alpha$, since the moment of a coil is IA). The controlling couple due to the earth is $AIH \sin \theta_1$ (compare Art. 20, viz. $MH \sin \theta = AIH \sin \theta$). The controlling couple due to the bifilar is $K \sin \theta_1$; hence

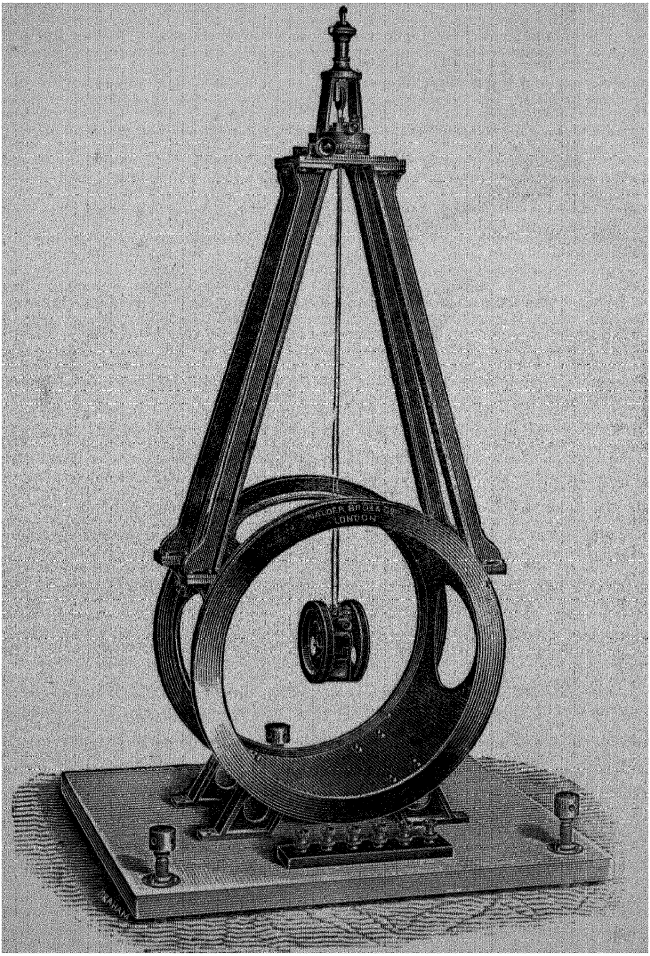


Fig. 352.

$$GI^2A\cos\theta_1 = AIH\sin\theta_1 + K\sin\theta_1,$$

$$\therefore\ \tan\theta_1 = \frac{GI^2A}{AIH+K} = \frac{\dfrac{GI^2A}{K}}{1+\dfrac{AIH}{K}}$$

$$= \frac{GI^2A}{K} - \frac{I^3HGA^2}{K^2}\ \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Now reverse the current and let θ_2 be the deflection. The deflecting couple and the controlling couple due to the bifilar will act as before, but the couple due to the earth will be reversed; hence

$$GI^2A\cos\theta_2 = -AIH\sin\theta_2 + K\sin\theta_2,$$

i.e.

$$\tan\theta_2 = \frac{GI^2A}{K} + \frac{I^3HGA^2}{K^2}\ \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Adding (1) and (2)—

$$I^2 = \frac{K}{2GA}(\tan\theta_1 + \tan\theta_2),$$

i.e.

$$I = k\sqrt{\tan\theta_1 + \tan\theta_2}.$$

By passing and reversing a known current k can be determined.

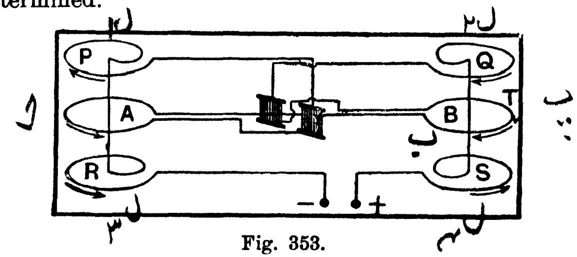


Fig. 353.

186. The Kelvin Current Balances.—The general principle of these will be gathered from Fig. 353. In them there are four fixed coils, P, Q, R, S, placed horizontally, and between these there are two movable ones, A, B, arranged at opposite ends of a light balance beam, the latter being hung at its centre by a number of very fine copper wires, which also serve to lead the current to and from the movable coils. With current passing as indicated it is clear that the beam will tilt upwards on the right, the left-hand side falling. A horizontal scale is fixed to the beam, and by means of a suitable sliding weight, which can be moved along the scale by means of cords and a corresponding counterpoise placed in the trough T on the right, the beam can be brought to its initial position and the current calculated from the position of the sliding weight and the known constants of the instrument. Each sliding weight has its own particular counterpoise, the latter being so constructed that it keeps the beam horizontal when no current is passing and the sliding weight is at zero on the scale. Clearly, if the sliding weight be at distance d from zero when the beam is brought into the horizontal, the restoring couple is proportional to d; the couple due to the current is proportional to I^2; hence

$$I^2 \propto d, \quad i.e.\ I \propto \sqrt{d},$$
$$\therefore\ I = K\sqrt{d}.$$

Each instrument is fitted with two scales, a fixed one of large divisions, the readings of which are *proportional to the square root of the distance from zero*, and a movable fine scale, which is used when great accuracy is required, each number on the fixed scale being double the square root of the number coinciding with it on the fine scale. Thus if in a test the pointer is not exactly below one of the divisions on the fixed scale, the fine scale reading is taken; if this be 301, then, since $\sqrt{301} = 17{\cdot}35$, the true reading of the fixed scale is 34·7, and this number multiplied by a particular constant supplied with the instrument gives the current required. The general appearance of the instrument is shown in Fig. 353*a*.

There are many types of these balances, all alike in principle, but differing slightly in construction according to their purpose.

187. Ammeters, Voltmeters, and Wattmeters.—Ammeters and voltmeters are commercial instruments for the measurement of current strength and potential difference respectively. Frequently the same design is utilised in both, but the essential difference is that an ammeter is a low resistance instrument, whilst a voltmeter

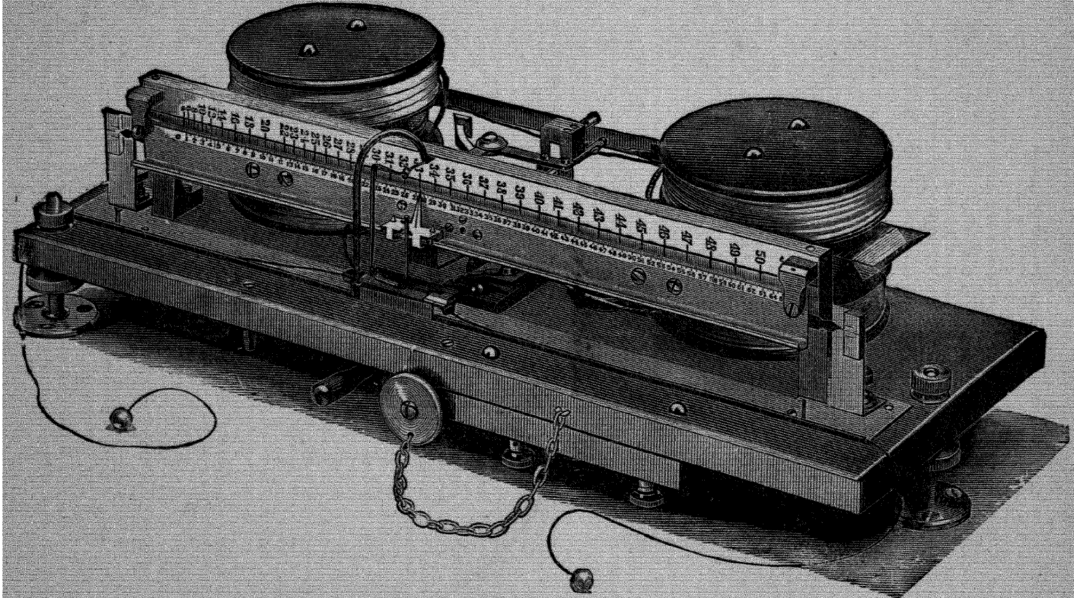


Fig. 353a.

must be of high resistance. Since an ammeter has to measure the current it must be placed *in the circuit* through which the current passes, and must of necessity be of low resistance; a voltmeter is always connected *in parallel* with that portion of the circuit for which the potential difference is required, and its resistance must, therefore, be large.

To increase the range and enable large currents to be measured ammeters are often provided with *specific shunts*; thus an ammeter for which the maximum permissible current is $\frac{1}{4}$ ampere may be employed on a 2 ampere circuit by using with it a shunt having $\frac{1}{9}$ of the ammeter resistance, for in this case only $\frac{1}{10}$ of the current passes through the instrument. Similarly, to increase the range of voltmeters *series resistances* are often employed with them; thus if V_2 be the total P.D. to be measured, V_1 the P.D. on the voltmeter, R_1 the resistance of the voltmeter, and R_2 the series resistance—

$$\frac{V_2}{V_1} = \frac{R_1 + R_2}{R_1}, \quad \therefore \; V_2 = \frac{R_1 + R_2}{R_1} V_1.$$

There are many ammeters and voltmeters which depend for their action on the principles dealt with in this chapter, but a very brief reference only to one or two types can be given here.

The *principle* of **Messrs. Siemens Bros. and Co.'s Gravity Control Instruments** will be gathered from Fig. 354. Here *I* is a thin soft-iron plate fixed on a horizontal axle *A*, which also carries the pointer *P* and a projection *p* to the end of which the curved wire *W* carrying the piston *S* is attached. The latter moves in the tube *TT*, thereby forming an "air damper" which renders the instrument dead beat. To *A* are attached two balance arms, each carrying a weight (not shown). On the passage of a current through the rectangular coil *CC* the iron *I* is drawn into the coil, thereby moving the pointer over the scale. The working parts are protected from external magnetic influence by a curved soft-iron shield, indicated by *XYZ* in the figure.

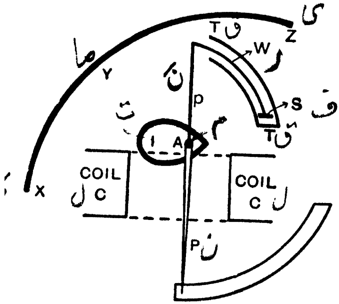


Fig. 354.

The **Weston Moving Coil Instrument** is shown in Fig. 355. Between the pole pieces, *P*, *P*, of a permanent horse-shoe magnet is fixed a cylinder, *I*, of soft iron. Embracing the cylinder is the moving coil, *CC*, of copper wire wound on an aluminium frame. One extremity of the upper hair spring, *S*, is attached to one end of the coil, the other extremity of the spring being fixed to a nonmoving part of the instrument. The other end of the coil is similarly attached to a second hair spring, *S*, seen in the lower part of the figure. These springs are coiled in opposite directions; they lead the current to and from the coil, and furnish the controlling couple which balances the deflecting couple when a current passes. The principle of action is that of the D'Arsonval galvanometer. The instruments are dead beat, damping being due to induced currents in the aluminium frame.

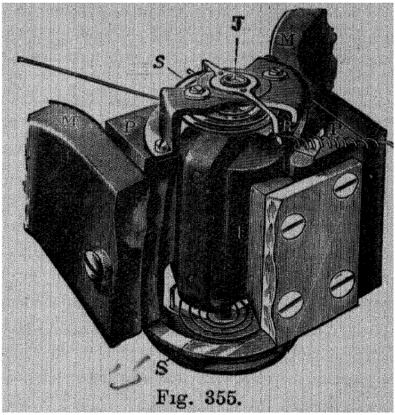


Fig. 355.

Wattmeters indicate the rate in watts at which energy is being utilised in any part of a circuit, *i.e.* they measure the "power." **Siemens' Wattmeter** (Fig. 356) is identical in principle with Siemens' electrodynamometer (Art. 184). The moving coil *C* (Fig. 356) is of low resistance, and is inserted in the main circuit; the high resistance fixed coil *V* is joined as a shunt to that part of the circuit for which the power consumption is required (a lamp *L* in figure). The planes of these coils are at right angles. On closing the circuit the main current *I* passes through the moving coil, and a small current proportional to the voltage *E* at the lamp terminals passes through *V*. The turning moment is proportional to the product of these currents, *i.e.* proportional to *EI* or the watts expended in *L*. When the movable coil is brought back to its normal position by turning the torsion head and its pointer through an angle $D°$,

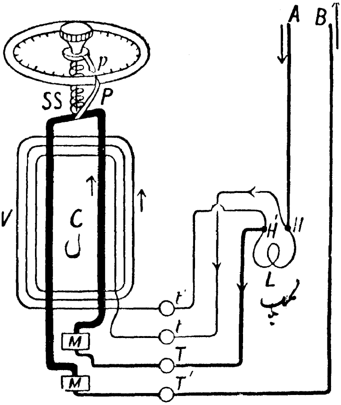


Fig. 356.

say, the turning moment is balanced by the torsional moment, which is proportional to D°. Hence

$$EI \propto D^\circ,$$

$$\textit{i.e.}\ \text{Watts expended in } L = KD,$$

where K is the constant of the instrument, which must be determined experimentally. (See *Technical Electricity*, Chapter XXIII.)

188. The Ballistic Galvanometer: Moving Magnet Type.—A ballistic galvanometer is an ordinary reflecting galvanometer with a moving system of large moment of inertia. It is used for measuring the *quantity* of electricity passed through it not as a continuous current but as a sudden discharge, and the moving system has a large moment of inertia, so that it may be slow in beginning to move under the impulse of the sudden discharge, and will therefore not have moved appreciably from its position of rest during the time the discharge takes to pass through the galvanometer. Further, in these instruments damping must be as small as possible, and for what exists a correction must be made (Art. 190).

Let G denote the constant of the galvanometer coil, H the intensity of the control field, K the moment of inertia of the needle, and M its magnetic moment. If the current at any instant during the discharge has the value I, then IG is the strength of the deflecting field, and, if the needle is supposed to be inappreciably deflected from its position of rest during the discharge, IGM is the moment of the couple tending to deflect the needle.

The angular acceleration due to this couple is $\frac{IGM}{K}$ and, therefore, during the very short time, δt, for which the current has the value I, the gain of angular velocity is $\frac{IGM}{K}\,\delta t$. During the whole discharge, therefore, the angular velocity imparted to the needle is given by $\Sigma\,\frac{IGM}{K}\,\delta t$, the summation being for the whole time of the discharge.

But $$\Sigma\,\frac{IGM}{K}\,\delta t = \frac{GM}{K}\,\Sigma I\,.\,\delta t$$

and $$\Sigma I\,.\,\delta t = Q,$$

where Q denotes the total quantity of electricity discharged through the galvanometer. Hence ω, the final angular velocity of the needle, is given by

$$\omega = \frac{MGQ}{K}$$

or
$$K\omega = MGQ \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

The kinetic energy of the needle is given by $\frac{1}{2}K\omega^2$, and this must be equal to the work done against the couple due to the control field H during the deflection throw of the needle. Let α be the angular deflection of the needle.

Then
$$\tfrac{1}{2}K\omega^2 = MH(1 - \cos\alpha)$$

or
$$K\omega^2 = 4MH\sin^2\frac{\alpha}{2} \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

for $MH\,(1 - \cos\alpha)$ is the work done in deflecting the small needle through the angle α in the field H.

Now if t denote the time of the swing of the needle

$$t = 2\pi\sqrt{\frac{K}{MH}}$$

and therefore
$$K = \frac{MHt^2}{4\pi^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Combining the relations (2) and (3) we get

$$(K\omega)^2 = \frac{M^2H^2t^2\sin^2\frac{1}{2}\alpha}{\pi^2}$$

or
$$K\omega = \frac{MHt}{\pi}\sin\frac{\alpha}{2}.$$

Substituting the value of $K\omega$ given by (1) in this, we get

$$MGQ = \frac{MHt}{\pi}\sin\frac{\alpha}{2}.$$

or
$$Q = \frac{Ht}{\pi G}\sin\frac{\alpha}{2}.$$

Thus the quantity of electricity discharged through the galvanometer is proportional to the sine of half the angle of the first swing of the needle. If the deflection of the spot of light on the scale be d, and D be the distance between the mirror and scale, $d/4D$ may be used for $\sin\frac{a}{2}$ and

$$Q = \frac{Ht}{\pi G} \cdot \frac{d}{4D}.$$

189. The Ballistic Galvanometer: Moving Coil Type.—If l be the length of the vertical side of the coil (Fig. 338), H the field in which it hangs, and I the current at any instant, the force on each vertical side is IHl (Art. 170) and $IHlb$ is the deflecting couple, where b is the breadth of the coil; but lb is the area A of the coil, so that the deflecting couple is IHA. This takes the place of IGM in the preceding investigation, and proceeding as before we obtain in place of (1) the expression

$$K\omega = HAQ \quad \text{.......................} \quad (4)$$

As in the previous case, the kinetic energy is $\frac{1}{2}K\omega^2$, but in this case the coil is brought to rest by doing work in twisting the suspension.

If c be the couple due to unit twist, ca is the couple due to twist a; the *work done* for an extra twist da is $c\,.\,a\,.\,da$ and the total work for a twist a is $\int_0^a c\,.\,a\,.\,da$, *i.e.* $\frac{1}{2}ca^2$; hence

$$\tfrac{1}{2}K\omega^2 = \tfrac{1}{2}ca^2,$$

$$\therefore \omega^2 = \frac{ca^2}{K} \quad \text{.......................} \quad (5)$$

But from (4)

$$\omega^2 = \frac{H^2A^2Q^2}{K^2},$$

$$\therefore \frac{H^2A^2Q^2}{K^2} = \frac{ca^2}{K},$$

i.e.

$$Q^2 = \frac{K}{H^2A^2}\,ca^2 \quad \text{..............} \quad (6)$$

Again, the time t of a torsional vibration is $2\pi\sqrt{\frac{K}{c}}$, i.e. $c = \frac{4\pi^2 K}{t^2}$; substituting in (6),

$$Q = \frac{2\pi K}{HAt}a,$$

or putting $K = \frac{ct^2}{4\pi^2}$,

$$Q = \frac{ct}{2\pi HA}a.$$

In this case the quantity is therefore proportional to the first angular swing a and not to the sine of half the angle, as in the previous case. Further, in this case H is the field to which the deflections are due and it appears in the denominator; in the previous case H is the controlling field and it appears in the numerator.

190. Correction for Damping in Ballistic Galvanometers.—If great accuracy is not required we can proceed as follows. Let a_1 be the first swing and a_2 the next swing of the needle in the same direction. Then the difference, $a_1 - a_2$, is due to the damping during *four* such successive swings and therefore $\frac{a_1 - a_2}{4}$ is approximately the correction for damping during the first swing. Hence the corrected value of the first swing is

$$a = a_1 + \frac{a_1 - a_2}{4}.$$

A more accurate treatment is as follows:—If $a_1\ a_2\ a_3 \ldots$ be successive swings *to left and right* it is found that

$$\frac{a_1}{a_2} = \frac{a_2}{a_3} = \frac{a_3}{a_4} = \qquad = \text{a constant} = k.$$

This constant k is called the **decrement,** and $\log_e k$ is called the **logarithmic decrement** and is denoted by γ; thus $\log_e k = \gamma$, *i.e.* $k = e^{\gamma}$; hence

$$\frac{a_1}{a_2} = \frac{a_2}{a_3} = \frac{a_3}{a_4} = e^{\gamma}.$$

Now the decrease from a_1 to a_2 takes place in half a complete vibration and $a_1/a_2 = e^{\gamma}$. Clearly for a whole vibration $a_1/a_3 = e^{2\gamma}$ and so on. Thus if a be the observed first swing and a^1 what it would have been if damping had been absent, then, as the period in question is a quarter vibration,

$$\frac{a^1}{a} = e^{\frac{\gamma}{2}} = 1 + \frac{\gamma}{2} + \text{terms in } \gamma^2 \text{ and higher powers}$$

$$= 1 + \frac{\gamma}{2} \text{ since } \gamma \text{ is always small,}$$

$$\therefore a^1 = a\left(1 + \frac{\gamma}{2}\right).$$

Thus to correct for damping the observed swing a must be multiplied by the factor $\left(1 + \frac{\gamma}{2}\right)$; the final expressions for the two types of galvanometers are therefore

$$Q = \frac{Ht}{\pi G} \sin \tfrac{1}{2} a \left(1 + \frac{\gamma}{2}\right); \quad Q = \frac{c \,.\, t}{2\pi HA} a \left(1 + \frac{\gamma}{2}\right).$$

Exercises XII.

Section B.

(1) Determine the resistance of a shunt which when joined to a galvanometer of resistance 3,663 ohms will result in $\frac{1}{34}$ of the total current passing through the galvanometer. Determine also (*a*) the joint resistance of the galvanometer and shunt; (*b*) the external resistance which must be added when the shunt is applied, so that the total current may be unaltered.

(2) The resistance of a shunted galvanometer is 75 ohms, that of the shunt being 100. A certain deflection of the galvanometer is obtained when the resistance in the rest of the circuit is 2,000 ohms. Find what additional resistance must be inserted that the galvanometer deflection may remain the same when the shunt is removed. What is the multiplying power of the shunt?

(3) A circuit contains a battery of 1 ohm resistance, a reflecting galvanometer of 4 ohms, and other conductors of 2 ohms resistance. The galvanometer deflection is 100 divisions. What will the deflection be (if deflection is proportional to current) when the galvanometer is shunted with a 4-ohm coil? What is the multiplying power of this shunt?

(4) A galvanometer of 4 ohms resistance is in a circuit where the total resistance is 80 ohms. The galvanometer is then shunted with a 4-ohm coil. Find the ratio of the current in the galvanometer before and after it is shunted.

Section C.

(1) Experiments are to be arranged to find out how a conductor carrying an electric current tends to move in a magnetic field. What experiment would you arrange? (Inter. B.Sc.)

(2) How may the intensity of the magnetic force inside a solenoid be approximately calculated? What is it in one of 300 turns, 15 cm. long, which carries a current of 0·2 ampere? What effect has the diameter of the solenoid? (Inter. B.Sc.)

(3) Explain how the current in a tangent galvanometer properly arranged is proportional to the tangent of the angle of deflection. Describe some form of tangent galvanometer, and explain how the sensitiveness can be varied by suitably placing a magnet outside a galvanometer. (Inter. B.Sc.)

(4) How would you show by experiment that the magnetic field due to a plane current circuit, at any distance great compared with the dimensions of the circuit, depends not on the form but only on the area and the current, and that it is equal to that of a certain magnet set with axis perpendicular to the plane of the circuit? Show, by considering the case of a plane circular current, that the moment of the equivalent magnet is (area × current).
(Inter. B.Sc. Hons.)

(5) State the rules by which the force acting on a conductor carrying a current in a magnetic field can be determined. A vertical circular ring, radius a, carrying a current i, is in equilibrium in the earth's magnetic field when perpendicular to the meridian. Find the work required to twist the ring round a vertical axis until its plane coincides with the meridian. (B.Sc.)

(6) Define the ampere, and find the direction and intensity of the force on a circular coil of n turns wound close together through which a current of A amperes is flowing due to a magnet whose poles lie on the axis of the coil. (B.Sc.)

(7) Describe the construction of the moving coil galvanometer, and explain how, with the addition of a shunt, it can be used as an ammeter for large currents. (B.Sc.)

(8) A small galvanometer needle swinging freely under the earth's force alone makes three oscillations per second. The control magnet of the galvanometer is replaced and adjusted until the needle makes one oscillation in 3 seconds. A millimetre scale is fitted at a distance of a metre from the mirror. Find the intensity of the magnetic field at the centre of the galvanometer due to unit current, if a current of 10^{-4} ampere produce a deflection of 20 divisions, the value of H being 0·172. (B.Sc.)

(9) Two single-needle galvanometers, A and B, are made geometrically similar in all respects, the linear dimensions of A being n times those of B. The magnetic fields are so adjusted by exterior

magnets that the periods of vibration of the needles are equal. What is the ratio of the angles of deflection when the same steady current is sent through each? (B.Sc. Hons.)

(10) Find an expression for the magnetic potential of a circular coil of n turns of mean radius r, carrying a current of c amperes, at a point situated on the axis at a distance x from the centre of the coil. Deduce from this an expression for the force exerted by the coil on a magnet pole of m units placed at the point in question. At what point on the axis is the rate of change of this force a maximum for a given small change of x? A coil of 40 turns like that of a tangent galvanometer of mean radius 12 cm. carries a current of 10 amperes. What is the intensity of the field in C.G.S. units which it produces at a point 50 cm. along its axis from the centre of the coil? (B.Sc. Hons.)

(11) Two circular coils of wire are placed with their planes parallel to each other at a distance 5 cm. apart. The larger coil has a radius of 10 cm. and 30 turns of wire, the smaller a radius of 2 cm. and 20 turns of wire. Calculate approximately in grammes weight the mechanical force between the coils when a current of 1 ampere is passed through both, proving any formula used. Show how the principles thus illustrated are applied practically in the ampere balance. (B.Sc. Hons.)

(12) Explain how to determine the absolute value of the current which produces a given deflection on the double coil tangent galvanometer. (D.Sc.)

(13) The wire of a galvanometer has the same resistance as that used for a telegraph line. At 100 miles from the battery the needle is deflected through a certain angle, the wire in the coil making 10 turns. The galvanometer being removed to 250 miles from the battery and the number of turns in the coil doubled, the deflection is the same as before. Prove that the internal resistance of the battery is equal to 50 miles of wire, and that to produce the same deflection at 610 miles the wire in the coil must make 44 revolutions. (Trinity Hall.)

(14) Assuming that the action of a magnetic pole on an element of an electric circuit varies as the sine of the angle between the distance of the pole from the element and the direction of the element directly, and as the square of that distance inversely, and is perpendicular to the plane through the pole and the element, prove that the action of such a pole on any closed circuit is in all respects the same as it would be upon an infinitely thin magnetic shell bounded by the circuit. (Tripos.)

(15) A rectangle of wire carrying a current is hung up in a given vertical plane, and one of its vertical sides is capable of sliding along the two horizontal sides Find the force necessary to keep that side from sliding. (Trinity College.)

CHAPTER XIII.

HEATING EFFECTS OF CURRENTS.

191. The Laws of Heating Effects of Currents: Theoretically.—The *unit of heat* is the **calorie,** defined as *the quantity of heat required to raise the temperature of one gramme of pure water from $9\frac{1}{2}°$ C. to $10\frac{1}{2}°$ C.*, but for practical purposes it may be taken as *the heat required to raise the temperature of one gramme of pure water one degree Centigrade.* Heat is a form of energy, and the **mechanical equivalent of heat** (J) may be defined, in this connection, as *the number of units of energy which is equal to one calorie*; experiment has proved it to be $4{\cdot}2 \times 10^7$ ergs; thus

$$1 \text{ calorie} = (4{\cdot}2 \times 10^7) \text{ ergs} = 4{\cdot}2 \text{ joules,}$$
$$\therefore\ 1 \text{ joule} = {\cdot}24 \text{ calorie.}$$

It is in order to have the statement 1 calorie $= (4{\cdot}2 \times 10^7)$ ergs *exact* (Rowland's work on J) that the calorie is defined above for a temperature range of $9\frac{1}{2}°$ C. to $10\frac{1}{2}°$ C.

We have seen that whenever a potential difference exists between two points in a circuit an *energy transformation* occurs between them. In the case of a battery the poles of which are joined by an ordinary conductor, the whole of the energy supplied by the former is transformed into heat partly in the battery and partly in the conductor. Should the external circuit contain a voltameter or motor (and therefore, as we shall see later, a back E.M.F.) a certain amount of the energy is utilised in chemical or mechanical work, the balance again appearing, however, as heat in the circuit. The transformation of electrical energy into heat always takes place when a current flows in a conductor; the laws relating to the development may now be established with exactness as follows:—

Consider a current-carrying wire; let E denote the P.D. and I the current, both in electromagnetic units, and let t denote the time in seconds during which the current flows; then (Art. 153)—

E = energy (in ergs) appearing as heat when unit electromagnetic current flows for one second.

$\therefore EI$ = energy (in ergs) appearing as heat when I electromagnetic units flow for one second.

And EIt = energy (in ergs) appearing as heat when I electromagnetic units flow for t seconds.

Now the mechanical equivalent of heat is $(4{\cdot}2 \times 10^7)$ ergs per calorie; hence if H be the heat in calories in the case above, $(4{\cdot}2 \times 10^7)H$ ergs will represent the total energy appearing as heat in the conductor. Clearly, then,

$$(4{\cdot}2 \times 10^7)H = EIt; \ \therefore H = {\cdot}24\frac{EIt}{10^7}.$$

Hence, when a current of I e.m. units flows for t seconds between two points of a conductor where the P.D. is E e.m. units, the heat produced is given by the expressions—

$$\textbf{Heat in calories} = {\cdot}\mathbf{24}\frac{\boldsymbol{EIt}}{\mathbf{10^7}} = {\cdot}\mathbf{24}\frac{\boldsymbol{I^2Rt}}{\mathbf{10^7}} = {\cdot}\mathbf{24}\frac{\boldsymbol{E^2t}}{\boldsymbol{R}.\mathbf{10^7}},$$

$$\textbf{Heat in ergs} = \boldsymbol{EIt} = \boldsymbol{I^2Rt} = \frac{\boldsymbol{E^2}}{\boldsymbol{R}}\boldsymbol{t},$$

R being the resistance of the conductor in e.m. units.

If the P.D. be E volts and the current I amperes, then, since E volts are equal to $(E \times 10^8)$ electromagnetic units and I amperes are equal to $(I \times \frac{1}{10})$ electromagnetic units, the first expression above becomes

$$H = {\cdot}24\frac{E \times 10^8 \times \mathrm{I} \times \frac{1}{10} \times t}{10^7} = {\cdot}24\ EIt.$$

Hence, when a current of I amperes flows for t seconds between two points of a conductor where the P.D. is E volts, the heat produced, in calories, is given by the expressions—

$$\textbf{Heat in calories} = {\cdot}\mathbf{24}\,\boldsymbol{EIt} = {\cdot}\mathbf{24}\,\boldsymbol{I^2Rt} = {\cdot}\mathbf{24}\frac{\boldsymbol{E^2}}{\boldsymbol{R}}\boldsymbol{t},$$

R being the resistance of the conductor in ohms. Again, since one joule is equal to ·24 calorie, the heat produced in joules is given by the expressions—

$$\textbf{Heat in joules} = \boldsymbol{EIt} = \boldsymbol{I^2Rt} = \frac{\boldsymbol{E^2}}{\boldsymbol{R}}\boldsymbol{t}.$$

Any of the above may be taken as the mathematical statement of Joule's law relating to the heating effects of a current; it is customary to take the second, which embodies the facts that—

(1) The heat produced is proportional to the square of the current strength.

(2) The heat produced is proportional to the resistance.

(3) The heat produced is proportional to the time the current flows.

192. The Laws of Heating Effects of Currents: Experimentally.

—The laws given in the preceding section may be verified experimentally as follows :—

Exp. 1. *To verify that the heat produced is proportional to the square of the current.*—Arrange apparatus as shown in Fig 357, where B is a battery of about 8 volts, R a variable resistance, G an ammeter or galvanometer of known reduction factor, and C a calorimeter. The latter (Fig. 357a) is a copper pot containing water, in which is immersed a spiral of German silver wire, the ends of which are attached to two stout copper leads, a, b; through the stopper also pass a stirrer S and a thermometer T, graduated, say, in fifths or tenths of a degree. The calorimeter C should be hung by threads inside a larger copper pot, and the inner surface of the latter and the outer surface of C should be smooth and polished. (In Fig. 357a C is drawn as if transparent, to show the inside.)

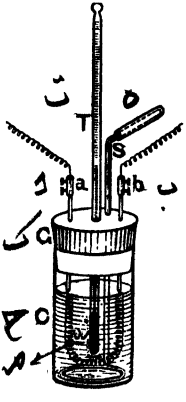


Fig. 357a.

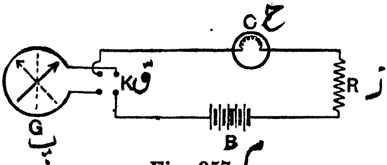


Fig. 357.

Note the temperature of the water, and then pass a current I_1 for, say, 5 minutes, stirring the water and keeping the current constant; note the rise in temperature of the water, say $\theta_1°$. Now cool the water down to approximately the same temperature as it had at the beginning, and repeat the experiment with a different current, I_2, flowing for 5 minutes; note the rise in temperature, $\theta_2°$. Since the mass of water and other conditions are constant, the rise in temperature is proportional to the heat produced, and it will be found that

$$\frac{\theta_1}{\theta_2} = \frac{I_1^2}{I_2^2},$$

i.e. the heat is proportional to the square of the current.

Exp. 2. *To verify that the heat produced by a current is proportional to the resistance.*—Arrange two Joule's calorimeters, A and B, and a battery in series. The calorimeters must be exactly alike, and contain equal quantities of water, but the heating spirals must be of different lengths, so that the resistance of the spiral in A is, say, R_1 ohms, and that in B, say, R_2 ohms. Pass the current for 5 minutes, gently stirring the water in each calorimeter. Note the rise in temperature of A ($\theta_1°$) and of B ($\theta_2°$). The same current has passed for the same time through both, and it will be found that

$$\frac{\theta_1}{\theta_2} = \frac{R_1}{R_2},$$

i.e. the heat produced is proportional to the resistance.

An extension of Exp. 1 enables the value of J to be determined experimentally.

Exp. 3. *To determine the Mechanical Equivalent of Heat.*—Arrange as in Exp. 1, placing a known mass (M grm.) of water in the calorimeter; the current must be kept constant throughout the experiment. If I be the current in e.m. units, R the resistance of the coil in e m. units, and t the time in seconds the current passes,

$$\text{Heat in ergs} = I^2Rt, \quad \therefore \text{ Heat in calories} = \frac{I^2Rt}{J},$$

where J is the mechanical equivalent (ergs per calorie).

Again, if w be the water equivalent of the calorimeter, heater, stirrer, and thermometer, and $\theta°$ the rise in temperature—

$$\text{Heat in calories} = (M + w)\theta,$$

$$\therefore \frac{I^2Rt}{J} = (M + w)\theta, \quad \textit{i.e. } J = \frac{I^2Rt}{(M + w)\theta}.$$

Many precautions and corrections (radiation, etc.) are necessary for an accurate determination.

193. Rise in Temperature of a Wire due to the Passage of a Current.—Joule's law refers to the *heat produced* in a conductor, and though the *rise in temperature* certainly depends on the amount of heat developed, sundry other factors exert an influence. It is obvious that were equal amounts of heat communicated to two wires *A* and *B*, of the same material, the former being short and thin, the latter long and thick, the rise in temperature of *A* would be greater than that of *B*, since the quantity of matter to be raised in temperature is smaller in *A* than in *B*. Again, each wire would be losing heat by radiation from its surface; experiment shows that, other things being the same, the greater the surface area the greater is this loss, and, further, the radiation is affected by the nature of the surrounding gas and its pressure, and by the character of the surface, black bodies, for example, emitting radiation more freely than white or transparent ones. Yet another factor, the capacity for heat of the material (*i.e.* the heat required to raise its temperature by unity), exerts an influence; thus, if the same current flows through equal pieces of platinum and copper the heat developed in the former is about seven times that in the latter, since the specific resistance of platinum is about seven times that of copper (annealed in both cases), but the temperature elevation of the platinum is *more than* seven times that of the other, for its capacity for heat is only about three-fifths that of an equal volume of copper.

Consider now a wire through which a current is flowing. The production of heat is accompanied by a rise in temperature until finally a steady condition is reached when *the heat lost per second* (*by radiation, conduction, and convection*) *is exactly equal to the heat gained per second.* By Newton's Law of Cooling, if T° C. be the elevation of temperature of the wire above the enclosure, the heat lost per second is proportional to T°, and experiment proves it to be proportional also to the surface area of the wire. If d cm. be the diameter of the wire its circumference is πd cm., and if l cm. be its length its surface area is πdl sq. cm.; hence—

Heat lost per second $\propto \pi dlT$,
,, ,, ,, $= a\pi dlT$ calories,

where a denotes the "emissivity" of the material, and may be defined as the heat (in calories) radiated in one second from unit area when the temperature difference between the body and the enclosure is 1° C.

Again, if I be the current in amperes and R the resistance of the wire in ohms,

$$\text{Heat gained per second} = \cdot 24 I^2 R \text{ calories.}$$

Hence—

$$a\pi d l T = \cdot 24 I^2 R,$$

i.e.

$$a\pi d l T = \cdot 24 I^2 S \frac{l}{\cdot 7854 d^2},$$

where S is the specific resistance of the material;

$$\therefore\ T = \cdot 097 \frac{S}{a} \frac{I^2}{d^3} \qquad \text{(1)}$$

and

$$I = \sqrt{\frac{T\,.\,a}{09\,7S}}\, d^{\frac{3}{2}} \qquad \text{(2)}$$

Thus—

(1) With a given current the elevation of temperature does not depend on the length of the wire.

(2) For the same current the elevation of temperature is directly proportional to the specific resistance and inversely proportional to the cube of the diameter (hence the high temperature of very thin wires).

(3) The elevation of temperature is proportional to the square of the current strength.

194. Wherever Work is done in the External Circuit other than the Generation of Heat a Back E.M.F. is developed.—Let E be the E.M.F. of a battery, R the total resistance (external and internal), and I the current in the circuit. By the action of the battery a quantity of electricity I is, in one second, raised in potential to the extent E, and therefore the work given out by the battery per second is EI; the battery supplies this energy per second at the expense of the chemical energy of the materials consumed in it.

Now the energy appearing per second as heat in the circuit is I^2R, and if w represents the energy per second devoted to *other* work (say chemical or mechanical) we evidently have

$$EI = I^2\mathrm{R} + w,$$

i.e.

$$E = IR + \frac{w}{I}.$$

But IR, the product of the current in the circuit and the resistance of the circuit, is, by Ohm's Law, the *effective* E.M.F. in the circuit; denoting this by E'—

$$E = E' + \frac{w}{I},$$

i.e.

$$E' = E - \frac{w}{I} = E - e.$$

Thus in cases where there is work other than the generation of heat, the effective or resultant E.M.F. (E') is less than the actual E.M.F. (E) by an amount e, where $e = w/I$; in other words, there is a *back E.M.F.*, e, and the actual current is given by the expression

$$I = \frac{E - e}{R}.$$

If w is zero, then $E = IR$, i.e. $I = E/R$.

Again, since $e = w/I$, $w = eI$; hence we have the result that *the power expended in additional work* (i.e. *other than the generation of heat*, e.g. *chemical decomposition*) *is given by the product of the back E.M.F. and the current strength.*

Example. *A copper sulphate vat has a resistance of* ·014 *ohm and a polarisation E.M.F. of* ·3 *volt. The total current required is* 1,000 *amperes. Find the total watts supplied to the vat.*

Watts spent in heat in the vat $= I^2r = (1000^2 \times {\cdot}014)$

Watts expended in chemical work $= eI = ({\cdot}3 \times 1000)$

$\therefore$ Total watts supplied to the vat $= (1000^2 \times {\cdot}014) + ({\cdot}3 \times 1000)$

$= 14300$ watts.

In dealing with the conservation of energy in a circuit there are thus three essential points to take into account. These are—(1) the work done in the battery, (2) the work done (if any) in the external circuit, and (3) the energy of the current. Of these the first—the chemical work done in the battery—is the source of energy in the circuit and gives rise to the other two, which are, therefore, together equivalent to it. The chemical work done in the battery per second is really the mechanical equivalent of the heat which would be generated by the total chemical action going on in the battery in that time, and is equal to EI; of this a portion eI is spent per second in additional work, while the remainder $EI - eI = I(E - e) = I \, . \, IR = I^2R$ is spent in maintaining the current I in the circuit, and appears as heat per second in the circuit. Thus *all the energy of the current* is dissipated as heat in the circuit, though *all the work done in the battery* may not be spent in producing current. Clearly if, say, the chemical work to be done in a voltameter is equal to or greater than that which can be done in the battery, then no current will flow. This explains why a single Daniell's cell cannot decompose water: the energy necessary to decompose water into oxygen and hydrogen is greater than that supplied by the chemical action going on in the Daniell, or, in other words, the back E.M.F. due to polarisation in the voltameter is greater than the E.M.F. of a single Daniell.

195. Theorems.—Some important theorems follow direct from the matter of the preceding section :—

(1) *For any given value of additional work* (w *per second*) *there are two current values.*

With the previous notation we have

$$EI = I^2R + w,$$

i.e.

$$I^2R - EI + w = 0.$$

$$\therefore \; I = \frac{E}{2R} \pm \sqrt{\frac{E^2}{4R^2} - \frac{w}{R}}.$$

This expresses the two values of I corresponding to any one value of w.

(2) *There is a maximum rate of additional work w, and this is equal to one quarter the rate at which energy would be given out if no additional work were done.*

The expression for I may be put in the form

$$I = \frac{1}{2}\frac{E}{R} \pm \sqrt{\frac{\frac{1}{4}\frac{E^2}{R} - w}{R}}.$$

Now if w exceeds $\frac{1}{4}\frac{E^2}{R}$ the expression under the root sign becomes negative; *hence the greatest value which w can have is $\frac{1}{4}\frac{E^2}{R}$.* But $\frac{E^2}{R}$ is the rate at which energy would be given out if no additional work were done, all the energy being dissipated as heat; which proves the theorem.

It should be noted that when w has this maximum value $I = \frac{1}{2}\frac{E}{R}$, i.e. *the current is half the current that would flow if no additional work were done.*

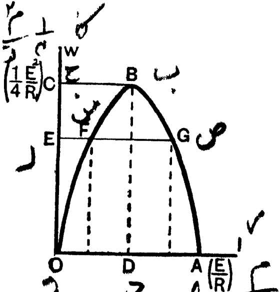


Fig. 358.

NOTE.—By taking a numerical example and working out the expression for I with varying values of w the above may be shown graphically; this is done in Fig. 358, values of w being taken as ordinates and values of I as abscissae.

When $w = 0$, $I = 0$ or $\frac{E}{R}$ $(= OA)$; when $w = OE$, $I = EF$ or EG; when $w = \frac{1}{4}\frac{E^2}{R}$ $(= OC)$, $I = \frac{1}{2}\frac{E}{R} (= CB = OD = \frac{1}{2}OA)$.

196. Battery Efficiency.—The efficiency of a battery is defined as *the ratio of the power in the external circuit to the total power developed.* If the external circuit does not contain any arrangement involving additional work (*e.g.* a voltameter) the whole of the power outside appears as heat; hence if r be the internal and R the external resistance,

$$\text{Efficiency} = \frac{I^2R}{I^2(R+r)} = \frac{R}{R+r} = \frac{\text{Terminal P.D.}}{\text{E.M.F.}}.$$

This is nearer unity (100 per cent.) the smaller the value of r.

If the battery has to do useful work—say chemical decomposition—the efficiency *of the system* refers to *the ratio of the power spent in this useful work to the total power*; hence if R be the *total* resistance and e the back E.M.F.,

$$\text{Efficiency} = \frac{eI}{EI} = \frac{EI - I^2R}{EI} = \frac{\frac{E}{R}I - I^2}{\frac{E}{R}I},$$

i.e.

$$\text{Efficiency} = \frac{e}{E} = \frac{\frac{E}{R} - I}{\frac{E}{R}}.$$

Hence—

(1) The efficiency becomes a maximum (*i.e.* unity or 100 per cent.) when the current I becomes zero; of course in this case the useful work is nil.

(2) The efficiency becomes a minimum, *i.e.* zero when $I = \frac{E}{R}$, *i.e.* when e is zero; the useful work is again nil.

(3) There is *maximum power* (w) when $I = \frac{1}{2}\frac{E}{R}$ (Art. 195, Theorem 2); *the efficiency is then* 1/2 *or* 50 *per cent.* and the back E.M.F. $e = \frac{1}{2}E$.

197. Calculation of E.M.F. from Thermo-Chemical Data.—In a voltaic cell the chemical reactions are essentially of an *exothermic* nature, that is, they are such as to produce under ordinary circumstances an evolution of energy in the form of heat. In a voltaic cell this energy is not evolved as heat, but the electrical energy evolved is the equivalent of this heat energy. In a voltameter the chemical reactions are of an *endothermic* nature, that is, they are usually accompanied by an absorption of heat energy, and the presence of a voltameter in a circuit therefore means the absorption of an amount of electrical energy from the circuit equivalent to this heat energy. From these considerations the E.M.F. of a given voltaic cell or the back E.M.F. in a voltameter can be calculated if the necessary data are known. In the following the figures are approximate only.

Case 1. *E.M.F. of Simple Cell.*—In this cell we may regard the action to consist in the oxidising of zinc which dissolves in sulphuric acid, and at the same time hydrogen is liberated ; the atomic weights of zinc and oxygen may be taken as 65 and 16 respectively, that of hydrogen being 1. Now—

Heat *evolved* by 65 grm. of zinc combining with oxygen = 85,400 calories.

Heat *evolved* by the 81 grm. of zinc oxide combining with sulphuric acid = 23,400 calories.

Heat *absorbed* when 18 grm. of water are decomposed = 69,000 calories.

∴ Total heat evolved for 65 grm. of zinc dissolved

$$= 85400 + 23400 - 69000 = 39800 \text{ calories,}$$

i.e. Total heat evolved for 1 grm. of zinc dissolved

$$= 39800/65 = 613 \text{ calories.}$$

The electro-chemical equivalent of zinc is ·00337 grm. per e.m. unit quantity ; hence the energy evolved during the consumption of this amount of zinc is $613 \times {\cdot}00337 \times 4{\cdot}2 \times 10^7$ ergs. But if E be the E.M.F. and Q the quantity of electricity in e.m. units, the work is EQ ergs; the quantity corresponding to the consumption of ·00337 grm. of zinc is, however, 1 e.m. unit ; hence

$$\begin{aligned} E &= 613 \times {\cdot}00337 \times 4{\cdot}2 \times 10^7 \text{ e.m. units} \\ &= {\cdot}9 \times 10^8 \text{ e.m. units} \\ &= {\cdot}9 \text{ volt.} \end{aligned}$$

Case 2. *E.M.F. of Daniell's Cell.*—In this case—

Heat *evolved* by 65 grm. of zinc combining with oxygen = 85400 calories.

Heat *evolved* by the 81 grm. of zinc oxide combining with sulphuric acid = 23400 calories.

Heat *absorbed* in the separation of the equivalent 79 grm. of copper oxide = 19045 calories.

Heat *absorbed* in the separation of the equivalent 63 grm. of copper from the oxide = 38260 calories.

∴ Total heat evolved for 1 grm. of zinc dissolved

$$= \frac{85400 + 23400 - 19045 - 38260}{65} = 792.$$

Hence

$$\begin{aligned} E &= 792 \times {\cdot}00337 \times {\cdot}42 \times 10^8 \text{ e.m. units} \\ &= 1{\cdot}12 \times 10^8 \text{ e.m. units} = 1{\cdot}12 \text{ volts.} \end{aligned}$$

Case 3. *The General Formula.*—Let h denote the heat evolved in the particular cell while 1 grm. of zinc is dissolved, z the electrochemical equivalent of zinc (per e.m. unit quantity), and J the mechanical equivalent of heat in ergs; then, from the preceding,

$$E = hzJ \text{ e.m. units} = \frac{hzJ}{10^8} \text{ volts.}$$

Case 4. *Back E.M.F. in Electrolysis of Water.*—We have to find the E.M.F. of hydrogen tending to recombine with oxygen. Now heat evolved when 1 grm. of hydrogen combines with oxygen $= 34000 = h$; z for hydrogen $= \cdot000104$; hence

$$\begin{aligned} e &= 34000 \times \cdot000104 \times 4\cdot2 \times 10^7 \text{ e.m. units} \\ &= 1\cdot49 \times 10^8 \text{ e.m. units} = 1\cdot49 \text{ volts.} \end{aligned}$$

The preceding calculations on the E.M.F.'s of cells are incomplete, for they ignore the fact that the E.M.F. of a cell depends on the temperature; the true formula (reversible cells) is

$$E = hzJ + T\frac{dE}{dT} \text{ (Art. 206),}$$

where T is the absolute temperature of the cell.

The "temperature coefficient" $\frac{dE}{dT}$ for a Daniell's cell is nearly zero, however; hence it is that the E.M.F. calculated in Case 2 agrees closely with the actual value.

198. Glow Lamps. General Remarks.—Allied to the heating effects of a current and the consequent rise in temperature of the conductor is the artificial production of light by electrical means. Investigation showed carbon to be a suitable conductor; but carbon *in air* is readily burnt if heated to the extent necessary; hence in one type of commercial incandescent lamp carbon in the form of a fine filament is heated in a globe from which the air has been exhausted.

The *efficiency* of a lamp is defined as *the ratio of the candle-power of the lamp to the watts absorbed.* The average power consumption of most of the carbon filament lamps in use is about 4 watts per candle, so that the efficiency is about ·25 candle per watt.

The main drawbacks to the carbon filament lamp are its large power consumption and the fact (associated with its negative temperature coefficient) that it is sensitive to changes in voltage, and many attempts have been made to improve upon it; as a result the **Nernst** lamp appeared in 1897, the **Osmium** lamp in 1902, and the **Tantalum** lamp in 1905 as commercial articles.

In the Nernst lamp the "glower" consists of a rod of zirconia mixed with the oxides of thorium, yttrium, and erbium; it is an insulator at ordinary temperatures, but becomes a conductor when its temperature is raised; hence the lamp is provided with a heating arrangement which is afterwards automatically cut out of circuit; this lamp "works" in air and absorbs about 1·3 watts per candle. The filaments of the osmium lamp were made of metallic osmium and the power consumption was about 1·6 watts per candle. The tantalum lamp has a filament of pure drawn tantalum wire and its power consumption is of the order 1·5 to 2 watts per candle. Metallic filament lamps of later introduction have filaments of **tungsten** and absorb from 1·1 to 1·3 watts per candle; in nitrogen they absorb ·5 watt per candle.

199. The Electric Arc. General Remarks.—If two carbon rods in contact end to end form part of a suitable continuous current circuit, and if when the current is flowing the carbons be drawn apart to a distance of $\frac{3}{16}$ or $\frac{1}{4}$ inch, a luminous "arc" will be formed between them, the arc constituting a conducting path from one carbon to the other. After a time the ends of the carbons become luminous, and more so than the arc; in fact, about 85 per cent. of the light is due to the positive carbon (*i.e.* the one joined to the positive lead or positive of generator), 10 per cent. to the negative carbon, and 5 per cent. to the arc. The positive carbon assumes a crater-like form as indicated in Fig. 359, the temperature of which is from 3,500° to 4,000° C.; the end of the negative becomes gradually pointed, incandescent matter being carried to it from the positive, and its temperature is about 2,500° C. The rate of consumption of the positive carbon is about twice that of the negative; hence for the former carbons about twice as thick are employed.

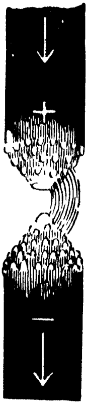


Fig. 359.

Experiment shows that in the case of continuous current arcs a P.D. of about 44 volts is necessary; they will not act at all with a pressure below 35 volts, and a frequent allowance is 50 volts for a 10-ampere arc. A considerable P.D. is found to exist between the positive carbon and a point slightly below the crater, and in comparison a small P.D. exists between this point and the negative carbon. To account for this it has been suggested that the volatilisation of the carbon in the crater results in the establishment of a back pressure of 35 to 38·9 volts in a manner similar to the back pressure consequent upon the separation of the ions in electrolysis, the result being, of course, that the applied pressure has to overcome this in addition to the ohmic resistance (of the order $\frac{1}{5}$ to $\frac{1}{2}$ ohm) of the arc; but although it is customary to speak of the "back E.M.F. of the arc," the question as to whether that is the correct explanation or not is still unsettled, the balance of opinion being rather to the effect that it is not.

The P.D. necessary to maintain an arc has been found by Mrs. Ayrton to be given by the following relation—

$$E = \left\{ a + bL + \frac{d + eL}{I} \right\} \text{ volts,}$$

where a, b, d, and e are constants, L the length of the arc in millimetres, and I the current in amperes; the values of the constants were found to be as follows:—

$$a = 38{\cdot}9,\ b = 2{\cdot}07,\ d = 11{\cdot}7,\ e = 10{\cdot}5.$$

Thus the P.D. for a 10-ampere arc of 5 mm. length is

$$E = \left\{ 38{\cdot}9 + (2{\cdot}07 \times 5) + \frac{11{\cdot}7 + (10{\cdot}5 \times 5)}{10} \right\} = 55{\cdot}67 \text{ volts.}$$

From the relation given above it follows that: (1) An increase in the current causes a decrease in the voltage if the length of the arc remains the same; (2) a decrease in the length of the arc causes a decrease in the voltage if the current remains the same. In consequence of the very high temperature the candle-power of an arc is high (of the order of 1,000); arcs absorb about ·8 watt per candle, and have therefore an efficiency of the order 1·25 candles per watt. A more perfect crater is formed if a "cored" carbon—*i.e.* one fitted with a core of softer carbon—be used as positive.

An *alternating current arc* requires about 1·12 watts per candle and a minimum E.M.F. of about 30 volts; the carbons (both cored) are equally consumed, equal in diameter, and both become pointed. *An enclosed arc* is one formed in a compartment from which the air is more or less excluded; the E.M.F. is about 80 volts and the power consumption about 1·5 watts per candle. In a *flame arc* special carbons are used and the arc is formed in a magnetic field which spreads it out into a fan-like shape. *Mercury vapour* lamps are vacuum tubes with mercury kathodes, the light being due to the incandescence of mercury vapour in the form of a luminous column filling the tube. (See *Technical Electricity*, Chapter XV.)

200. Hot-wire Ammeters and Voltmeters.—Fig. 360 gives a diagrammatic view of the hot-wire ammeter made by Messrs. Johnson and Phillips. The brass plate *B* carries a clamp *D* to which one end of the measuring wire *W* of platinum silver is attached. The other end of the wire is joined to a pillar *E* connected to the plate *B*. Near the middle of *W* a wire *P* of phosphor bronze is attached, its other end being fixed to the insulated pillar *F*. Not far from the centre of *P* a cocoon silk fibre *Z* is attached; this passes round a metal pulley fixed on a pivoted steel spindle, and is finally attached to a flat steel spring *S*. The whole is thus subjected to tension, and any slackening of *W* is immediately taken up by *S*, resulting in a rotation of the pulley, the spindle of which carries the pointer. Mounted on the spindle is an aluminium disc *A*, which, when the pointer is deflected, moves between the poles of the magnet *M*, thus rendering the instrument dead beat. *C* is a copper strip in conducting communication with the middle of *W* by means of a fine spring; this strip is connected to one terminal t_1, the other terminal t_2 being joined to the brass plate *B*. The terminals are also connected by a shunt T_1T_2 of constantin. When a current flows the wire *W* is heated and expands and the sag is taken up by *S*, thereby moving the pointer over the scale. The voltmeters are similar, but are fitted with series resistances instead of shunts.

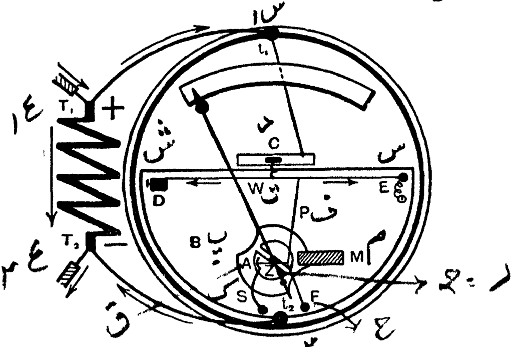


Fig. 360.

Exercises XIII.

Section B.

(1) Two circuits whose resistances are respectively 1 ohm and 10 ohms are arranged in parallel. Compare the amount of current passing through each of these circuits with that through the battery. Compare also the amount of heat developed in the same time in the two circuits. (B.E.)

(2) A wire of resistance r connects A and B, two points in a circuit, the resistance of the remainder of which is R. If, without any other change being made, A and B are also connected by $(n-1)$ other wires, the resistance of each of which is r, show that the heat produced in the n wires will be greater or less than that produced originally in the first wire according as r is greater or less than $R\sqrt{n}$. (B.E.)

(3) The E.M.F. of a battery is 18 volts and its resistance 3 ohms. The P.D. between its poles when they are joined by a wire A is 15 volts, and falls to 12 volts when A is replaced by another wire B. Compare the resistances of A and B, and the amounts of heat developed in them in equal times. (B.E.)

(4) Heat is generated in a wire of resistance 10 ohms which forms part of a circuit containing a battery and having a total resistance of 25 ohms. Find the resistance of a shunt which when applied to the extremities of the wire will cause the heat generated in it per second to diminish in the ratio 4 to 1, assuming that the resistance of the wire does not alter sensibly with its temperature. (B.E.)

(5) Determine the final temperature of a copper wire ·166 cm. in diameter through which a current of 10 amperes is flowing. Specific resistance of copper = 1·65 microhms per centimetre cube, emissivity of copper = ·00025, temperature of room = 15° C.

Section C.

(1) If a cell has an E.M.F. of 1·08 volts and 0·5 ohm internal resistance, and if the terminals are connected by two wires in parallel of 1 ohm and 2 ohms resistance respectively, what is the current in each, and what is the ratio of the heats developed in each? (Inter. B.Sc.)

(2) A battery of 5 cells, each of which has an E.M.F. of 2 volts and an internal resistance of ·03 ohm, is connected (a) in series and (b) in parallel, the current passing through a wire of resistance 0·1 ohm. Calculate the heat developed in the wire in each case. (Inter. B.Sc.)

(3) A jacketed vessel contains a liquid in which a spiral of wire is immersed. An E.M.F. of 20 volts is applied to the ends of the

spiral, and a current of 2 amperes passes through it. Five grammes of the liquid are boiled away every minute after steady boiling has begun. What is the latent heat of vaporisation of the liquid? (Inter. B.Sc. Hons.)

(4) The heat of combustion of hydrogen and oxygen to water is 34,200 water gramme units for each gramme of hydrogen burnt. A C.G.S. unit current decomposes in one second 0·000945 gm. of water. The mechanical equivalent of heat being $4{\cdot}2 \times 10^7$, find in volts the smallest E.M.F. which can decompose water. (B.Sc.)

(5) A normal Daniell's cell has an E.M.F. of 1·07 volt and resistance 2 ohms. Its terminals are connected by two wires in parallel of 3 and 4 ohms. Assuming that the electrochemical equivalent of copper is ·000328 gramme per coulomb, calculate the weight of the copper deposited in the cell, and also the heat developed (*a*) in the cell, (*b*) in each of the wires, during one hour of working of the cell. (B.Sc.)

(6) If the chemical action going on in a battery results in the formation of 500 calories of heat for every gramme of metal dissolved, and if the quantity of electricity set in motion by the solution of a centigramme of metal is 0·5 C.G.S. unit, what is the E.M.F. of the battery (assumed to be calculable from the above data)? (B.Sc.)

(7) The reactions within a cell generate electrical energy at the rate of 1 watt per ampere; a current of 10 amperes is being generated, with the result that energy is dissipated within the cell in the form of heat at the rate of 1 watt. What is the difference of potential between the terminals of the cell? also what is the internal resistance of the cell? (B.Sc.)

(8) The poles of a cell whose E.M.F. is 1·2 volts and internal resistance 0·4 ohm are connected by a copper wire 10 metres long and 1 square mm. in cross-section. Determine the amount of heat developed per minute. (Specific resistance of copper $= 1{\cdot}6 \times 10^{-6}$ ohms per centimetre cube.) (B.Sc.)

(9) Show that when a battery of constant electromotive force is employed to drive an electromagnetic engine the greatest efficiency will be secured if the speed of the engine be such that the current is indefinitely small, but that the greatest horse-power will be obtained if the current be reduced to one-half the value it has when the engine is at rest. (D.Sc.)

(10) If p grammes of zinc be consumed in a galvanic battery per unit time when no external work is done by the current, and if q grammes be consumed when w units of external work are done per unit time, prove that

$$W = Eq\left(1 - \frac{q}{p}\right),$$

where E is the energy developed by the battery per gramme of zinc consumed. (Tripos.)

CHAPTER XIV.

CHEMICAL EFFECTS OF CURRENTS.

201. Faraday's Laws of Electrolysis.—These simple laws are two in number :—

(1) *The amount of chemical decomposition which takes place in a given time in a cell or voltameter is proportional to the total quantity of electricity which passes in that time;* thus if w denote the mass of the particular ion liberated in time t seconds, and Q the quantity of electricity which has passed—

$$w \propto Q \propto It$$
$$\text{or } w = zIt,$$

in which I is the current and z the electrochemical equivalent (see Art. 151).

(2) *If the same current flows through several electrolytes the masses of the ions liberated are proportional to their chemical equivalents* (Appendix, 2). Thus, if the current from 20 Daniell's cells in series be passed through four voltameters in series containing respectively $CuSO_4$, H_2O, $AgNO_3$, and HCl, it will be found that while 1 gramme of hydrogen is liberated 31·5 grammes of copper, 7·94 grammes of oxygen, 107·12 grammes of silver, and 35·18 grammes of chlorine will be liberated in the voltameters, 31·5 grammes of copper will be deposited in *each* cell, and 32·45 grammes of zinc will be used up in *each* cell of the battery. If the cells be arranged in two rows in parallel, 10 cells in series per row, only half the external current traverses each cell, and therefore the chemical action per cell will be half the chemical action in the voltameters; thus, while 1 gramme of hydrogen is liberated in the voltameter, $\frac{32{\cdot}45}{2}$ grammes of zinc will be used up per cell.

Taking the chemical equivalent of hydrogen as 1, the electrochemical equivalent of any ion can be obtained by multiplying the electrochemical equivalent of hydrogen (·0001044 grm. per e.m. unit quantity) by the chemical equivalent of the ion. Further, it must be remembered that the chemical equivalent of an element is its atomic weight divided by its valency.

The figures given above are approximate chemical equivalents, taking $H = 1$. In chemical science it is now usual to refer atomic weights to that of oxygen taken as 16, in which case the atomic weight of H is 1·008 (Appendix, 2); this leads to certain confusion, but in the details which follow the more recent data are employed.

A *gramme-equivalent* of a substance is a quantity in grammes equal to the chemical equivalent. Thus the atomic weight of silver is 107·88 and its valency is 1; hence the chemical equivalent is 107·88 and the gramme-equivalent of silver is therefore 107·88 grammes. The atomic weight of zinc is 65·37 and its valency is 2; hence the chemical equivalent is 32·68 and the gramme-equivalent of zinc is 32·68 grammes. Similarly a *gramme-atom* of an element is a quantity in grammes equal to its atomic weight, and a *gramme-molecule* of a substance is a quantity in grammes equal to its molecular weight.

Since 1 e.m. unit quantity liberates ·0001044 grm. of hydrogen, 1·008/·0001044, *i.e.* 9,650 e.m. units, will liberate 1·008 grm.; and this same quantity will, by the second law above, liberate 107·88 grm. of silver, 32·68 grm. of zinc, and so on. *Hence 9,650 e.m. units of electricity will liberate one gramme-equivalent of any substance.*

Modern theory on electrolysis indicates that the atoms are themselves the carriers of the charges. Now the number of atoms in one gramme-equivalent of hydrogen is about $6{\cdot}16 \times 10^{23}$; hence the charge carried by a hydrogen atom (ion) in electrolysis is $9650/6{\cdot}16 \times 10^{23}$, *i.e.* $1{\cdot}57 \times 10^{-20}$ e.m. units. Further, since the gramme-equivalent of all monovalent substances contains the same number of atoms, this will be the charge carried by all monovalent ions, *i.e.* charge carried by a monovalent ion $= 1{\cdot}57 \times 10^{-20}$ e.m. units $= 1{\cdot}57 \times 10^{-19}$ coulombs $= 4{\cdot}71 \times 10^{-10}$ e.s. units.

Again, since the gramme-equivalent of a divalent substance contains *half* the number of atoms indicated above, it follows that the charge carried by a divalent ion is *twice* that carried by a monovalent ion, the charge carried by a trivalent ion is *three* times the above, and so on.

The charge carried by the monovalent ion is the natural "atom" of electricity already referred to (vol. i, 176; ii, 34): it is identical in magnitude with the free negative charges known as "electrons"; as will be seen later, recent observations on this electronic charge give results ranging from $4{\cdot}65 \times 10^{-10}$ to $4{\cdot}77 \times 10^{-10}$ e.s. units.

Example. *What is Faraday's Law regarding electrodeposition? How much caustic soda is produced per ampere hour, and how much lead, silver, and mercury (from mercurous nitrate) would be deposited per ampere hour? One coulomb evolves, say,* ·0104 *milligramme of hydrogen, and the atomic weights of sodium, lead, silver, and mercury are respectively* 23, 207, 108, *and* 200. (C. and G.)

(*a*) See above.

(*b*) Valency of sodium = 1 ∴ its chemical equivalent = 23.
,, lead = 2 ∴ ,, ,, ,, = 103·5
,, silver = 1 ∴ ,, ,, ,, = 108.
,, mercury(ous) = 1 ∴ ,, ,, ,, = 200.

Now 1 coulomb liberates ·0104 m.g. of H, ∴ 1 ampere hour (3600 coulombs) liberates (·0104 × 3600) = 37·44 m.g. of H.

Hence by Law 2—

(1) Amount of sodium liberated
per ampere hour = **37·44** × **23** = **861·12 m.g.**

(2) Amount of lead liberated
per ampere hour = **37·44** × **103·5** = **3875** ,,

(3) Amount of silver liberated
per ampere hour = **37·44** × **108** = **4043·52** ,,

(4) Amount of mercury liberated
per ampere hour = **37·44** × **200** = **7488** ,,

Caustic soda = NaHO.

Atomic weights of Na, H, and O are respectively 23, 1, and 16 (say); hence in 40 grm. of caustic soda there are 23 grm. of sodium, 1 grm. of hydrogen, and 16 grm. of oxygen.

In the problem the 861·12 m.g. (·86112 grm.) of sodium will combine with water, forming caustic soda; clearly then

23 grm. of sodium form 40 grm. of caustic soda;
∴ 1 grm. ,, forms $\frac{40}{23}$,, ,, ,,
and ·86112 grm. ,, ,, ($\frac{40}{23}$ × ·86112) = **1·497 grm. of caustic soda—say 1·5 grm.**

202. Back E.M.F. in Electrolysis.—When an electrolyte is decomposed by the passage of a current, the separated ions possess potential energy and tend to reunite, setting up a back E.M.F. (Art. 194); thus the back E.M.F. in a water voltameter is about 1·5 volts and in a copper sulphate voltameter *fitted with platinum electrodes* about 1·17 volts.

Consider, however, a copper sulphate voltameter fitted with copper electrodes, *the latter being in such a condition that they are readily acted on by the sulphion* SO_4; the chemical actions are:—

(Kathode) $CuSO_4 = Cu + SO$ .... (1)
(Anode) $Cu + SO_4 = CuSO_4$ ....(2)

Equation (2) is similar to that of Art. 142. As in that case *energy is liberated*, and further, as in the simple cell, *a forward E.M.F. is the result.* Equation (1) is the converse of (2); it represents a condition in which *energy is absorbed*, and, like the polarisation of the high-potential plate in the simple cell, *a back E.M.F. is the result.* These E.M.F.'s are equal, and cancel each other; hence *when copper sulphate is subjected to electrolysis, the electrodes being copper plates which can readily be acted on by the* SO_4, *there is no back E.M.F. of any importance.* The only E.M.F. which need be mentioned in this case is a very slight one, due to a difference in concentration in the electrolyte, the copper solution becoming slightly stronger near the anode, and weaker near the kathode as electrolysis continues.

It was shown in Art. 194 that the power spent in chemical decomposition is given by the product of the back E.M.F. and the current strength.

203. Electrolytic Conduction.—The modern theory of electrolytic conduction is based upon the theory of dissociation. In an electrolyte the dissolved substance is supposed to be either partially or completely dissociated into its constituent *ions* (Art. 143). The more dilute the solution the more complete is the dissociation or ionisation, so that in a very dilute electrolytic solution practically all the molecules of the dissolved substance dissociate into free separate ions. Each of these dissociated ions carries its own appropriate charge of electricity, and therefore the chemical and general properties of an ion differ essentially from those of the same atom or group of atoms when free of charge. Thus in a dilute solution of KCl, the KCl molecule is dissociated into K and Cl ions. The K ion carries a positive charge and the Cl ion a negative charge, and so differ from free K and free Cl.

The behaviour and properties of a particular ion are found to be independent of the other ion with which it may be associated, so that many of the properties of an electrolytic solution are additive properties determined by the properties of the ions present. It is reasonable to predict ionisation in electrolytes, for the high specific inductive capacity of the solvent will result in a marked decrease in the electric attraction which binds together the positive and negative ions in the molecule (Art. 80).

The explanation of the process of electrolysis and electrolytic conduction on this basis is a simple one. When an external E.M.F. is applied so as to give a current through an electrolyte it is assumed that the influence of the E.M.F. is merely directive. It causes the free positively charged ions (kations) to travel in one direction through the solution and the free negatively charged ions (anions) to travel in the opposite direction. The charges carried by the ions are delivered up at the electrodes and the ions themselves liberated.

The *experimental* evidence on which this theory rests is found along a number of widely divergent paths of research:—

Kohlrausch and, later, Fitzgerald and Trouton have shown that the flow of current through an electrolyte is subject to Ohm's law. This implies that *in the electrolyte* no back E.M.F., due to molecular decomposition, is set up, and this is exactly what ought to obtain in a solution containing free dissociated ions. The gradient of potential set up in the liquid urges the positively charged ions in one direction and the negatively charged ions in the opposite direction without the development of back electromotive force. *The back E.M.F. due to polarisation takes place at the electrode. where the ions give up their charges, and not in the mass of the liquids*

The colour of electrolytic solutions is always an additive property. Thus, the colour of copper chloride molecules is yellow, a concentrated solution of the salt is green, and a dilute solution blue. This is explained by assuming that in the concentrated solution dissociation is incomplete, and the green colour results from the combination of yellow due to undissociated copper chloride molecules and blue due to the copper ions, while in the dilute solution where the dissociation is complete the colour is the blue of the copper ions.

Measurements of osmotic pressure in electrolytic solutions also support the dissociation theory. Direct determination of osmotic pressure and determinations of the associated constants, the lowering of the vapour pressure, and the lowering of the freezing point give abnormally high results for all electrolytes. These results are at once explained and become normal if it is assumed that the number of active molecules per unit volume of the solution is given, not by the number of molecules of the dissolved substance, *but by the number of free ions and molecules per unit volume.*

The chemical activity of electrolytes is also found to depend upon the dissociation of the dissolved molecules.

In the experimental investigation of electrolysis the quantity which is most directly measured is the *specific resistance*; methods of doing this are explained in Chapter XVI. If an electrolytic solution contains m gramme-equivalents per litre and the specific resistance of the solution be ρ, then $1/\rho$ is the *conductivity* κ of the solution and

$$\text{Equivalent Conductivity} = \frac{\text{Conductivity}}{\text{Concentration}} = \frac{\kappa}{m}$$

This constant is an important one in electrolysis. It is found that for solutions of salts the equivalent conductivity gradually increases as the concentration decreases, and for very dilute solutions is practically constant. That is, for very dilute solutions the conductivity is proportional to the concentration. In the case of acids and alkalies the equivalent conductivity increases as the concentration decreases, attains a maximum for very low concentration, and then rapidly decreases. For all salt solutions the value of κ/m is practically the same; for acids and alkalies the maximum value is about three times that for neutral salts. The value of κ for a solution of KCl containing 0·1 gramme-equivalent per litre is $1{\cdot}119 \times 10^{-12}$ C.G.S. units.

The ratio of the number of active molecules in a dilute electrolyte to the number there would be if there were no dissociation can evidently be determined from the ratio of the value of κ/m for the solution to its limiting value for an infinitely dilute solution. This ratio can also be determined from the lowering of the freezing point of the solution. The table given below, due to Arrhenius, shows the agreement of the results obtained. The first column gives the ratio as determined by measurements of conductivity by Kohlrausch and Ostwald. The second gives the values as determined by Raoult from the lowering of the freezing point. The third column gives the coefficients of ionisation, indicating the proportion of dissociated molecules present in the solutions.

NaOH	1·88	1·96	·88
HCl	1·90	1·98	·90
HNO_3	1·92	1·94	·92
H_2SO_4	2·19	2·06	·60
KCl	1·86	1·82	·86
NaCl	1·82	1·90	·82

The conductivity of electrolytes increases with rise of temperature, but the value of the coefficient of increase decreases with rise of temperature and increase of concentration; varying from ·035 for dilute solutions at 0° C. to ·015 for concentrated solutions at 18° C.

204. Ionic Velocities.—It is known that changes in concentration occur in an electrolyte during the process of electrolysis; Hittorf has explained this in terms of the difference between the velocities of the positive and negative ions, and has shown how the ratio of the velocities may be determined from the changes in concentration.

Consider Fig. 361, in which the dots represent positive ions (kations) and the circles negative ions (anions), and imagine the velocity of the former to be double that of the latter. Charges must of course be given up to the electrodes at the same rate. The first two rows indicate the condition before the current passes; there are eight molecules in each compartment. The second two rows indicate the condition an instant later, when the current is passing: at the kathode 3 ions have been deposited, the kathode compartment contains 7 molecules and *has therefore lost* 1 *molecule*; at the anode 3 ions have been deposited, the anode compartment contains 6 molecules and *has therefore lost* 2 *molecules.* Still later, the third two rows indicate the condition: 6 ions have been deposited at each electrode; the kathode compartment has 6 molecules and *its loss is therefore* 2 *molecules*; the anode compartment has 4 molecules and *its loss is therefore* 4 *molecules.* Clearly—

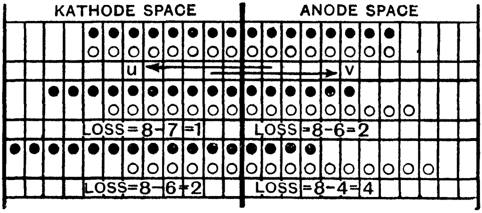


Fig. 361.

$$\frac{\text{Loss in concentration at kathode}}{\text{Loss in concentration at anode}} = \frac{1}{2}$$

$$= \frac{\text{Velocity of anion}}{\text{Velocity of kation}},$$

i.e.

$$\frac{\textbf{Kathode space loss}}{\textbf{Anode space loss}} = \frac{v}{u} \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

where v is the velocity of the negative ion and u the velocity of the positive ion. A chemical analysis of the solution in the neighbourhood of the kathode and anode after the passage of a current will therefore give the ratio of v to u. Further—

$$\frac{\text{Kathode space loss}}{\text{Total loss}} = \frac{v}{u+v}; \quad \frac{\text{Anode space loss}}{\text{Total loss}} = \frac{u}{u+v}.$$

These fractions are known as the *transport ratios* or *migration constants*; the former, which gives the loss from the neighbourhood of the kathode as a fraction of the total loss, is the one usually given.

It will be noted in the above that no chemical action at the electrodes is involved. It corresponds in fact to the decomposition of (say) copper sulphate with platinum electrodes; here the *weakening* on the anode side is to the *weakening* on the kathode side as u is to v. Consider, however, the decomposition of copper sulphate with copper electrodes. If an experiment be arranged, the electrodes being horizontal in the solution with the anode below and the kathode above to prevent the observations being interfered with by convection currents, it will be found that the solution becomes *lighter near the kathode and darker near the anode.* In this case the concentration as a whole remains the same, the solution is *strengthened* near the anode and *weakened* near the kathode (by an amount proportional to v), and copper (proportional to $u + v$) is taken from the anode, an equal amount being deposited on the kathode.

Thus imagine u gramme molecules of copper pass from the anode side to the kathode side, and therefore v gramme molecules of SO_4 from the kathode side to the anode side. On the anode side these v of SO_4, together with the u of SO_4 left by the copper which has migrated, take $(u + v)$ of copper from the anode, forming $(u + v)$ of $CuSO_4$. There is against this increase a decrease u of $CuSO_4$ due to the copper which has migrated; the solution on the anode side is therefore *stronger* by v of $CuSO_4$. On the kathode side the u of copper which has entered and the v of copper left behind by the SO_4 which has migrated are deposited, and the solution on the kathode side is therefore *weaker* by v of $CuSO_4$. Thus, in this case as in the preceding, the ratio $v : u$ can be found experimentally by an analysis of the solution (say) at the anode, and by finding the increase in weight of the kathode.

One form of apparatus is shown in Fig. 362: *A* is a silver anode, *B* a silver kathode, and the solution is $AgNO_3$. When the current has passed for a time half the contents is removed from the burette and analysed and the strengthening (proportional to v) is found; the increase in weight of *B* (proportional to $(u + v)$) is then found. The ratio $v : u$ is therefore known.

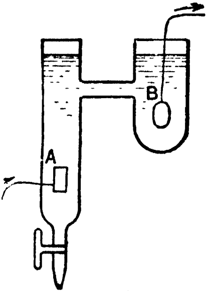


Fig. 362.

A further step, due to Kohlrausch, enables the value of $(u + v)$ to be determined. It will be seen that by the ionic theory the current through an electrolyte depends upon three things, (1) the number of ions involved, (2) the charge carried by each ion, (3) the velocity of the ions through the electrolyte. Thus, in a dilute solution of potassium chloride containing n gramme-equivalents per cubic centimetre, let the charges carried by a gramme-equivalent of the K and Cl ions be e and $-e$ units, and let the velocities of these ions be u and v cm. per sec. respectively. The current across one square centimetre in the electrolyte will then consist of the transfer of neu units of positive electricity in one direction, and nev units of negative electricity in the opposite direction, that is, the current is measured by $ne(u+v)$ or $9650n(u+v)$, since $e = 9650$ e.m. units. The current is also given by $\kappa\frac{dV}{dx}$, where κ is the conductivity of the liquid and dV/dx the gradient of potential in the direction of the current. Hence

$$9650n(u+v) = \kappa\frac{dV}{dx} \text{ or } u+v = \frac{\kappa}{n}\cdot\frac{1}{9650}\cdot\frac{dV}{dx}.$$

Here, if the concentration of the solution be expressed as m gramme-equivalents per litre, we have $m = n \times 10^3$, and the value of $(u+v)$ for a potential gradient of one volt per cm. $= 10^8$ e.m. units of potential per cm. is given by

$$(u+v) = \frac{\kappa}{m} \times 10^3 \times \frac{1}{9650} \times 10^8$$

$$= \frac{\kappa}{m} \times 10^{11} \times \cdot 00010362,$$

$$\therefore\ (u+v) = \frac{\kappa}{m} \times \mathbf{1{\cdot}0362} \times \mathbf{10^7} \ \ldots\ldots\ldots\ldots \ (2)$$

Hence $(u+v)$, the sum of the velocities of the ions, can be calculated from κ/m, the equivalent conductivity of the electrolyte, and this latter is known since m is the known concentration and $\kappa = 1/\rho$, where ρ is the specific resistance found by the methods of Art. 234. Knowing $(u+v)$ and v/u, the separate values v and u are determined.

To be exact, the theory can only be applied to cases where *all* the molecules are dissociated. Further, these "drift" velocities in an electric field are small, and must not be confused with the velocities of ions moving indiscriminately in all directions with which "osmotic pressure" is associated. The velocity of an ion under unit electric force (say one volt per centimetre) is called its ***mobility***.

The table below gives the values of $u + v$, u, and v in 10^{-6} cm. per sec. for K, Na, and Cl in solutions of KCl and NaCl of different concentrations.

	KCl			NaCl		
m	$u + v$	u	v	$u + v$	u	v
0·0000	1350	660	690	1140	450	690
·0001	1335	654	681	1129	448	681
·001	1313	643	670	1110	440	670
·01	1263	619	644	1059	415	644
·03	1218	597	625	1013	390	623
·1	1153	564	589	952	360	592
·3	1088	531	557	876	324	552
1·0	1011	491	520	765	278	487

This table indicates (1) that the ionic velocity increases as the dilution increases and tends to a constant maximum value at infinite dilution; (2) that the limiting velocity of the same ion (Cl) is the same in different electrolytes, that is, the velocity is a specific constant of the ion.

The specific ionic velocities of some of the commoner ions are given below in 10^{-5} cm. per sec.:—

H	320	OH	162
K	66	Cl	69
Na	45	I	69
Ag	57	NO_3	64

From these velocities it is evidently possible to calculate the conductivity of dilute electrolytes. For example, for a dilute solution of $AgNO_3$ the value of $(u + v)$ is 121×10^{-5} cm. per sec., and substituting this value in the relation

$$u + v = \kappa/m \times 1{\cdot}0362 \times 10^7$$

the value of κ/m can be determined.

This exposition of ionic velocity was given by Kohlrausch in 1879. Since that date many direct experimental determinations of the velocities of ions have been made, and the results have confirmed Kohlrausch's theory.

The first determination was made by Lodge in 1885. Two vessels containing dilute sulphuric acid were connected by a tube containing a slightly alkaline agar-agar jelly solution of sodium chloride and a trace of phenol phthalein. A current was passed from one vessel to the other through the tube, and as electrolysis went on the velocity of transfer of the H ion was indicated and measured by the rate at which the phenol phthalein indication of the formation of HCl travelled along the tube.

Similar determinations of ionic velocity have more recently been made by W. C. D. Whetham and by Orme Masson. The methods adopted were slight modifications of Lodge's original method. In one experiment the rate of displacement of the boundary between equivalent solutions of potassium chloride and potassium bichromate, as indicated by the advance of the coloured ion Cr_2O_7, was observed. In another a coloured kation, such as Cu from a solution of copper chloride, and a coloured anion, such as Cr_2O_7 from a solution of potassium bichromate, were made to travel in opposite directions along a tube containing a jelly solution of KCl, and the rate of progress of each ion observed.

205. Theory of the Simple Cell (continued). Chemical and Contact Theories.—Before proceeding with this section the student should again read Art. 143, which outlines the **chemical theory** of the simple cell. It is there indicated that both the zinc and the copper attract the negatively charged oxygen ions within a very narrow (molecular) film, that equilibrium is quickly attained since the plates become negatively charged and soon repel the free oxygen ions electrically as strongly as they attract them chemically, and that both plates are thus reduced in potential below the outer surfaces of the films; it is further indicated that, since the attraction of the zinc for oxygen is greater than that of the copper, the potential slope e_1 at the zinc is greater than the slope e_2 at the copper, so that the potential of the zinc is below the potential of the copper. The E.M.F. (E) of the cell is $e_1 - e_2$; Figs. 275 (*a*), 275 (*b*) indicate the conditions referred to.

The same idea may be conveyed as follows. When the zinc is placed in the acid the positive zinc ions combine with the negative SO_4 ions, forming $ZnSO_4$ and leaving the zinc plate negatively charged. Each negative SO_4 ion which combines leaves two positive hydrogen ions; these are attracted to the plate, so that around the latter we have an *electric double layer*, consisting of the negative zinc on the one side and the positive hydrogen on the other (Fig. 363), and forming a molecular condenser, of which the zinc is the low and the acid the high potential element. Equilibrium is soon established between the tendency of the zinc ions to combine with the SO_4 ions and the tendency ~~for~~ positive ions to be driven from the acid to the negative zinc plate, so that with an infinitesimal consumption of zinc the P.D. e_1 is established. Similar actions occur at the copper, but the tendency of copper to combine with the acid is less than that of zinc, so that the P.D. e_2 is less than e_1.

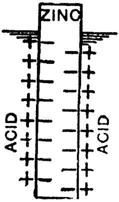


Fig. 363.

If now a copper wire be attached to the zinc plate *it acquires* (*practically*—see Chapter XV.) *the potential of the zinc plate*, and the P.D. between this copper wire and the copper plate of the cell is therefore (on open circuit) the E.M.F. (E) of the cell, viz. $e_1 - e_2$.

There are, however, many experiments which show that *two metals in contact are at different potentials*, and Volta believed that the *main cause* of the working of a cell was **the contact of the metals**; on this is based the **contact theory** of the simple cell. When zinc touches copper there appears to be a flow of electricity from the copper to the zinc which causes the zinc to be of higher potential. Experiments were designed by Volta and others to test this, but most of these earlier experiments are capable of other explanations and are inconclusive; Ayrton and Perry's more recent experiment, which is a modification of Kelvin's experiment, is comparatively free from objection.

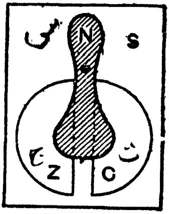


Fig. 364.

KELVIN'S EXPERIMENT (**Fig.** 364).—Z and C are plates of zinc and copper placed horizontally on an insulating support, and N is an electrometer needle charged *positively* and suspended above the plates. When the plates Z and C are joined by a copper wire N is deflected, *being attracted by the copper*. If the wire be removed and Z and C be joined by a drop of acid *the needle is not deflected*. This seems to indicate that when zinc and copper are in contact there is a P.D. between them, the zinc being positive, but that when immersed in acid they are at the same potential.

AYRTON AND PERRY'S EXPERIMENT (Fig. 365).—In the figure *G* represents diagrammatically a quadrant electrometer, whose two pairs of quadrants are connected to flat horizontal brass plates *A* and *B*. *L* and *M* are two other plates parallel to and at the same distance from *A* and *B*, and connected to one another by a wire. It was argued that if *L* and *M* be at different potentials, *A* and *B* will have a proportional difference of potential of the same kind. The experiment clearly showed a P.D., when zinc and copper were used, of about ·75 volt, the zinc being positive.

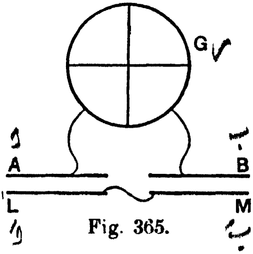


Fig. 365.

These results appear to indicate that zinc and copper have a contact P.D. approaching a volt. Modern work, however, goes far to show that this theory must be abandoned: contact of dissimilar metals or other substances does give rise to electrical separation, but not to *the* separation we have to deal with in a voltaic cell. For metals the difference of potential established by contact is exceedingly minute compared with the electromotive force of a cell, and has been held to have little or nothing to do with voltaic currents, in the ordinary meaning of the term. The most probable theory of the voltaic cell is the chemical theory given above, and an explanation of the experiments just described can be given in an exactly similar way.

Consider a piece of zinc (Fig. 366) surrounded by air—here, just as when immersed in dilute acid, it attracts the neighbouring oxygen atoms, but in the liquid these atoms *while dissociated* are free to obey this attraction, whereas in air, *where there is practically little dissociation*, they are not free to do so.

The molecules of free oxygen are uncharged, but each molecule may be supposed to be resolvable into two atoms, one charged positively and the other negatively. Under the influence of the attraction of the zinc the layer of oxygen molecules adjacent to it may become polarised, forming an inner layer of negatively charged atoms in contact with the zinc and an outer layer of positively charged atoms surrounding this inner layer. Equilibrium will be attained when the attraction between the zinc and the adjacent layer of negatively charged oxygen atoms equals the attraction between the two layers of atoms, (and the work done in effecting the polarisation of the oxygen molecules is equal to the energy of the charged condenser constituted by the two layers of oppositely charged atoms.

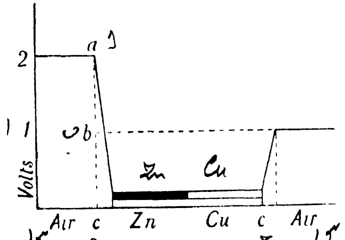


Fig. 366.

Whether the inner layer of oxygen atoms actually combines with the zinc and so transfers its charge to the metal or simply remains in close contact with it, the zinc is practically negatively charged, and a difference of potential is set up between it, or the layer in contact with it, and the adjacent layer of positively charged atoms. The magnitude of this difference of potential will depend upon the force of attraction between the zinc and the oxygen atoms. The relation between these two quantities may perhaps be expressed by means of the formula for the attracted disc electrometer plates. If v be the difference of potential set up, δ the distance between the two layers of atoms, and f the force of attraction per unit area of the zinc on the inner oxygen layer, then we have $v = \delta\sqrt{8\pi f}$.

In the case of the copper the details would be the same as for the zinc, but as the chemical attraction between copper and oxygen is less than between zinc and oxygen the difference of potential set up between the two layers of oxygen atoms would be less in the case of the copper. In the case of zinc this difference is about 1·8 volts and in the case of copper about ·8 volt.

When the zinc and copper are put in contact *they acquire the same potential,* but there will now be a difference of potential between the outer layer of oxygen in contact with the zinc and that in contact with the copper. The potential of the air just above the zinc will be higher than that above the copper by a quantity of the order of one volt; *and it is this difference which is actually measured by the plates A and B* (Fig. 365) *in Ayrton and Perry's experiment.* The distribution of potential difference, described above, is indicated graphically in the diagram of Fig. 366. In this diagram the metals are in contact, and the difference of potential between the films of oxygen on the two metals is represented by *ab* and is here shown to be about one volt.

The contact experiments of Volta and others are thus merely illustrations of the action of a voltaic cell in which the medium which surrounds the plates is an insulator, and not an *electrolyte*, which conducts by virtue of the dissociation which goes on in it.

Fig. 275 (*a*) for a simple cell on open circuit may now be extended as shown in Fig. 367, which gives the distribution of potential when the plates are merely placed in the acid. The differences of potential between the air and the metal, and between the acid and the metal, are shown to be the same for each metal, because in each case the magnitude of the differences depends upon the same chemical affinity. This explains why there is no slope of potential in the intervening air, and accounts for the second part of Kelvin's experiment above. If, however, a copper wire is attached to each plate of the cell, the distribution changes as shown in Fig. 368. This figure is really a combination of Fig. 366 and Fig. 367.

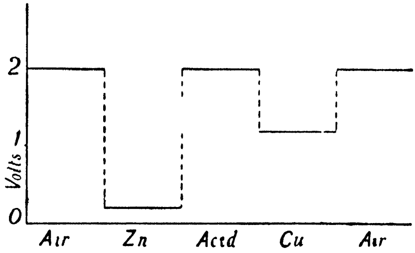


Fig. 367.

The essential point of difference between the two theories of the voltaic cell—Volta's contact theory and the chemical action theory—is the explanation of the source of the electromotive force of the cell. The contact theory states that the electromotive force results from metallic contact between the two metallic elements of the cell. The chemical theory states that it results in the way detailed above from the chemical action going on in the cell. The evidence of Volta's experiment and others gave strong support to the contact theory, until it was shown that the result of the experiments could be explained by the chemical action theory. The chemical theory is also strongly supported by the fact that it supplies a satisfactory explanation of the source of energy in a working cell. On the contact theory it would be necessary to suppose the energy to be derived from the heat of the elements of the cell, and not, as is obviously the case, from the chemical reactions going on in the cell.

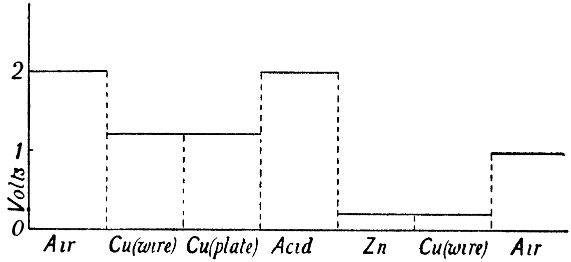


Fig. 368.

In thermo-electricity, considered in a later chapter, we have a true case of electromotive force resulting from contact between dissimilar metals, but the electromotive forces developed are extremely small compared with those due to chemical action in a cell.

In the chemical theory above the *sign* of the P.D. between the plates and the acid is the same for *both* plates, and the E.M.F. (E) is the *difference* of these two, viz. $e_1 - e_2$. Experiments on the single potential differences at the surfaces of metals and solutions made by means of "dropping electrodes" and capillary electrometers (Art. 207) seem to indicate that the single potential differences at the surfaces of zinc and copper have opposite signs, *i.e.* that the acid is above the zinc in potential, *but that the copper is above the acid*, so that the E.M.F. is the *sum* of the two. It is still uncertain which of these views represents the truth; there is much in favour of the former, and there is some doubt on the accuracy of single potential difference determinations by dropping electrodes and capillary electrometers.

206. E.M.F. of Reversible Cells. The Gibbs-Helmholtz Equation.—Cells are frequently classified into reversible and non-reversible types. Consider, for example, a Daniell's cell balanced by an equal and opposite external E.M.F. If the latter be reduced by a very small amount the cell will drive a small current in the normal direction, zinc will pass into solution at the anode, and copper will be deposited at the kathode; on reversing the current by increasing the external E.M.F. the cell may be brought back to its original condition, copper being passed into solution from the copper plate and zinc being deposited on the zinc plate. Such a cell may be taken as a reversible cell (neglecting diffusion and the I^2Rt heating, both of which are irreversible effects). The simple cell, as previously mentioned, is a non-reversible cell; hydrogen escapes and cannot be replaced by a reverse current.

The second law of thermodynamics can be applied to reversible processes. Imagine a reversible cell (balanced by an equal and opposite E.M.F.) in an enclosure maintained at absolute temperature T, and let E be its E.M.F. at this temperature. Reduce the balancing E.M.F. slightly so that the cell drives a charge q round the circuit. The line traced on the indicator diagram is SR (Fig. 369); this is parallel to the axis of q, for the cell takes in heat h to keep its temperature constant and the E.M.F. (E) is constant at constant temperature.

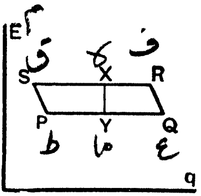


Fig. 369.

Now isolate the cell thermally, and let it drive an infinitesimal charge; the cell draws upon its own energy, and we will suppose that this produces a cooling effect. The temperature is now $T-\delta T$, the E.M.F. is $E-\frac{dE}{dT}\delta T$, the term $\frac{dE}{dT}$ denoting the rate of change of E.M.F. with temperature, and the line traced on the diagram is the adiabatic RQ.

Place the cell in an enclosure maintained at absolute temperature $T-\delta T$ and, with a reverse E.M.F. infinitesimally greater than that of the cell, let a reverse charge q be passed; the line traced is QP. Finally, thermally isolate the cell and pass a charge which will bring the cell to its original temperature T; the line traced is PS.

The balance of useful work done by the cell is represented by the area $SRQP$, that is—

$$\text{Work done} = \overline{SR} \times \overline{XY} = q \frac{dE}{dT} \delta T.$$

In the thermodynamics of reversible engines it is shown that the ratio of the useful work during a cycle to the heat energy taken in at the higher temperature is equal to the ratio of the difference in the two temperatures to the higher absolute temperature; hence

$$\frac{dE}{dT} \qquad \frac{q \frac{\cancel{Ed}}{\cancel{Td}} \delta T}{h} = \frac{\delta T}{T},$$

$$\therefore h = qT \frac{dE}{dT}$$

Now let h^1 be the heat in energy units due to chemical changes in the cell when unit e.m. quantity passes, so that $h^1 q$ will be the heat for a transfer q. Thus *during the process* SR the total heat in energy units is $h^1 q + h$ and the work done is Eq; hence

$$Eq = h^1 q + h,$$

$$\therefore E = h^1 + \frac{h}{q},$$

i.e.

$$\boldsymbol{E = h^1 + T \frac{dE}{dT}}.$$

This is known as the equation of Helmholtz or of Willard Gibbs. If the temperature coefficient dE/dT be zero, $E = h^1$; this is assumed so in the calculations of Art. 197. If dE/dT be positive $E > h^1$ and therefore h is positive; heat must therefore be taken in to keep the temperature constant and such a cell will cool on giving a current. If dE/dT be negative the opposite will be the case.

207. Solution Pressure. Electrode Potentials. Capillary Electrometer.—Consider a zinc plate in a solution of zinc sulphate. The zinc ions in solution exert a pressure (osmotic pressure) tending to drive the ions upon the metal, but the ions in the metal also exert a pressure (solution pressure) tending to drive zinc ions into solution; there will be equilibrium when the opposing influences balance each other. Thus for a metal placed in a solution of the same metal there is a particular value of the osmotic pressure of the metallic ions in solution for which the metal will neither be deposited from the solution nor dissolved from the plate; this measures what is termed the **electrolytic solution pressure** of the metal for the solvent.

Consider again the zinc plate in zinc sulphate and let the osmotic pressure of the zinc ions in solution be less than the solution pressure of the zinc. Positively charged zinc ions pass from the plate, and an electric double layer is formed, the zinc plate being negative and the solution positive. This will continue until the solution pressure of the metal is balanced by the osmotic pressure of the zinc ions in solution and the electric double layer. If the osmotic pressure of metallic ions in solution be greater than the solution pressure metallic ions will be driven on the metal and the electric double layer will be of opposite sign, *i.e.* the plate will be positive and the solution negative. If the osmotic pressure be equal to the solution pressure no double layer is formed and no P.D. exists between the metal and the solution.

The P.D. between a metal and an electrolyte is referred to as an **electrode potential** and the E.M.F. of a cell, as usually measured, is the algebraic sum of all the single electrode potentials and other P.D.'s due to contact of dissimilar substances. To measure a *single* electrode potential another metallic junction must be introduced to connect the liquid to the measuring instrument and this introduces a difficulty and sources of error. One method is to take the series *metal plate* (*say Zn*)/*electrolyte* (*say* H_2SO_4)/*mercury*, and as the *capillary electrometer* is considered to give the P.D. H_2SO_4/Hg and the total P.D. can be measured, the remaining single potential difference Zn/H_2SO_4 is found. Another method is to use a "dropping electrode," *e.g.* mercury dropping into a solution from a fine nozzle.

The **Capillary Electrometer** is an application of the effect of electric currents on the surface tension or surface energy of the boundary between mercury and an acid. At the contact an E.M.F. is set up such that the mercury is positive to the acid, and the surface tension depends upon this E.M.F., becoming a maximum when this E.M.F. is reduced to zero. In Fig. 370 the vessel *B* contains mercury and a quantity of dilute sulphuric acid; *A* is a tube drawn out to a fine point and containing mercury. *B* is joined to the high potential end and *A* to the low potential end of an adjustable resistance through which a current is passing, so that this *applied* potential difference, viz. current × resistance, may be varied. Now if the surface tension increases the meniscus will rise in *A*, while if the surface tension decreases it will fall. By adjusting the resistance the applied potential difference is altered until the mercury in *A* reaches its highest point, in which case the mercury in *A* and the acid are at the same potential; the P.D. given by the product of the resistance and the current is then equal to the P.D. between the mercury and the acid since the two are exactly balancing.

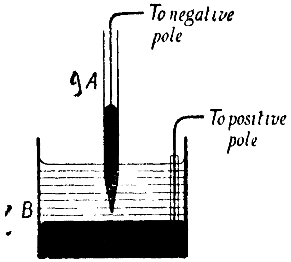


Fig. 370.

The instrument may be used as an electrometer. When the mercury in *A* is connected to the mercury in *B*, and both are therefore at the same potential, the mercury surface in the lower part of the tube *A* will have a definite position which may be marked. If, however, *A* and *B* be connected to the two points the P.D. for which is required —*A* to the low and *B* to the high so that a current passes from *B* to *A*—the surface tension at the lower end of the mercury in *A* increases, and the surface rises in the tube.

The extent of this change of position, or the change of pressure necessary to bring the surface back to the same position, may be taken to indicate the difference of potential between the mercury in *A* and that in *B*.

Fig. 371 shows a form of instrument constructed to measure the increase of the pressure on the upper surface of the mercury in the tube necessary to adjust the lower surface to the position indicating no difference of potential between the two masses of mercury. The connections are so arranged that the lower mass of mercury is at a higher potential than the upper. This causes the lower level of

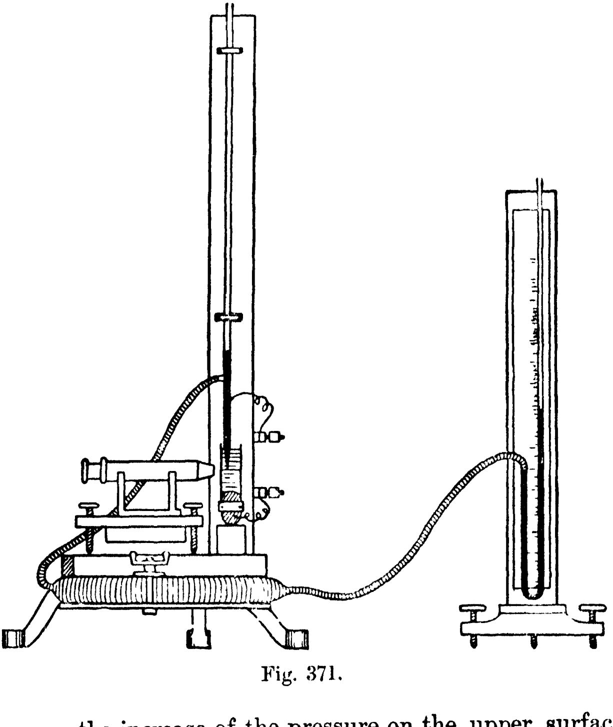


Fig. 371.

the mercury in the tube to rise and the pressure in the upper part of the tube has to be increased to bring it back to the zero position. The increase of pressure is proportional to the difference of potential, but the instrument should be calibrated for use and cannot be used for a difference of potential greater than about ·9 volt. This form of electrometer is due to Lippmann.

A simple electrometer in which a P.D. is indicated by the motion of a liquid may be referred to in passing (Fig. 371*a*). *AB* is a bubble of dilute sulphuric acid, in a capillary tube, between two mercury surfaces. If *C* be connected to a positive electrode and *D* to a negative, a small current flows which increases the surface tension at *B* and diminishes it at *A*. The bubble therefore moves in the direction *AB* till brought to rest by the difference of mercury level produced at *C* and *D*.

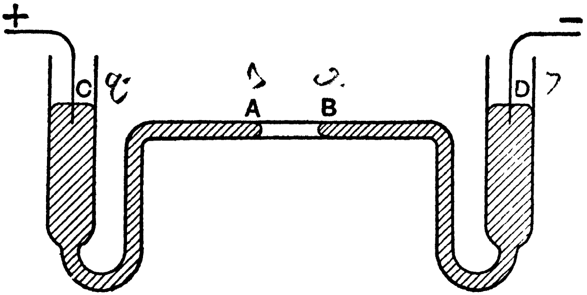


Fig. 371*a*.

Neumann gives the following for the single potential differences for metals in contact with normal solutions of their salts; plus signs indicate that the metals are *above* the liquids in potential.

Metal.	*Sulphate.*	*Nitrate.*
Zinc	− ·524	− ·473
Copper	+ ·515	+ ·615
Silver	+ ·974	+ 1·055
Mercury	+ ·980	+ 1·028

In a Daniell's cell we have the arrangement

$$Cu/CuSO_4/ZnSO_4/Zn/Cu,$$

and of these the contacts $CuSO_4/ZnSO_4$ and Zn/Cu are small and may be neglected. Assuming the above correct, the contact $Cu/CuSO_4$ is ·515 volt, the copper being *above* the $CuSO_4$, and the contact $ZnSO_4/Zn$ is ·524, the $ZnSO_4$ being *above* the zinc; the E.M.F. is therefore ·515 + ·524 = 1·039 volts. Fig. 363 for the zinc plate in sulphuric acid will also apply to the present case of the zinc plate in zinc sulphate; the osmotic pressure of the zinc ions in solution is less than the solution pressure of the zinc, so that positive zinc ions leave the plate, and the electric double layer has the sign indicated, *i.e.* the zinc is lower than the solution in potential. Fig 371*b* would, according to the above, represent the case of the copper plate in copper sulphate: the osmotic pressure of the copper ions in solution is greater than the solution pressure of the copper, so that positive copper ions are driven on the plate and the electric double layer has the sign indicated, *i.e.* the copper is above the solution in potential. As previously indicated, however, there is still some doubt as to the accuracy of these single P.D. determinations by capillary electrometers and dropping electrodes.

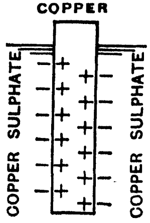


Fig. 371*b*.

208. E.M.F. due to Difference in Concentration in an Electrolyte. Concentration Cells.—When two different electrolytes or two solutions of a salt of different concentrations are in contact a P.D. is usually produced; this is due to the natural process of diffusion which takes place.

According to modern theory the molecules of a substance in solution are more or less free of each other, are in continual motion, and, in many respects, are under the same conditions as the molecules of a gas or vapour. In dilute solutions it is believed that the relation between pressure (osmotic), volume, and absolute temperature is identical with the relation in the case of a gas, viz. $P\bar{v}=RT$, the value of R depending on the quantity of the substance (gas or solution) taken. Consider, for example, the *gramme-molecule.* For this mass of any gas at normal temperature and pressure the volume ($\bar{v}$) is 22,320 c.cm., the pressure (P) is $76 \times 13{\cdot}6 \times 981$ dynes per sq. cm., and the absolute temperature (T) is 273; substituting in the above we get $R = 8{\cdot}28 \times 10^7$.

Now in the case of an electrolyte the molecules are dissociated into charged ions and the pressure is greater; taking dissociation to be complete for a very dilute solution we may write $P = \frac{nRT}{\bar{v}}$, where n is the number of ions arising from each molecule.

Consider now an electrolyte consisting (say) of two solutions of silver nitrate of different concentrations separated by a porous diaphragm. Diffusion takes place from the more to the less concentrated solution, each ion carrying its charge with it, and it is clear that if the opposite ions travel with different velocities there will be a P.D. established between opposite sides of the diaphragm.

Let A denote the area of the diaphragm and consider a thickness dx, the osmotic pressures at opposite faces being P and $P+dP$ respectively; the resultant pressure is AdP, the number of ions within the space considered is $NAdx$, where N is the total number of ions per c.cm. within the space, and the force on each ion is therefore $AdP/NAdx$, i.e. $\frac{1}{N}\frac{dP}{dx}$.

Again, if dV be the P.D. established as indicated above between opposite sides of the layer the potential gradient is dV/dx, and if e be the charge on an ion the force exerted per ion is $e\frac{dV}{dx}$ or $-e\frac{dV}{dx}$ according to the kind of ion considered; thus we have

$$\text{Total force on kation} = \frac{1}{N}\frac{dP}{dx} + e\frac{dV}{dx},$$

$$\text{Total force on anion} = \frac{1}{N}\frac{dP}{dx} - e\frac{dV}{dx}.$$

Let u = velocity of the kation and v = velocity of the anion when dV/dx is unity, *i.e.* when the electric force $\left(e\frac{dV}{dx}\right)$ is e, so that the velocities *under unit force* will be u/e and v/e respectively; hence for the velocities under the above forces we have

$$\text{Velocity of kation} = \frac{u}{e}\left(\frac{1}{N}\frac{dP}{dx} + e\frac{dV}{dx}\right),$$

$$\text{Velocity of anion} = \frac{v}{e}\left(\frac{1}{N}\frac{dP}{dx} - e\frac{dV}{dx}\right).$$

Now when a steady condition is reached under the combined influences referred to, the opposite ions must migrate through the diaphragm with equal velocities, in which case

$$\frac{u}{e}\left(\frac{1}{N}\frac{dP}{dx} + e\frac{dV}{dx}\right) = \frac{v}{e}\left(\frac{1}{N}\frac{dP}{dx} - e\frac{dV}{dx}\right),$$

$$\therefore dV = \frac{v-u}{v+u}\cdot\frac{1}{Ne}\cdot dP. \checkmark$$

The expression $P = \frac{nRT}{\overline{v}}$ may be written $P = n\frac{M}{\overline{v}}\frac{RT}{M}$, where M is the number of grammes in a gramme-molecule, and as $M/\overline{v}$ = mass in 1 c.c. = concentration = C, it may be expressed $P = nC\frac{RT}{M}$. Further, as N = the number of ions per c.c. and n = the number of ions per molecule, N/n = the number of molecules per c.c.;

hence if m = actual mass of a single molecule, $\frac{N}{n} m$ = mass per c.c. = C and

$$P = nC\frac{RT}{M} = n\frac{N}{n}m\frac{RT}{M} = N \,.\, m\,\frac{RT}{M},$$

$$\therefore\ dP = m\frac{RT}{M}dN,$$

i.e.
$$dV = \frac{v-u}{v+u}\,\frac{m}{e}\,.\,\frac{RT}{M}\frac{dN}{N}.$$

Let N_1 and N_2 be the number of ions per c.c. in the concentrated and dilute solutions; integrating between these limits—

$$V_1 - V_2 = \frac{v-u}{v+u}\,.\,\frac{m}{e}\,.\,\frac{RT}{M}\log_e\frac{N_1}{N_2}$$

$$= \frac{v-u}{v+u}\,.\,\frac{M}{e\,.\,6{\cdot}16 \times 10^{23}}\,.\,\frac{RT}{M}\log_e\frac{C_1}{C_2},$$

i.e.
$$E = \frac{v-u}{v+u}\,.\,\frac{1}{9650}\,.\,RT\log_{10}\frac{C_1}{C_2}\,.\,2{\cdot}303.$$

This is the E.M.F. arising from difference in concentration.

Example. Taking, for example, deci- and centi-normal solutions of silver nitrate, the temperature being 18° C., we get

$$E = \frac{64-57}{64+57} \times \frac{1}{9650} \times 8{\cdot}28 \times 10^7 \times 291 \times \log_{10} 10 \times 2{\cdot}303$$
$$= {\cdot}0333 \times 10^7 \text{ e.m. units} = {\cdot}00333 \text{ volt.}$$

In dilute solutions the osmotic pressures are proportional to the concentrations, so that if P_1 and P_2 denote the osmotic pressures above we may write

$$E = \frac{v-u}{v+u}\,.\,\frac{1}{9650}\,.\,RT\log_e\frac{P_1}{P_2}.$$

This may be extended to the case of a metal, say silver, in a solution of $AgNO_3$; here only the kation is to be considered, for no anion crosses the layer, and v is zero; hence

$$E = -\frac{RT}{9650}\log_e\frac{P^1}{P_1},$$

where P^1 is the osmotic pressure of the kations in the metal, *i.e.* the solution pressure. Ostwald and others have constructed **Concentration Cells** consisting of similar electrodes in two different concentrations of the same solution, *e.g.* two silver plates, one in each of the $AgNO_3$ solutions above; in this case we have for *the E.M.F. of the concentration cell*—

$$E = \frac{RT}{9650}\left(\log_e \frac{P^1}{P_2} + \frac{v-u}{v+u}\log_e \frac{P_1}{P_2} - \log_e \frac{P^1}{P_1}\right)$$
$$= \frac{RT}{9650} \cdot \frac{2v}{u+v} \cdot \log_e \frac{P_1}{P_2} = \frac{RT}{9650} \cdot \frac{2v}{u+v} \cdot \log_e \frac{C_1}{C_2}.$$

Example. Taking the concentration cell referred to, viz. two silver plates in the previous solutions of silver nitrate at 18° C.,

$$E = \frac{8{\cdot}28 \times 10^7 \times 291}{9650} \times \frac{128}{121} \times \log_{10} 10 \times 2{\cdot}303$$
$$= {\cdot}604 \times 10^7 \text{ e.m. units} = {\cdot}0604 \text{ volt.}$$

Nernst measured the E.M.F. experimentally and obtained the result ·055 volt.

209. Secondary Cells or Accumulators.—The following experiment will illustrate the principle of the secondary cell.

Exp. In Fig. 372 A and K are two lead plates immersed in dilute sulphuric acid, V is a voltmeter, and a battery is connected as shown. A current is passed through the voltameter in the direction A to K by closing the key on the left. The result is that hydrogen appears at the kathode K and oxygen at the anode A. The oxygen at A combines with the surface lead forming the dark brown peroxide of lead (PbO_2); the hydrogen at K mostly rises to the surface, so that this plate remains in the metallic state. When this "charging" process, as it is termed, has continued for some time the battery is disconnected and the key on the right closed. The voltameter will give a current through the *outside* circuit in the direction A to K, the voltmeter will indicate about 2 volts at first, but will gradually drop, the current falling off: the peroxide will be found to have disappeared from A, and *both* plates will have lead sulphate ($PbSO_4$) formed on them with traces of the monoxide (PbO). The charging process may now be repeated: the oxygen at A will convert the $PbSO_4$ (and any PbO) into PbO_2, and the hydrogen at K will reduce the products there to the metallic state, so that the electrodes will again be in their "formed" condition, viz. Pb at K, PbO_2 at A.

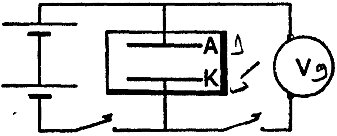


Fig. 372.

Such an arrangement is termed a **secondary cell, storage cell,** or **accumulator;** the *anode* is called the *positive plate* and the *kathode* the *negative plate* of the cell. It should be particularly observed, however, that there is no accumulation or storing of electricity; fundamentally, it is again the transformation of electrical energy into the potential energy of separated ions, *i.e.* into "chemical potential energy"; and when the cell gives a current the energy transformation is merely reversed.

In the preceding it was mentioned that on the first "charge" the plate *K* remained more or less in its initial condition, and as such it does not readily combine with the sulphion SO_4 to form $PbSO_4$ at discharge. This defect is eliminated by adopting the "alternate" charge process introduced by *Planté.* The current is first passed through the electrolyte in the direction *A* to *K*, with the result that *A* is peroxidised; it is then reversed, in which step the oxygen at *K* forms PbO_2 there, while the hydrogen at *A* reduces the existing PbO_2 to porous spongy lead. This operation is several times repeated, with the final result that the last anode has a thick coating of dark brown lead peroxide, while the last kathode is coated mainly with metallic lead of a greyish colour in a porous spongy condition, which is readily acted on by the SO_4 during discharge.

To obviate the tedious formation of the Planté plates *Faure* coated the plates, prior to charging, with a paste of red lead (Pb_3O_4) and sulphuric acid, the adherence of the paste to the plates being assisted by a covering of paper; on charging the red lead on the kathode becomes quickly reduced to spongy metal, while that on the anode becomes peroxidised. Later the *Sellon-Volckmar* plates were introduced, which, being constructed in the form of grids, more effectively secured the paste. Thus accumulators follow in general two specific types—(1) the Planté or naturally "formed" cell, and (2) the Faure or pasted grid cell; frequently they are of a composite character, having Planté positives and pasted negatives, since pasted positives, particularly in central station batteries, are liable to disintegration (falling of paste).

The chemical changes which occur during the discharge and charge of an accumulator may be briefly summarised as follows. Consider a charged storage cell of the Planté type: the positive plate is coated, as we have seen, with PbO_2, while the negative plate is coated with spongy metallic lead.

(1) *Chemical Changes during "Discharge."*—In discharging hydrogen will appear at the positive plate and oxygen at the negative plate. The hydrogen at the positive combines with the peroxide there with the following results—

$$PbO_2 + H_2 = \mathbf{PbO} + H_2O \;\ldots\ldots\ldots\ldots\; (1)$$
$$PbO + H_2SO_4 = \mathbf{PbSO_4} + H_2O \;\ldots\ldots\ldots\; (2)$$

The oxygen at the negative acts on the lead according to the equations

$$Pb + O = \mathbf{PbO} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; (3)$$
$$PbO + H_2SO_4 = \mathbf{PbSO_4} + H_2O \;\ldots\ldots\ldots\; (4)$$

(2) *Chemical Changes during "Charge."*—In charging oxygen is liberated at the positive plate and hydrogen at the negative plate. The oxygen at the positive combines with any oxides there, producing higher oxides, and particularly PbO_2; thus

$$PbO + O = \mathbf{PbO_2} \;\ldots\ldots\ldots\ldots\ldots\ldots\; (5)$$

while the $PbSO_4$ is converted into PbO_2 according to the equation

$$PbSO_4 + O + H_2O = \mathbf{PbO_2} + H_2SO_4 \ldots\ldots\; (6)$$

The hydrogen at the negative combines with the products there, reducing them to the metallic state, thus—

$$PbSO_4 + H_2 = \mathbf{Pb} + H_2SO_4 \ldots\ldots\ldots\ldots\; (7)$$
$$PbO + H_2 = \mathbf{Pb} + H_2O \ldots\ldots\ldots\ldots\ldots\; (8)$$

It will be observed that the actions represented by equations (6) and (7) increase the density of the electrolyte, while those represented by (2) and (4) weaken it. In modern accumulators the specific gravity of the acid is a good test of the condition of the cells: when fully charged the specific gravity ranges from 1·205 to 1·215, and when discharged from 1·17 to 1·19.

In the pasted type a preliminary chemical action takes place as follows—

$$Pb_3O_4 + 2H_2SO_4 = \mathbf{PbO_2} + \mathbf{2PbSO_4} + 2H_2O \ldots (9)$$

so that prior to charging *both* plates contain the peroxide (PbO_2) and the sulphate ($PbSO_4$). On charging, the action at the positive plate will be as indicated by (6) above, whilst the action at the negative plate will be (7) above, and

$$PbO_2 + 2H_2 = \mathbf{Pb} + 2H_2O \ldots\ldots\ldots\ldots (10)$$

so that, as before, the positive is coated with PbO_2 and the negative with spongy metallic lead. The discharge equations are those already given.

Frequently the preliminary treatment consists in coating the positive plate with the red lead and sulphuric acid paste, and the negative plate with a paste made of the monoxide PbO (litharge), in which case the preliminary action at the latter plate is

$$PbO + H_2SO_4 = \mathbf{PbSO_4} + H_2O \ldots\ldots\ldots (11)$$

The **efficiency** of an accumulator is given by the expression

$$\text{Efficiency} = \frac{\text{Watt-hours given out at discharge}}{\text{Watt-hours put in at charge}},$$

and in a general way is of the order 65 to 75 per cent.

The **capacity** of an accumulator is measured in *ampere-hours*; thus if a cell has a capacity of 605 ampere-hours and the maximum discharge current, as stated by the makers, is 55 amperes, it will be able to give this current for 11 hours. In a general way the ampere-hours given out vary from 85 to 90 per cent. of the ampere-hours put in.

The E.M.F. of a fully charged accumulator is about 2·1 volts, and as the internal resistance is very small a large current can be obtained from it. Fig. 373 depicts the arrangement in a thirteen plate cell, consisting of six positives and seven negatives. In practice the plates are very near each other, internal contact being prevented by glass tube separators or thin sheets of wood.

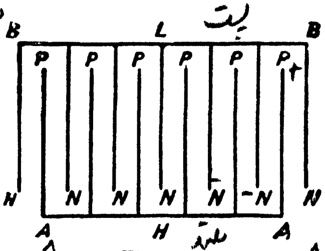


Fig. 373.

In most cases an accumulator should be regarded as "discharged" when its E.M.F. has fallen to about 1·85 volts and the specific gravity to about 1·18; if worked below this an insoluble lead sulphate is formed which ruins the cell. Further, an accumulator should never be short-circuited, for an excessive discharge current will flow, resulting in sulphating, disintegration of active material, and buckling of the plates.

The very many practical applications of accumulators cannot be referred to here; briefly their advantages and disadvantages as compared with primary cells are :—

(*a*) *Advantages.*—(1) They have a high E.M.F. and low resistance, and can therefore supply large currents; (2) when "run down" they can be recharged; (3) they can be used for lighting, traction, etc., where primary cells are useless.

(*b*) *Disadvantages.*—(1) Their initial cost is high; (2) they require careful attention to maintain them in good condition; (3) their efficiency is only low; (4) their weight renders them not very portable for laboratory purposes; (5) unless care be exercised they are subject to sulphating, disintegration, buckling, and short-circuiting.

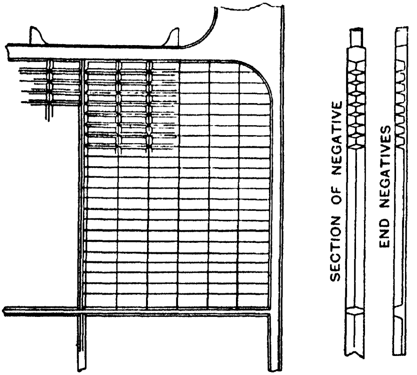


Fig. 374.

Fig. 374 depicts a portion of a typical (negative) grid for a modern accumulator.

In the *Edison accumulator* the plates are steel grids (nickel-plated), the positive grids carrying alternate layers of metallic nickel and nickel hydrate, and the negative grids oxide of iron; the electrolyte is a solution of potassium hydrate; the E.M.F. is about 1·3 volts.

Exercises XIV.

Section A.

(1) Develop expressions for v/u and $(v+u)$, where v and u are the velocities of the anions and kations respectively in an electrolytic solution.

(2) Write a short essay on the chemical and contact theories of the simple cell.

(3) Develop the Gibbs-Helmholtz equation for the E.M.F. of a reversible cell.

(4) Describe the construction and action of the capillary electrometer.

(5) Develop expressions for the E.M.F. due to difference in concentration in an electrolyte and for the E.M.F. of a concentration cell.

Section B.

(1) Explain the term "electrochemical equivalent." If 3 amperes deposit 4 grammes of silver in 20 minutes, what is the electrochemical equivalent of silver? (B.E.)

(2) If 10,000 kilocoulombs are passed through cells arranged in series containing solutions of Cu_2Cl_2, $Hg_2(NO_3)_2$, $CuSO_4$, H_2SO_4, and NaCl, how much copper, mercury, hydrogen, and caustic soda can be obtained? (City and Guilds of London.)

(3) Give an account of the chemical changes which occur in a storage cell during charge and discharge. (B.E.)

Section C.

(1) Give the elementary theory of the capillary electrometer and describe its application to the measurement of the potential difference between a solution and mercury. (B.E. Hons.)

(2) Give a short account of the ionic theory of electric conduction in electrolytes, and show why a difference in potential should in general be expected when diffusion of a salt takes place. (B.E. Hons.)

(3) Find an expression for the E.M.F. due to difference in concentration in an electrolyte. (B.E. Hons.)

(4) A tangent galvanometer has a current passed through it which deflects it 45° The same current passes through a copper voltameter, where it deposits 0·3 gramme of copper in 30 minutes.

If the electrochemical equivalent of copper is 0·00033 gramme/ampere-second, find the value of the current, and show how to determine the current for any other reading of the galvanometer. (Inter. B.Sc.)

(5) How do you account for the fact that an E.M.F. of about $1\frac{1}{2}$ volts is needed to electrolyse water at an appreciable rate? An accumulator 2 volts E.M.F. maintains a current in a circuit of total resistance 2 ohms. An electrolytic cell with back E.M.F. 1·5 volts is then inserted, the resistance being adjusted again to 2 ohms. Compare the currents and the rate of working in the two cases. (Inter. B.Sc.)

(6) A current of electricity driven by an electromotive force of 10 volts traverses a water voltameter in which there is a resistance of 2 ohms and a back electromotive force of 1·5 volts. Calculate the weight of hydrogen, and the number of calories developed in the voltameter per hour, assuming the resistance of other parts to be negligible. [The electrochemical equivalent of hydrogen is 0·000010384 gramme per coulomb, and the mechanical equivalent of one calorie is 42,000,000 $\text{cm.}^2\,\text{gm.}^1\,\text{sec.}^{-2}$.] (Inter. B.Sc. Hons.)

(7) Explain the meaning of the expression "v and u the mobilities of the ions in electrolysis."

Show that if in the electrolysis of a solution $10{\cdot}36 \times 10^{-6}$ gramme-equivalents of each ion are liberated by the passage of an ampere for a second,

$$u + v = 10{\cdot}36 \times 10^{-6}\,k/\cdot N,$$

when k is the conductivity of the electrolyte and N the number of gramme-equivalents of dissolved salt per cubic centimetre of the solution. (B.Sc.)

(8) Two liquid resistances, A and B, of 5 and 10 ohms respectively are connected in parallel, and a battery of electromotive force 8 volts and 2 ohms internal resistance is used to send a current through them. Find the currents in the two liquids, being given that the electromotive force of polarisation is 0·1 volt in A and 1·8 volts in B. (B.Sc.)

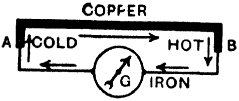
(9) Show how the velocity of electrolytic ions in an electric field can be calculated from measurement of the specific resistance and of the transport ratio. Describe, mentioning necessary precautions, experiments by which this velocity is directly measured. (B.Sc.)

(10) Find a relation between the rate of change with temperature of the electromotive force of a reversible cell and the other constants of the cell. (B.Sc. Hons.)

CHAPTER XV.

THERMO-ELECTRICITY.

210. Seebeck Effect. Thermo-Electric Currents.—Fig. 375 represents pieces of copper and iron wire joined together at their ends *A* and *B*, and *G* is a low resistance mirror galvanometer included in the circuit. If both junctions be initially at 0° C., and the junction *B* be then gradually heated, a current will flow in the circuit in the direction indicated, viz. copper to iron through the hot junction, and iron to copper through the cold junction. This current will increase in strength until the hot junction is at a temperature of about 270° C. (different for different specimens of iron and copper), at which stage the maximum current will be flowing. If heating be continued the current will decrease in strength, and will be zero when the hot junction is at 540° C. On heating still further the junction *B*, the current will increase again, but *it will be reversed in direction*, i.e. it will flow from iron to copper through the junction *B*, and from copper to iron through the junction *A*. Currents produced in this way by heating junctions of different metals are known as *thermo-electric currents*, and were discovered by **Seebeck** in 1821. The phenomenon of *inversion* of the E.M.F. was discovered later by Cumming.



SEEBECK

Fig. 375.

If both junctions of the copper-iron couple be initially at 10° C. (say) instead of at 0° C., there will again be maximum current when the hot junction is at 270° C., but there will be zero current and reversal in this case when the hot junction is at 530° C. *The temperature of the hot junction at which maximum current flows is a constant for a given couple, and is known as the neutral temperature for that couple.* The temperature of the hot junction at which there is zero current and reversal is a variable one, being always as much above the neutral temperature as the cold junction is below it; thus, if the cold junction be at $T°$ C. and the neutral temperature be $\theta°$ C., there will be inversion of current when the hot junction is at a temperature $\{\theta + (\theta - T)\}°$ C.

Let the two junctions of a couple be at $T°$ C. initially, and then let one of them be raised in temperature by a very small amount dT; if dE be the corresponding small E.M.F. generated, the ratio $\frac{dE}{dT}$ is known as the Thermo-Electric Power of the two metals at the temperature T. If one junction of a couple be at $T_1°$ C. and the other at $T_2°$ C., then, *provided that one of these is not above and the other below the neutral temperature θ,* the total E.M.F. is given by the product of the thermo-electric power at the mean temperature $\frac{1}{2}(T_1 + T_2)$ and the difference in temperature $(T_2 - T_1)$. If T_1 be *below* and T_2 *above* the neutral temperature θ, the actual E.M.F. in the circuit is found thus:—

$$\text{Total E.M.F.} = (\text{E.M.F. with junctions at } T_1 \text{ and } \theta) - (\text{E.M.F. with junctions at } \theta \text{ and } T_2).$$

These points will be more clearly grasped after further reading of the present chapter.

In the above copper and iron are chosen simply because the couple they form is a convenient one for experimental purposes, but any other pair of metals will indicate similar results, the numerics being of course different. It may be noted here that these E.M.F.'s are very small; thus with a copper-iron couple with junctions at 0° C. and 100° C. the E.M.F. is only about ·0013 volt.

211. Experimental Laws.—There are two simple laws established by experiment. The *law of successive temperatures* states that, for a given couple, the electromotive force for any specified range of temperature is the sum of the electromotive forces for any number of successive steps into which the given range of temperature may be divided. That is,

$$E_{t_1}^{t_n} = E_{t_1}^{t_2} + E_{t_2}^{t_3} + \ldots . E_{t_{n-1}}^{t_n}$$

where t_1, t_2, t_3 .... are successive temperatures intermediate between t_1 and t_n.

The *law of successive contacts* states that if, at a given temperature, a number of metals are in successive contact so as to form a chain of elements connected in series, the electromotive force between the extreme elements, if placed in direct contact, is the sum of the electromotive forces between successive adjacent elements. That is, for metals A, B, C, D .... N in successive contact

$$E_{\mathrm{A}}^{\mathrm{N}} = E_{\mathrm{A}}^{\mathrm{B}} + E_{\mathrm{B}}^{\mathrm{C}} + E_{\mathrm{C}}^{\mathrm{D}} + \ldots . E_{\mathrm{M}}^{\mathrm{N}}$$

provided all *the junctions* in the series are at the same temperature. This law evidently states that for given fixed junction temperatures, if $E_{\mathrm{A}}^{\mathrm{B}}$, $E_{\mathrm{B}}^{\mathrm{C}}$, $E_{\mathrm{C}}^{\mathrm{D}}$, etc., denote the electromotive forces for circuits with the metals A and B, B and C, C and D, etc., then the electromotive force for metal A and N is given by

$$E_{\mathrm{A}}^{\mathrm{N}} = E_{\mathrm{A}}^{\mathrm{B}} + E_{\mathrm{B}}^{\mathrm{C}} + E_{\mathrm{C}}^{\mathrm{D}} + \ldots . E_{\mathrm{M}}^{\mathrm{N}}.$$

From this second law it follows that the junction of two metals may be soldered or a galvanometer may be inserted as in Fig. 375 without interfering with the result.

Examples. (1) *The thermo-electric powers of iron and nickel with respect to lead are* + 12 *and* − 20 *microvolts respectively at* 50° *C. Find the E.M.F. of an iron-nickel couple with junctions at* 0° *C. and* 100° *C.* (B.E.)

A metal A is said to be positive to another metal B if the thermo-current flows from A to B through the *cold* junction. In the example the thermo-electric power for iron and nickel at 50° C. is 32 microvolts, and the E.M.F. with junctions at 0° C. and 100° C. is $32 \times 100 = 3200$ microvolts (Art. 210).

(2) *The thermo-electric powers of iron and copper with respect to lead are* + 10·5 *and* + 3·5 *microvolts respectively at* 100° *C. Find the E.M.F. of a copper-iron couple with junctions at* 50° *C. and* 150° *C.*

In this example the thermo-electric power for copper and iron at 100° C. is 7 microvolts, and the E.M.F. with junctions at 50° C. and 150° C. is 7 × (150 − 50) = 700 microvolts.

212. Peltier Effect.—Fig. 376 again depicts a copper-iron couple, but with a battery included in the circuit. Experiment shows that with current passing in the direction indicated heat is *absorbed* at the junction *B* and *generated* at the junction *A*, and by comparing Figs. 375 and 376 it will be seen that that particular junction is cooled which must be heated in order to give a thermo-current in the same direction as the battery current; thus in Fig. 376 the junction *B* would have to be *heated* in order to give a thermo-current in the direction of the arrows, and when the battery gives a current in this direction the junction *B* is *cooled.* This discovery was made by Peltier in 1834, and it is known as the **Peltier Effect.** It follows directly from this that when a thermo-electric current flows in a circuit such as Fig. 375 heat will be absorbed at the hot junction and liberated at the cold junction.

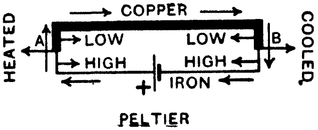


Fig. 376.

The explanation of the Peltier Effect lies in the fact that contact of dissimilar substances gives rise to potential differences. When iron touches copper there is a contact *PD*, the iron being *above* the copper; thus at *B* (Fig. 376) the current traverses a junction where there is an up-gradient of potential, so that it gains energy which is absorbed as heat and a cooling effect results; at *A* the current traverses a junction in the direction of the down-gradient of potential, so that it gives out energy and a heating effect results. *The energy absorbed or evolved at a junction when the unit e.m. current flows for one second,* i.e. *when the unit e.m. quantity passes, measures what is termed the* **Peltier Coefficient**; this coefficient is not a constant, but depends upon the temperature of the junction.

If Π be the Peltier Coefficient at a junction, then the energy absorbed or evolved when a current I e.m. units flows for t seconds is ΠtI ergs; but this energy is also equal to EtI ergs, where E is the potential difference at the junction in e.m. units; thus *the Peltier coefficient is numerically equal to the potential difference at the junction in e.m. units.*

In practice the Peltier Effect is masked by the Joule Heating Effect, but the existence of the Peltier Effect may be shown and the value of the coefficient determined by an experiment of the following type. Let (say) a copper-iron junction be immersed in water, and let a current I pass for t seconds in the direction iron to copper; if H_1 be the heat produced and J the mechanical equivalent of heat,

$$JH_1 = I^2Rt + \Pi tI.$$

If H_2 be the heat produced when the same current passes for the same time in the opposite direction,

$$JH_2 = I^2Rt - \Pi tI.$$

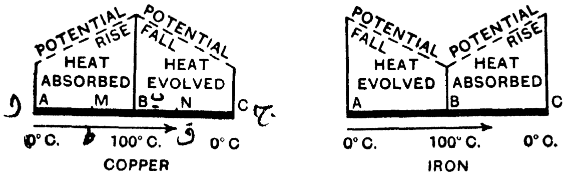
The Peltier Effect is a heating effect in the first case and a cooling effect in the second case. By subtraction

$$\Pi = \frac{J(H_1 - H_2)}{2It}.$$

213. Thomson Effect.—It has been indicated that when a thermo-electric current passes in a couple circuit heat is absorbed at the hot junction and evolved at the cold junction, and this seemed to indicate the source of energy in a thermo-electric couple—the heat absorbed at the hot junction may be greater than that evolved at the cold junction, and the difference between the two may be the source of energy to which the current in the circuit is due; further investigation, however, indicates that this statement requires some modification and extension.

From theoretical considerations Sir William Thomson was led to assume that the Peltier Effect is zero when the thermo-electric power is zero. Now if the temperature of a junction is at the neutral point for the couple, then at that junction there is, as will be better seen later, no thermo-electric power and therefore there is no Peltier Effect. If, however, this junction is the hot junction, then we meet with a difficulty in explaining where the energy of the current is derived from; for if the Peltier Effect at the hot junction is zero *no heat is absorbed there,* and at the cold junction *heat is given out,* hence heat must be absorbed in a thermo-electric circuit at other points than at the junctions. This result was first pointed out by Sir William Thomson, and he showed that *heat must be absorbed in one or both of the wires* in virtue of the difference of temperature between the ends, or that *heat may be absorbed in one wire and generated in the other,* but that the quantity of heat absorbed in the couple circuit must on the whole be greater than that given out. This effect—the absorption or evolution of heat due to the flow of a current in an unequally heated conductor—is known as the **Thomson Effect,** and an explanation similar to that for the Peltier Effect may be given.

In an unequally heated conductor different parts are at different potentials. In the case of copper the hotter parts are at a higher potential than the colder parts; hence if a current passes as indicated along the copper wire of Fig. 377 (*a*) heat will be absorbed in the part *AB* and evolved in the part *BC*; on the whole the Joule Heating Effect will predominate, but if two points *M* and *N* equidistant from *B* be selected, *N* will be at a higher temperature than *M* owing to the effect we are considering. In the case of iron the colder parts are at the higher potential; hence if a current passes as indicated along the iron wire of Fig. 377 (*b*) heat will be evolved in the part *AB* and absorbed in the part *BC.*



(*a*) Fig. 377. (*b*)

To summarise: heat is *absorbed* when a current flows from "cold" to "hot" in copper, and heat is *evolved* when the current flows from hot to cold; heat is *evolved* when a current flows from cold to hot in iron, and heat is *absorbed* when the current flows from hot to cold.

The Thomson Effect in silver, zinc, antimony, and cadmium resembles that in copper, and is said to be positive; bismuth, cobalt, platinum, and nickel resemble iron, and the Thomson Effect in them is said to be negative. In lead the Thomson Effect is probably *nil.*

In any conductor consider two points very close together at temperatures T and $T + dT$. The *difference of potential* between the two points may be expressed as σdT, and the *energy absorbed* (or evolved) when a current I flows for a time t is $It\sigma dT$, where σ may be called the **Thomson Coefficient**; from the first expression we may say that *the Thomson Coefficient is numerically equal to the difference of potential per degree Centigrade*, and from the second that *it is numerically equal to the energy absorbed or evolved per unit quantity per degree Centigrade.*

The coefficient σ is not a constant, but a function of the temperature. It has been called the **specific heat of electricity**, the term being derived from the following analogy. Imagine AC in Fig. 377 (a) to represent a copper tube heated in the same way as the bar, and that a liquid flows through the tube under the condition that it takes the temperature at each point of the tube as it flows along. Under this condition of flow it is evident that the liquid will absorb heat in flowing from A to B and give out heat from B to C. Also, if s denote the specific heat of the liquid, the heat absorbed or evolved by unit mass in passing from one point to another through a difference of temperature dT is $s \cdot dT$. That is, the thermal effect, when a current flows along a conductor for which the Thomson Effect is positive, is analogous to this case of liquid flow along a tube, and σ the coefficient of the Thomson Effect corresponds to s the specific heat of the liquid. For metals in which the Thomson Effect is positive σ is positive, and for metals with a negative Thomson Effect σ is negative. Hence, in the former class of metals the specific heat of electricity is said to be positive, and in the latter class it is said to be negative. That is, in the positive class electricity is supposed to behave like a real liquid in the absorption and evolution of heat, but in the negative class it behaves like a hypothetical liquid which absorbs heat in cooling and gives out heat in heating.

To determine the energy absorbed *by unit quantity* of electricity in passing along a conductor from a point where the temperature is T_1 to one where the temperature is T_2 it is evident that, since σ is not a constant but varies with the temperature, the difference of temperature $T_2 - T_1$ must be divided into infinitely small steps each denoted by dT; then the energy gained at each step is $\sigma . dT$, where σ has its proper value for each step and $\int_{T_1}^{T_2} \sigma dT$ is the energy gained by unit quantity of electricity in passing from a point at temperature T_1 to one at temperature T_2.

Now consider the thermo-electric circuit shown in Fig. 375. The current passes from cold to hot along the copper, and therefore energy is absorbed as heat (Thomson); in traversing the junction B there is a further absorption of energy as heat (Peltier); in passing along the iron from hot to cold there is again an absorption of energy as heat (Thomson); in traversing the junction A heat is evolved (Peltier). On the whole more heat is absorbed than evolved and the difference is the source of energy to which the current is due.

214. The Electromotive Force in a Thermo-Electric Circuit.—By considering the energy absorbed and evolved in a thermo-electric circuit it is possible to find an expression for the electromotive force in the circuit. Take the case of a circuit with junctions at temperature T_1 and T_2, the latter temperature being the higher. Let Π^1 and Π_2 denote the Peltier coefficients at T_1 and T_2, and σ_A and σ_B the Thomson coefficients for the metals A and B of the circuit. Then, assuming the current to pass from A to B at the hot junction, the energy *gained by unit quantity* of electricity in passing round the circuit is

Π_2 for the Peltier Effect at the junction at temperature T_2,

$-\Pi_1$ for the Peltier Effect at the junction at temperature T_1,

$\int_{T_1}^{T_2} \sigma_A dT$ for the Thomson Effect in metal A,

$-\int_{T_1}^{T_2} \sigma_B dT$ for the Thomson Effect in metal B.

Hence the total gain for the complete circuit is

$$\Pi_2 - \Pi_1 + \int_{T_1}^{T_2} (\sigma_A - \sigma_B)\, dT,$$

and since this total must be numerically equal to the total electromotive force in the circuit we have

$$E = \Pi_2 - \Pi_1 + \int_{T_1}^{T_2} (\sigma_A - \sigma_B) dT \text{............} (1)$$

If we consider a circuit with junctions at temperatures T and $T + dT$, where dT is infinitely small, the result is more simply expressed, for the Peltier coefficients are Π and $\Pi + d\Pi$, where $d\Pi$ is the increment or difference in Π corresponding to the difference in temperature dT, and σ_A and σ_B are the values of these coefficients for the temperature T. Hence the four gains of energy given above are respectively $\Pi + d\Pi$, $-\Pi$, $\sigma_A dT$, and $-\sigma_B dT$, and we therefore have for dE the infinitely small electromotive force in the circuit

$$dE = d\Pi + (\sigma_A - \sigma_B) dT \text{..............} (2)$$

215. Preliminary Ideas on the Thermo-Electric Diagram.—The student will more readily grasp the details of succeeding sections if one or two preliminary statements be made at this stage on the *thermo-electric diagram*; these statements will be *proved* later.

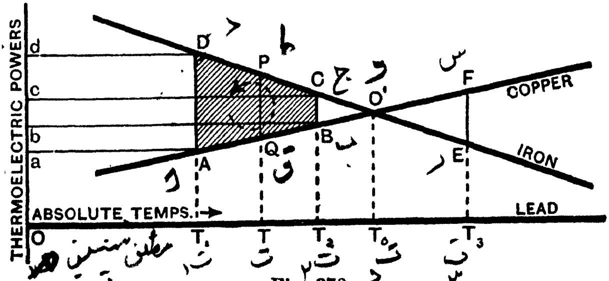


Fig. 378.

In the diagram *absolute temperatures* are taken as abscissae and *thermo-electric powers* as ordinates, and lead is taken as the base line since the Thomson Effect in lead is *nil*; the thermo-electric power lines are straight lines and Fig. 378 gives the diagram for copper, iron, and lead. The figure is *not* drawn to scale.

In Fig. 378 T_1A = the thermo-electric power for lead and copper at absolute temperature T_1 and T_1D = the thermo-electric power for lead and iron at the same temperature T_1; hence AD = the thermo-electric power for copper and iron at absolute temperature T_1. Similarly BC = the thermo-electric power for copper and iron at T_2; at T_0 (270° C.) the thermo-electric power for copper and iron is *nil.*

Consider now a copper iron couple with junctions at T_1 (0° C.) and T_2 (200° C.); a thermo-electric current will be flowing in the direction copper to iron through the hot junction. On the diagram the total E.M.F. under these conditions is represented by the area of the trapezium $ABCD$ and as this area $= \overline{PQ} \times \overline{T_1T_2}$, PQ being the thermo-electric power at 100° C. and T_1T_2 being the difference in temperature of the junctions, we have the fact previously given, viz.—

The total E.M.F. is given by the product of the thermo-electric power at the mean temperature of the junctions and the difference in temperature of the junctions.

The current passes along the copper from cold to hot (T_1 to T_2) and *energy is absorbed.* On the diagram this Thomson Effect (energy absorbed per unit quantity) is represented by the area $aABb$, and as this is given by $ab \times OT = (Ob - Oa) \times OT$ we have the rule—

The Thomson Effect in the copper is given by the product of the mean absolute temperature (OT) *and the difference between the thermo-electric powers (copper-lead couple) at the temperatures of the junctions* ($Ob - Oa = T_2B - T_1A$).

The current passes from copper to iron through the hot junction and *energy is absorbed.* On the diagram this Peltier Effect (energy absorbed per unit quantity) is represented by the area $bBCc$, and as this is given by $BC \times bB = BC \times OT_2$, we have the rule—

The Peltier Effect at the hot junction is given by the product of the thermo-electric power of copper and iron at this temperature and the absolute temperature.

The current passes along the iron from hot to cold (T_2 to T_1) and in this case *energy is again absorbed.* On the diagram this Thomson Effect is represented by the area $cCDd$, and the rule for its calculation is similar to that given above for the copper.

The current passes from iron to copper through the cold junction and *energy is evolved.* On the diagram this Peltier Effect is represented by the area $aADd$, and the rule for its calculation is similar to that given for the hot junction.

Clearly the total energy absorbed per unit quantity is represented by

$$aABb + bBCc + cCDd - aADd = ABCD,$$

and this, as previously indicated, represents the E.M.F.

If the copper-iron couple has cold junction 0° C. and hot junction 270° C., the E.M.F. is represented by the area of the triangle $AO'D$ and is a maximum (Art. 210). If the hot junction be T_3 the E.M.F.

is represented by the area *AO'D minus* the area *EO'F* (the copper line is now above the iron line) and is therefore less (Art. 210). If T_3 be 540° C. the two areas *AO'D* and *EO'F* will be equal, and the E.M.F. will be zero (Art. 210). If the hot junction be above 540° C. (cold junction still 0° C.) the resultant area will appear on the right of the diagram, *i.e.* the E.M.F. will now be reversed (Art. 210).

In the sections which follow the facts merely stated here will be *proved.*

216. Application of Thermo-Dynamics.—If the principles of thermo-dynamics be applied to a thermo-electric circuit several important relations may be deduced. In such a circuit the operations are reversible if we neglect the Joule Heating Effect in the conductors, and when the electromotive force in the circuit is infinitely small the current is infinitely small, and therefore the Joule Heating Effect, being proportional to the square of the current, is negligible. The Peltier and Thomson Effects being directly proportional to the current are reversible in the thermo-dynamic sense. Hence, in the thermo-electric circuit (Art. 214), with junctions at *absolute* temperatures T and $T + dT$, since quantities of energy $\Pi + d\Pi$, $-\Pi$, $\sigma_A dT$, and $-\sigma_B dT$ are absorbed at temperatures $T + dT$, T, T, and T, it follows thermo-dynamically that

$$\frac{\Pi + d\Pi}{T + dT} - \frac{\Pi}{T} + \frac{(\sigma^A - \sigma_B)}{T} dT = 0 \,\ldots\ldots\ldots (1)$$

or

$$d\Pi - \frac{\Pi}{T} dT + (\sigma_A - \sigma_B)dT = 0.$$

$$\therefore\ (\sigma_A - \sigma_B)dT = \frac{\Pi}{T} dT - d\Pi \,\ldots\ldots\ldots\ldots (2)$$

and substituting this value in the expression given in Art. 214 for dE, viz. $dE = d\Pi + (\sigma_A - \sigma_B)dT$, we get

$$dE = \frac{\Pi}{T} dT \,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

and

$$\Pi = T\frac{dE}{dT} \,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

This relation indicates the only practicable method of measuring Π; it means that for any two substances *the*

Peltier Coefficient at any temperature is given by the product of the thermo-electric power at that temperature and the absolute temperature (Art. 215).

From (2) it follows that if for metal A the value of σ be zero (lead), then

$$-\sigma_B = \frac{\Pi}{T} \quad \frac{d\Pi}{dT} \qquad \text{.................. (5)}$$

This result expresses the coefficient of the Thomson Effect at temperature T for the metal B in terms of the Peltier Coefficient for B and a metal in which the Thomson Effect is zero.

Again, from (2)—

$$\sigma_A - \sigma = \frac{\Pi}{T} - \frac{d\Pi}{dT}.$$

Now
$$\Pi = T\frac{dE}{dT},$$

and, by differentiating,

$$\frac{d\Pi}{dT} = T\frac{d^2E}{dT^2} + \frac{dE}{dT}.$$

Hence

$$\sigma_A - \sigma_B = \frac{dE}{dT} - \left(T\frac{d^2E}{dT^2} + \frac{dE}{dT}\right),$$

$$\therefore \ \sigma_A - \sigma_B = -T\frac{d^2E}{dT^2} \qquad \text{.................. (6)}$$

That is, if A is a metal for which σ is zero,

$$\sigma_B = T\frac{d^2E}{dT^2} \qquad \text{.................... (7)}$$

and if dE/dT be denoted by y,

$$\sigma_B = T\frac{dy}{dT} \qquad \text{.................... (8)}$$

$$\therefore \ \sigma_B dT = T \, . \, dy \qquad \text{.................... (9)}$$

This indicates that *the Thomson Effect in a metal with ends at temperatures T and $T + dT$ is given by the product of the absolute temperature and the difference of the thermo-electric powers at the ends* (Art. 215). If the ends are at

T_1 and T_2 respectively, the Thomson Effect is given by $\int_{T_1}^{T_2}\sigma_B dT$. Now it is shown in Art. 217 that the curves obtained by plotting temperatures as abscissae and *total E.M.F.'s* as ordinates are parabolic, and can be represented by an equation of the form $E = aT + bT^2$; hence $dE/dT = y = a + 2bT$, and we may write $T = A + By$. Let y_1 = thermo-electric power at T_1 and y_2 = thermo-electric power at T_2; then—

$$\int_{T_1}^{T_2}\sigma_B dT = \int_{T_1}^{T_2}T dy = \int_{y_1}^{y_2}(A + By)dy$$

$$= \left[Ay + B\frac{y^2}{2}\right]_{y_1}^{y_2}$$

$$= A(y_2 - y_1) + \tfrac{1}{2}B(y_2^2 - y_1^2)$$

$$= (y_2 - y_1)\left(A + \frac{B}{2}(y_2 + y_1)\right)$$

$$= (y_2 - y_1)\cdot\tfrac{1}{2}(A + By_2 + A + By_1)$$

$$= (y_2 - y_1)\frac{T_2 + T_1}{2} \qquad\text{(10)}$$

Thus *the Thomson Effect in a metal with junctions at T_1 and T_2 is given by the product of the difference of the thermo-electric powers at the ends and the mean absolute temperature* (Art. 215). Remember that the second metal for the couple is one for which σ is zero.

آصفیہ [illegible]

217. Thermo-Electric Curves.—If one junction of a couple with the metals *A* and *B* be kept at 0° C. and the E.M.F. in the circuit be measured, by methods to be described later, when the other junction is adjusted to a number of known successive temperatures it will be possible to plot a curve showing **the relation between the total electromotive force in the couple and the difference of the temperatures at the junctions.** The form of this curve in practically all cases is that shown in Fig. 379, and it is found to approximate very closely to a parabola with its axis parallel to the axis of electromotive force.

The curve *OAB* in the figure indicates that with one junction at 0° C. the electromotive force in the circuit increases as the temperature of the other junction is raised from 0° C. to 200° C., and thence decreases till its temperature is 400° C., where the electromotive force is reversed. In normal cases it goes on increasing in the reversed direction indefinitely, the curve beyond *B* being a geometrical continuation of the parabolic segment *OAB*.

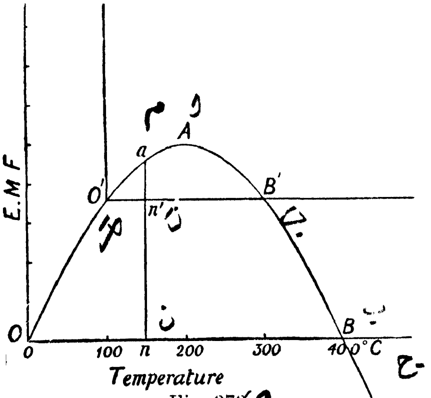


Fig. 37~~8~~9

If, with this couple, the junction of constant temperature be maintained at 100° C. instead of 0° C., then the curve of electromotive force is obtained from *OAB* simply by changing the origin *O* to *O'* and taking *O'B'* instead of *OB* as the axis of temperature. This is evidently in accordance with the law of successive temperatures, for the relation $E_0^{150} = E_0^{100} + E_{100}^{150}$ is correctly represented in the figure by the geometrical relation $na = nn' + n'a$, where na represents E_0^{150}, $nn' = E_0^{100}$, and $n'a = E_{100}^{150}$.

So far we have considered the case of a couple made with the metals *A* and *B*, and *OAB* in Fig. 379 is the curve for the couple. If now, in the same way, we obtain the curves for other couples made with the metals *A* and *C*, *A* and *D*, *A* and *E*, and so on, we shall have a set of curves similar to the three shown in Fig. 380, the curve *OAB* being for the metals *A* and *B*, *OAC* for the metals *A* and *C*, and *OAD* for the metals *A* and *D*. The curve *OAD* is drawn with its ordinates negative to indicate that for the metals A and D the direction of the current, under given conditions, is opposite to that in the A, B and A, C circuits under the same conditions. If, for example, the B conductor in the A, B couple be replaced by the D conductor the direction of the current in the circuit is reversed. When the current passes in the standard metal,

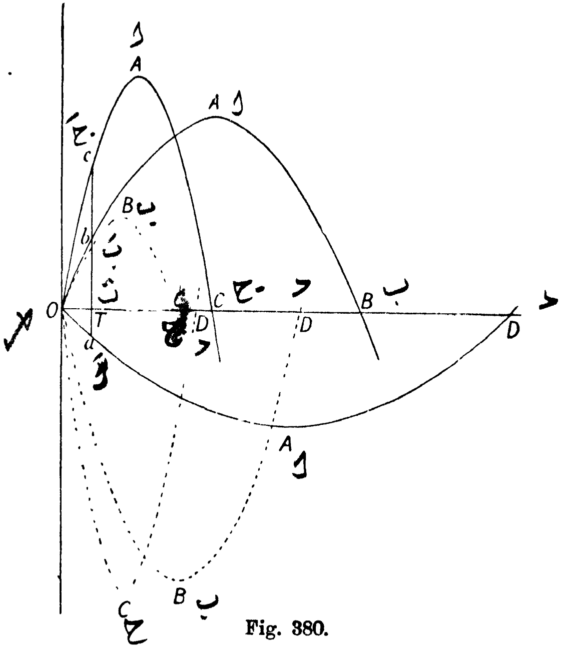


Fig. 380.

A, from cold to hot, the direction of the current is to be taken as positive.

From these curves by application of the law of successive contacts the curves for a B, C, a B, D, or a C, D couple may be obtained. By this law we have $E^B_A + E^C_B = E^C_A$, or $E^C_B = E^C_A - E^B_A$. In the figure E^C_A is represented, for the range of temperature represented by OT, by Tc, E^B_A for the same range of temperature by *Tb*, and therefore $E^{\mathrm{C}}_{\mathrm{B}}$ for this range by *bc*. That is, the differences of the ordinates of the curves *OAB* and *OAC* give the ordinates of the curve for the couple *B*, *C*. This curve is shown by the dotted line *OBC* in the figure. Similarly we have $E^{\mathrm{D}}_{\mathrm{C}} = E^{\mathrm{D}}_{\mathrm{A}} - E^{\mathrm{C}}_{\mathrm{A}}$, that is, $E^{\mathrm{D}}_{\mathrm{C}}$ is represented by $Td - Tc = -(dT + Tc) = -dc$; that is, the algebraic differences of the ordinates of the curves *OAC* and *OAD* give the ordinates for the curve for the couple *C*, *D*. This curve is shown by the dotted line *OCD*, and the dotted curve *OBD* for the couple *B*, *D* is obtained in the same way.

From the symmetry of these curves it will be evident that the neutral point, or temperature of maximum electromotive force represented by the abscissa of the point *A*, in each curve is a constant temperature equal to the arithmetic mean of the temperatures of the two junctions at reversal. Also, the temperature of reversal, given by the point where the curve cuts the axis of temperature, is variable and evidently depends upon the temperature of the lower junction, which, as shown in Fig. 379, fixes the position of the origin of the axes of temperature and electromotive force. The reversal temperature is as much above the neutral point as the cold junction is below it (Art. 210).

218. Thermo-Electric Power Lines.—If for each of the preceding parabolic curves a differential curve be drawn (by any graphic method) showing **the relation between the thermo-electric power, *dE/dT*, and the temperature *T* for each couple,** it will be found that this curve is practically a straight line. For example, if for each degree of temperature the *difference* of electromotive force be plotted as the approximate value of *dE/dT* at the middle of the degree, it will be found that the curve obtained approximates very closely to a straight line.

In Fig. 381 let *OAB* be the electromotive force curve for an *A*, *B* couple. Then between the points *a*, *b* on the curve, *dE* is represented by *cb* and *dT* by *ac*, and *dE/dT* therefore by *cb/ac*, that is by tan *bac*, where *bac* is the direction that the curve at *ab* makes with the axis *OB*. Inspection of the curve will show that the tangent of this angle is positive at *O* and decreases from *O* to *A*, where it becomes zero, then, changing sign, it again increases from *A* to *B*. For the curve *OAB*, therefore, *the thermo-electric power line*, *LNM*, starts with a positive ordinate, *OL*, at *O*, crosses the axis of temperature at *N*, the foot of the perpendicular from *A* on *OB*, and continues from *N* onwards with negative ordinates. The point *N*, where the line crosses the axis of temperature and where the thermo-electric power is zero, evidently indicates the neutral point for the given couple.

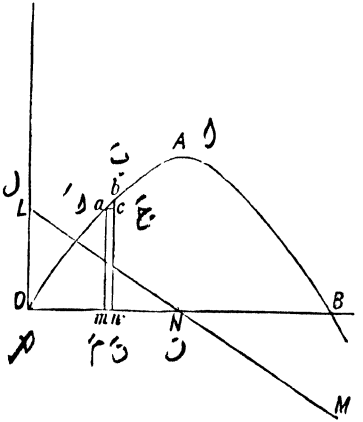


Fig. 381

The electromotive force in a circuit for which the thermo-electric power line is known is readily found. Let *LNM*, Fig. 382, be the thermo-electric power line for a given couple. At any temperature represented by *OA* the thermo-electric power, that is, the value of dE/dT, is represented by the ordinate *Aa*. For a very small increment of temperature, dT, at this temperature the increment in the electromotive force of the couple is given by $(dE/dT) \, . \, dT$, the product of the *rate* of change of electromotive force with temperature and the change of temperature. Hence, if *AB* represent dT, the increment of electromotive force is represented by the rectangle determined by *Aa* and *AB*, and if dT be infinitely small this rectangle is practically equal to the area of the strip *ABba* standing on *AB*. It follows from this that, if one junction of a couple be maintained at a temperature represented by *OP* while the temperature of the other is raised to one represented by OQ, the electromotive force set up in the couple will be represented by the area $PQqp$.

If the temperatures of the junctions of the couple be represented by OP and OQ', then the electromotive force in the circuit is, as before, represented by the area bounded by Pp, $Q'q'$, LNM, and the axis of temperature; but, of this area, PNp to the left of N is positive and $Q'Nq'$ to the right of N is negative, therefore the electromotive force is represented by the *difference* between the two areas PNp and $Q'Nq'$. Hence, if these two areas were equal, that is if N were midway between P and Q', the electromotive force would be zero. That is, when the temperature of the neutral point represented by ON is the arithmetic mean of the temperatures of the junctions represented by OP and OQ' the point of reversal is reached. When PNp is the greater area the electromotive force is positive, and when it is the smaller area the electromotive force is negative (Art. 215).

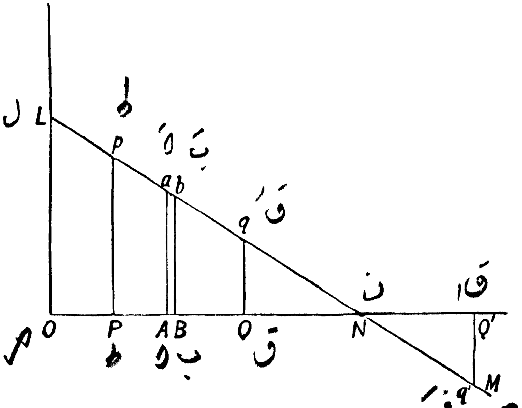


Fig. 382.

If we take the curves OAB, OAC, and OAD in Fig. 380 and draw the corresponding thermo-electric power lines we get lines such as are shown in Fig. 383, AB, AC, and AD being the thermo-electric power lines for the A, B the

A, C and the A, D couples. The points N_1, N_2, N_3, where these lines cross the axis of temperature, are the neutral points for these couples.

The electromotive force for an A, C couple for junction temperatures represented by OP and OQ is represented by the area $PQqp$, and the electromotive force for the A, B couple for the same junction temperatures is represented by $PQsr$· therefore, by the law of successive contacts, the electromotive force in the B, C couple is represented by the area $rsqp$. It follows from this that the differences of corresponding ordinates of the A, C line and the A, B line give the ordinates for the thermo-electric power line of the B, C couple (Art. 215).

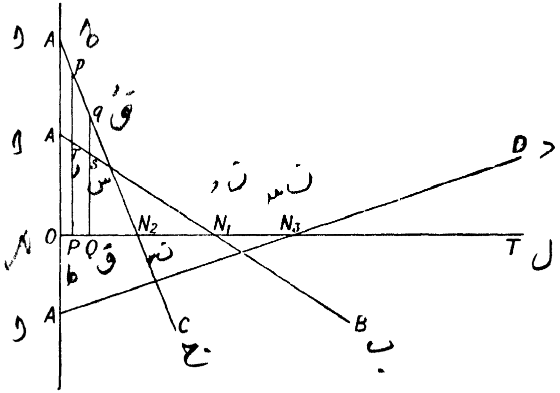


Fig. 383.

Similarly, from the differences of the ordinates of the lines AB, AC, and AD the ordinates of the thermo-electric power lines for the B, D and C, D couples may be obtained. From these differences separate thermo-electric power lines might be plotted from the axis OT for the B, C, the B, D, and the C, D couples, giving lines corresponding to the dotted electromotive force curves OBC, OBD, and OCD in Fig. 380 It would, however, only needlessly complicate the diagram to do this. In representing the relative thermo-electric powers for a number of metals A, B, C, D, etc., it is most convenient to take one of the metals, say A, as a standard and plot the thermo-electric power lines for the couples A and B, A and C, A and D, and so on Then, for any two metals P and Q, the relative thermo-electric power at any temperature may be found, as explained above, from the difference of the ordinates of the A, P and the A, Q lines at that temperature, and the electromotive force for a P, Q couple between any two temperatures may also be found, as already described, from these lines. It will also be evident that the point of intersection of the A, P and the A, Q lines gives the neutral point of the P, Q couple.

If, therefore, we have a diagram with thermo-electric power lines for the couples B and A, C and A, D and A, etc., the axis of temperature may be associated with the metal A, and the lines may be called the thermo-electric power lines for the metals B, C, D, etc., and for a couple made up of the metals P and Q the P and Q lines serve to determine the constants of the couple. A diagram drawn in this way and giving the thermo-electric powers of the metals B, C, D, etc., supplies, with proper conventions as to sign and regard to the position of the origin, all the necessary thermo-electric data for a couple of any two given metals included in the diagram. Such a diagram is called a *thermo-electric diagram.*

219. The Thermo-Electric Diagram. Sign Conventions.

—For theoretical purposes it is most convenient to *take the origin of the axes of temperature and thermo-electric power at the absolute zero of temperature and to express temperature on the absolute scale.* The necessary sign conventions of the diagram are most simply understood by considering the case of a particular couple. Take the case of a couple made with the metals A and B, and suppose both junctions to be at the same temperature T. If at the junctions the metal B is at a higher potential than the metal A, then B is said to be positive to A. If this difference of potential is denoted by Π, then we have at each junction a seat of electromotive force Π tending to send a current from B to A, *not* across the junction, but round through the circuit of the couple. These two electromotive forces are equal and opposite in the same circuit, and therefore balance each other. Similarly, since the two ends of each element of the couple are at the same temperature T, the algebraic sum of the electromotive forces associated with the Thomson Effects in the A and B conductors is zero in each conductor. Hence, when both junctions of this couple are at the same temperature the total electromotive force in the circuit is zero, although at each junction the potential of B is higher than that of A.

Let one of the junctions be now raised in temperature from T to $T + dT$. The electromotive forces set up at the junctions are no longer equal, for the increment of temperature dT at one junction causes an increment $d\Pi$ of the electromotive force at that junction, and their difference is therefore $d\Pi$.

At the same time the electromotive forces associated with the Thomson Effect cease to balance, for there is now a difference of temperature dT between the ends of each conductor, and this gives a difference of potential $\sigma\ .dT$ between the ends of the A conductor tending to send a current through B, and a difference of potential $\sigma_B . dT$ between the ends of the B conductor tending to send a current through A. If σ_A and σ_B are assumed to be positive these differences of potential are opposed to each other, and as in Art. 214 the algebraic sum of the electromotive forces in the circuit is given by

$$dE = d\Pi + (\sigma_A - \sigma_B)\, dT,$$

and is seen to be made up of $d\Pi$, the increment of Π at the junction, and $(\sigma_A - \sigma_B)\, dT$, the increment due to the Thomson Effect in the A and B conductors.

This electromotive force is for diagram purposes taken as positive or negative according as the current passes in the standard metal of the diagram from cold to hot or hot to cold. As the current always passes (except in the case illustrated by Fig. 385) at the hot junction from the negative to the positive metals, this convention involves that dE is positive when the metal coupled with the standard metal is positive to the standard, and negative when this metal is negative to the standard.

From what has been said we can now deduce the sign conventions that apply to thermo-electric power lines. When, at any temperature T, the increment of electromotive force dE for a positive increment of temperature dT is positive, then the thermo-electric power dE/dT is positive. If the line AB in Fig. 383, giving the thermo-electric power line for the A, B couple, be taken as the line for the metal B and the axis of temperature as the line for the metal A taken as a standard, then the diagram is evidently so drawn that at any temperature for any two metals the thermo-electric power of the positive metal of the two is the greater. Thus in Fig. 384 the thermo-electric power lines BB and CC for the metals B and C indicate that up to the neutral point, represented by On, the metal C is positive to B, but for temperatures above the neutral point B is positive to C.

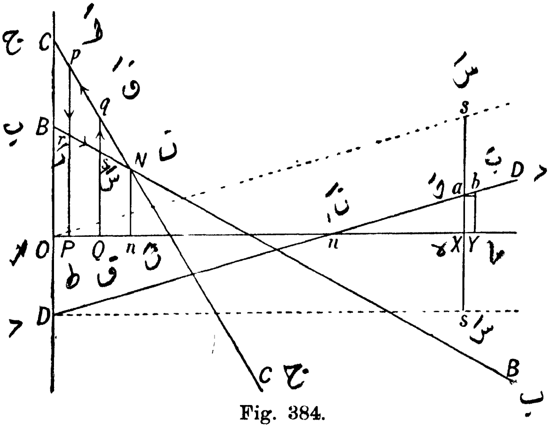


Fig. 384.

Also, if the temperatures of the junctions of a B, C couple be represented by OP and OQ (Fig. 384), then the direction of the current at the hot junction will, in accordance with what has been said above, be from B to C. This may be indicated on the diagram by an arrow from

B to *C*, and, in the same way, the direction of the current in each element of the couple and at the cold junction may be indicated by arrows round the area *sqpr*, representing, as previously explained, the electromotive force in the circuit.

In Fig. 385, if the temperatures of the junctions are represented by *OP* and *OQ* and arrows be drawn, as above, to indicate the direction of the current in the circuit, we get the area *sqnrpns* to represent the electromotive force, and, of this area, the part *npr* is positive and *nqs* negative, so that the electromotive force is represented by the difference of the two areas. It should be noticed in this figure that the arrow in *sq* is in the direction opposite to that given above for the hot junction. In this case, where the neutral point lies between the junction temperatures, the current passes at the hot junction in a direction opposed to that in which the electromotive force at the junction tends to send it so long as the area *pnr* is greater than *nqs*, that is, so long as *rp* is greater than *sq*.

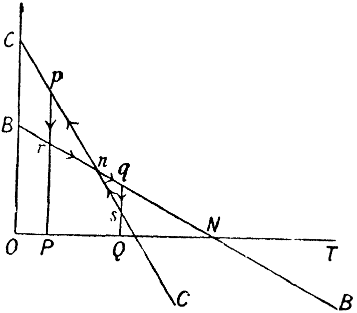


Fig. 385.

220. The Thermo-Electric Diagram. Representation of the various Quantities on the Diagram.

(1) *The Thomson Coefficient.*—Since the thermo-electric power of the more positive of two metals is the greater, it is evident that on the diagram a path with increasing ordinates is also a path of increase of potential. Hence, in going along the *B* and *C* lines, Fig. 384, from cold to hot, the potential falls, and therefore, by Art. 213, σ, the coefficient of the Thomson Effect, is negative. In the case of the *D* line, however, the potential rises from cold to hot, and therefore σ is positive. It must be remembered, how-

ever, that these lines are supposed to be drawn relative to metal A as a standard, and that, therefore, *unless the metal A is one for which σ is zero*, all that can be deduced from the above argument is that $(\sigma_B - \sigma_A)$ and $(\sigma_C - \sigma_A)$ are negative and $(\sigma_D - \sigma_A)$ positive.

From Art. 216 we get

$$\sigma_A - \sigma_B = - T\frac{dy}{dT} \text{ or } \sigma_B - \sigma_A = T\frac{dy}{dT},$$

where y denotes dE/dT, the thermo-electric power. Now, taking the A, D line (Fig. 384) for which $(\sigma_D - \sigma_A)$ is positive, the value of y at temperature T, represented by OX, is represented by Xa, and for a small increment of temperature, dT, represented by XY, the increment dy is represented by bc. Hence dy/dT is represented by bc/ac, the tangent of the angle which the A, D line makes with the positive direction of the axis of temperature.

Hence, if through the origin O, *taken at the absolute zero of temperature*, a line Os is drawn parallel to DD, then *the value of $(\sigma_D - \sigma_A)$ at any temperature T, represented by OX, is given by the ordinate Xs*, for

$$\frac{Xs}{OX} = \tan XOs,$$

$$\therefore\ Xs = OX \,.\, \tan XOs = T \,.\, \tan anX = T\frac{dy}{dT}.$$

By an evident alternative construction *the line da may also be taken to represent $\sigma_D - \sigma_A$*. It will be seen here that when dy/dT is positive, that is, when the thermo-electric power line slopes upwards like DD, the value of $(\sigma_D - \sigma_A)$ is positive.

Experiment has shown that for lead the value of σ is zero or negligibly small, so that by taking lead as the standard metal A the value of σ_A may be taken as zero, and $(\sigma_B - \sigma_A)$, $(\sigma_C - \sigma_A)$, etc., become simply σ_B, σ_C, σ_D, etc., the Thomson Coefficients for the metals for which the lines are drawn; *thus Xs or da represents the Thomson Coefficient for the metal D.* It must be remembered that the magnitude of σ for any metal can be obtained from the diagram in the way described, only if the origin of the axes is taken at the absolute zero of temperature.

(2) *The Thomson Effect.*—The absorption or evolution of energy associated with the Thomson Effect can also be represented. A path with increasing ordinates is one of absorption of energy; thus energy is absorbed in C and evolved in B (Fig. 386), for with junction temperatures OP and OQ the 'path' is q to p in C and r to s in B. Further, for any very small difference of temperature, dT, represented by XY, in say the C conductor, the energy absorbed for unit quantity of electricity is given by $-\sigma_C dT$. Now σ_C is negative, so that $-\sigma_C dT$ is a positive quantity, and it is represented by ax, and XY, which represents dT, is equal to ab, therefore $\sigma_C dT$ is represented by the area $ax \times ab$, that is, by the strip $axyb$, when dT is infinitely small. From the geometry of the diagram the strip $cxyd$ is equal to $axyb$, and may therefore be taken to represent the energy absorbed in the element xy of the C conductor. It follows at once from this that *the total energy absorbed in the C conductor, the junction absolute temperatures being OP and OQ, must be represented by the area mqpl.* Similarly *the energy evolved in the B conductor is represented by the area nrso.* (In Art. 215, Fig. 378, energy is *absorbed* in *both* conductors.)

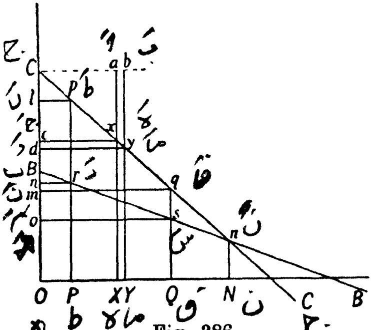


Fig. 386.

(3) *The Peltier Effect.*—Energy is absorbed at the hot junction and evolved at the cold junction. The energy absorbed at the hot junction is, for unit quantity of electricity, measured by Π_2, the Peltier Effect at temperature T_2, and as, by Art. 216,

$$\Pi = T\frac{dE}{dT} \quad \text{we have}$$

$$\Pi_2 = T_2\frac{dE}{dT}.$$

Now in the diagram (Fig. 386), at temperature T_2, dE/dT is represented by sq and T_2 by OQ or os, therefore Π_2 *is* *represented by the area os × sq, that is, by the rectangle osqm.* Similarly Π_1, the Peltier Effect at the cold junction, is given by $T_1\frac{dE}{dT}$, and *is represented by the area nr × rp or the rectangle nrpl.*

(4) *Total E.M.F.*—The total energy absorbed in the above circuit (Fig. 386) is represented by *osqm* + *mqpl*, that is, by the area *osqpl*, and the total energy evolved is represented by *lprn* + *nrso*, that is, by the area *lprso*. The energy dissipated in the circuit is consequently represented by *osqpl* — *lprso*, that is, *by the area rsqp*. This area therefore represents the energy spent in the circuit for each unit quantity of electricity travelling round it. *It therefore represents the electromotive force in the circuit,* in accordance with the result given above.

The magnitude of the electromotive force in the circuit can also be determined from the diagram by finding an expression for the measure of the area *pqsr*. This area is measured by the product of one half the sum of the parallel sides and the perpendicular distance between them, that is, by

$$PQ \times \tfrac{1}{2}(pr + sq).$$

If the diagram is drawn to scale this value is readily determined. It is numerically the thermo-electric power at the mean temperature multiplied by the difference in temperature.

It can also be reduced to a formula, for PQ represents $T_2 - T_1$, and, from the figure, $sq/rp = NQ/NP$, where ON represents T_n, the neutral point of the B, C couple. That is $\frac{sq}{rp} = \frac{T_n - T_2}{T_n - T_1}$ or $sq = k(T_n - T_2)$ and $rp = k(T_n - T_1)$, where k is a constant. This gives for E, the electromotive force in the circuit, the expression

$$E = (T_2 - T_1)k\left(T_n - \frac{T_1 + T_2}{2}\right)$$

or

$$E = k(T_2 - T_1)\left(T_n - \frac{T_1 + T_2}{2}\right).$$

This expression shows that E is zero when $T_2 = T_1$, that is, when both junctions are at the same temperature, and also when $T_n = (T_1 + T_2)/2$, that is, at the point of reversal when the neutral point is the arithmetic mean of the junction temperatures.

Fig. 391*a*, page 199, gives a thermo-electric diagram for a number of metals.

221. The Thermo-Electric Couple treated Analytically. —The total E.M.F. curves being practically parabolas may, when the origin is at the absolute zero, be expressed by the equation

$$E = aT + bT^2 \quad\text{.............................}\quad (1)$$

where E denotes the electromotive force in a couple with one junction at absolute zero and the other at temperature T on the absolute scale, and a and b are constants depending upon the metals of the couple.

It follows from this that the electromotive force for a couple with junctions at temperatures T_1 and T_2 is given by

$$E_{T_1}^{T_2} = a(T_2 - T_1) + b(T_2^2 - T_1^2) \quad\text{..............}\quad (2)$$

Further, from the equation

$$E = aT + bT^2$$

we get, by differentiating,

$$\frac{dE}{dT} = a + 2bT.$$

Or, representing dE/dT by y, we have

$$y = 2bT + a \quad\text{................................}\quad (3)$$

This is the equation of a straight line in which $2b$ represents, in the usual way, the tangent of the angle made by the line and the positive direction of the axis of temperature, and a is the intercept on the axis of thermo-electric power.

Also from $$y = 2bT + a$$

we get $$\frac{dy}{dT} = 2b,$$

and since $$\sigma_B - \sigma_A = T\frac{dy}{dT}$$

we have $$\sigma_B - \sigma_A = 2bT,$$

or, if $\sigma_A = 0$, we have

$$\sigma_B = 2bT \quad\text{....................................}\quad (4)$$

Thus at any temperature

$$\sigma_B = y - a,$$

where y is the thermo-electric power at that temperature and a the intercept of the B line on the axis of thermo-electric power.

At the neutral point for any two metals the thermo-electric power dE/dT is zero, and therefore from

$$y = 2bT + a$$

we get $$2bT_n + a = 0,$$
where T_n denotes the neutral point.

This gives $$T_n = -\frac{a}{2b} \quad \text{.......} \quad (5)$$

and $$a = -2bT_n \quad \text{.......} \quad (6)$$

Substituting this value of a in the expression for $E_{T_1}^{T_2}$ we get

$$E_{T_1}^{T_2} = 2b(T_2 - T_1)\left(\frac{T_2 + T_1}{2} - T_n\right) \quad \text{.......} \quad (7)$$

This corresponds with the formula given above if $k = -2b$.

For the value of Π we have, from the relation

$$\Pi = T\frac{dE}{dT},$$

by substituting for dE/dT, the equation (3)

$$\Pi = aT + 2bT^2 \quad \text{.......} \quad (8)$$

Also in a couple with junctions at temperature T_1 and T_2 the energy absorbed in the circuit per unit quantity of electricity is given by

$$\Pi_2 = aT_2 + 2bT_2^2 \text{ at the } T_2 \text{ junction,}$$
$$-\Pi_1 = -(aT_1 + 2bT_1^2) \text{ at the } T_1 \text{ junction,}$$
$$\int_{T_1}^{T_2} \sigma_A dT - \int_{T_1}^{T_2} \sigma_B dT = \int_{T_1}^{T_2} (\sigma_A - \sigma_B) dT$$
$$= -2b\int_{T_1}^{T_2} TdT = -b(T_2^2 - T_1^2)$$

in the two conductors.

Hence the total energy absorbed in the circuit is given by the sum of these quantities, and is equal to

$$a(T_2 - T_1) + b(T_2^2 - T_1^2) \quad \text{.......} \quad (9)$$

This is the measure of the electromotive force in the circuit. (See (2).)

If the thermo-electric power at 0° C. be denoted by a_0, the thermo-electric power at t° C. is evidently given by $a_0 + 2bt$. The values of a_0 and $2b$ for certain metals are given on p. 197. They apply to a range of temperature extending from -200° C. to 100° C. The values are such as to give the thermo-electric power in microvolts per degree centigrade.

	a_0	$2b$
Antimony	1·800	·02817
Bismuth	−72·630	−·08480
Copper	2·815	·00683
Cobalt	−15·582	−·07340
Iron	15·087	−·01330
Mercury	−4·460	−·00860
Nickel	−16·050	−·05639
Silver	2·960	·00714
Zinc	2·713	·01040

222. Abnormal Metals.—The axes of the parabolas in Figs. 380 and 381 are assumed parallel to the E.M.F. axis. Recent experiments indicate, however, that (when coupled with lead) copper, nickel, aluminium, platinum, cadmium, and manganese give *inclined* axes, and hence the thermo-electric power lines will not be straight lines. The general shape of the nickel line is shown in Fig. 387 and of the iron line *at high temperatures* in Fig. 388. Other normal metals may become abnormal at sufficiently high temperatures.

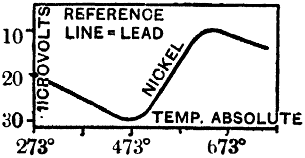


Fig. 387.

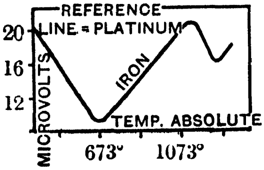


Fig. 388.

223. Practical Applications.—The **Thermopile** (Fig. 389) consists of a number of bismuth-antimony couples arranged in series so as to multiply the effect. If one set of junctions be protected as indicated, while heat be allowed to fall on the other set, the galvanometer joined to the instrument will be deflected, and the deflection may be used as a measure of the heat falling on the exposed junctions. The main defect of the thermopile is its large mass, resulting in a sluggish response to changes in temperature.

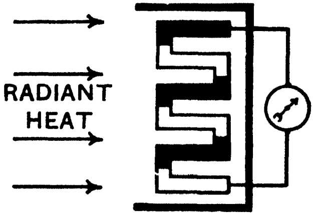


Fig. 389.

Boys' Radio-Micrometer is similar to the **Duddell Thermo-Galvanometer** (Art. 180), the heater being omitted. The moving coil consists of a single loop of copper suspended between the poles by a quartz fibre. The two lower ends of the coil hang below the magnets and carry a small bismuth-antimony couple attached to a very thin blackened copper disc; the couple is screened from the magnetic field by a jacket of soft iron. When radiation falls upon the couple a thermo-current flows in the coil and the latter is deflected. This instrument is extremely sensitive; it gives quite a good deflection with an amount of heat equal to that which would fall upon a disc $\frac{1}{2}''$ diameter from a candle 1,600 feet distant.

In **Callendar's Radio-Balance** for the measurement of radiant heat a blackened copper disc is fixed to the junction of four iron and four constantin wires (Fig. 390), and another iron-constantin couple leads to the galvanometer *G*. Another circuit, as indicated, contains a battery, an adjustable resistance, and a milliammeter. When radiation falls on the disc a thermo-current flows and the galvanometer is deflected, but *R* is adjusted and such a current passed that the Peltier cooling effect cancels this and the galvanometer deflection is reduced to zero. Knowing the current and the Peltier Coefficient the rate of absorption of energy by the disc is known. In the actual instrument there are two similar discs, and in later patterns the discs are replaced by small cups; it is used in experiments on radio-activity.

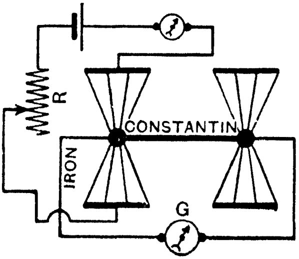


Fig. 390.

The **Thermo-Couple Pyrometer** for the measurement of temperatures up to about 1,200° C. consists of a platinum and platinum-rhodium alloy couple. The circuit includes a galvanometer, and the temperature of the "cold junction," *i.e.* the connections to the galvanometer, is kept constant. The galvanometer is calibrated so that the deflections give the temperatures to which the "hot" junction is subjected. Frequently the couple consists of platinum and platinum-iridium alloy.

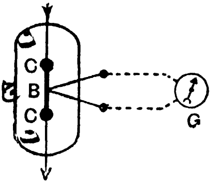


Fig. 391.

Fleming's Thermo - Milliammeter (Fig. 391) consists of a fine wire, *CC*, of constantin (which carries the small current to be measured), a bismuth-tellurium couple, *B*, soldered to *CC*, and a galvanometer, *G*. The wire *CC* and the couple *B* are in a vacuum. When a current passes CC is heated, the junction B is raised in temperature, a thermo-current flows, and the galvanometer is deflected. The instrument is calibrated by passing known currents through CC.

Thermo-Electric Generators are not, so far, a commercial success. Clamond uses, in one form, couples of iron and antimony-zinc alloy; 60,000 of these with one set of junctions heated by coke give an E M.F. of 109 volts.

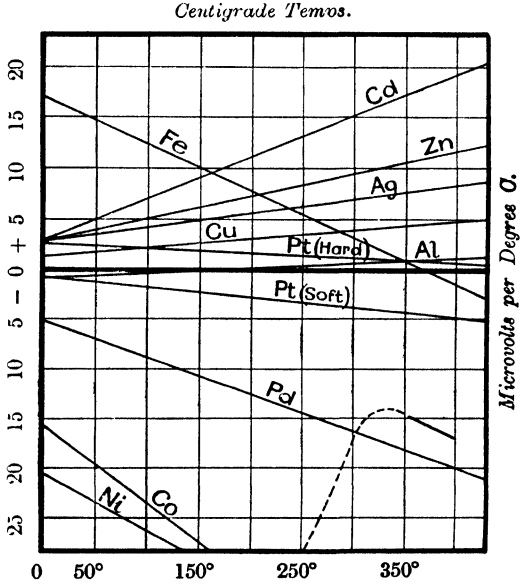


Fig. 391*a*.

Note that the copper, silver, zinc, and cadmium lines slope upwards to the right (Thomson Effect positive), whilst the iron, steel, and platinum slope downwards (Thomson Effect negative).

Exercises XV.

Section B.

(1) A thermopile is joined up in series with a Daniell's cell and the current allowed to flow for a short time. The thermopile is then removed from the circuit and connected with the terminals of a galvanometer, the needle of which is thereupon considerably deflected but gradually returns to its undisturbed position. Explain this. (B.E.)

Section C.

(1) Give Kelvin's theory of the thermo-electric circuit, and find an expression for the E.M.F. if the Specific Heat of electricity varies inversely as the absolute temperature. (B.E. Hons.)

(2) Suppose that at some point in an electric circuit heat was being developed by the passage of the current. Describe how you would determine whether the heating was due to a resistance or to a thermo-electric (Peltier) effect. (Inter. B.Sc. Hons.)

(3) What is meant by thermo-electric power, and how can the data for a diagram representing it be obtained? (B.Sc.)

(4) The thermo-electric power of iron is 1,734 micro-volts per degree at 0° and 1,247 at 100°, and that of copper is 136 at 0° and 231 at 100°. Construct a thermo-electric diagram for these metals, lead being the standard, and state how the amounts of heat absorbed and given out in the different parts of a copper-iron circuit with its junctions at 0° and 100° when there is a current of C amperes are shown in the diagram. Calculate also the electromotive force in volts. (B.Sc.)

(5) Explain clearly what is meant by the "specific heat of electricity."

Along a metal rod whose area of cross-section is 1 sq. cm. there is a uniform temperature gradient of 1° C. per centimetre. The specific resistance of the material of the rod is 150 microhms per centimetre cube.

When a current of 0·05 ampere is sent from the hot to the cold end the temperature gradient is unaltered. Calculate the specific heat of electricity for this metal. (B.Sc.)

(6) What is meant by the specific heat of electricity? Assuming that the E.M.F. of a circuit of two metals with the cold junction kept at constant temperature varies with the temperature of the hot junction according to a parabolic law, show that the difference of the specific heats of electricity in the two metals is proportionate to the absolute temperature. (B.Sc. Hons.)

(7) Prove that the coefficient of the Peltier Effect at a given junction is the product of the absolute temperature of the junction and the rate of change of the whole E.M.F. of the circuit with the temperature of that junction. (B.Sc. Hons.)

(8) The E.M.F. in a simple thermo-electric circuit one junction of which is heated while the other is kept at 0° C. is given by the expression $E = bt + ct^2$, where t is the temperature of the hot junction. Determine the neutral temperature and the Peltier and Thomson Effects in the circuit. Explain the theory on which these determinations are made. (B.Sc. Hons.)

CHAPTER XVI.

ELECTRICAL MEASUREMENTS.

224. Principle of the Wheatstone Bridge.—The principle underlying this method of measuring resistance has been referred to (Art. 162). With the arrangement indicated in Fig. 392 it is clear that selecting a point *D* on the upper branch *ADC*, there must be on the lower branch *AEC* some point (say *E*) at the same potential as *D*, so that on connecting *D* and *E* through a galvanometer there will be no deflection. Connection between *D* and *Y* will result in a current *from Y to D*, for *Y* is above *E* and therefore above *D* in potential; similarly, connection between *D* and *Z* will result in a current *from D to Z*.

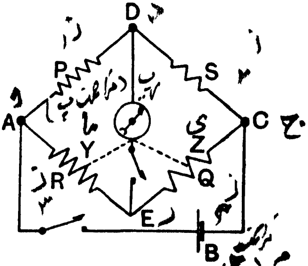


Fig. 392.

As *D* and *E* are at the same potential we have

$$\frac{\text{P.D. between } A \text{ and } D}{\text{P.D. between } D \text{ and } C} = \frac{\text{P.D. between } A \text{ and } E}{\text{P.D. between } E \text{ and } C},$$

i.e.

$$\frac{\text{Curr. in } AD \times \text{Res. } AD}{\text{Curr. in } DC \times \text{Res. } DC} = \frac{\text{Curr. in } AE \times \text{Res. } AE}{\text{Curr. in } EC \times \text{Res. } EC}.$$

$$\therefore \frac{\text{Res. } AD}{\text{Res. } DC} = \frac{\text{Res. } AE}{\text{Res. } EC},$$

$$\text{i.e. } \frac{P}{S} = \frac{R}{Q},$$

where *P*, *Q*, *R*, and *S* denote the four resistances. Thus an unknown resistance (say P) can be found if the other three are known, *or* if one of the adjacent resistances (R or S) be known and the *ratio* of the other two be also known. The Metre Bridge and Post Office Box are practical applications of the Wheatstone Bridge.

225. Sensitiveness of the Bridge.—An examination of Fig. 392 will show that if the positions of the galvanometer and battery be interchanged the relation established above will still hold; the *sensitiveness* may, however, be quite different in the two cases. The greater the galvanometer current due to a small lack of balance the more sensitive is the arrangement. Applying the method of Art. 162 to Fig. 392, the expression for the galvanometer current is

$$\frac{E(RS-PQ)}{BG(P+Q+R+S)+B(P+R)(Q+S)+G(P+S)(Q+R)+PS(Q+R)+RQ(P+S)}$$

In considering the effect on this of interchanging B and G the first term in the denominator may be neglected since it contains the product BG and will not be altered by interchanging B and G. The last two terms do not contain B or G and may be neglected. The part to be considered is therefore

$$B(P+R)(Q+S)+G(Q+R)(P+S) \quad \ldots\ldots\ldots\ldots \quad (a)$$

If B and G be interchanged this becomes

$$G(P+R)(Q+S)+B(Q+R)(P+S) \quad \ldots\ldots\ldots\ldots \quad (b)$$

Hence $\quad (a)-(b)=(B-G)(P-Q)(S-R)$.

Now assume B greater than G and let P and R be great in comparison with Q and S: the factors $(B-G)$ and $(P-Q)$ are positive, whilst $(S-R)$ is negative, so that the expression on the right is negative, *i.e* (b) is greater than (a). This means that the *denominator* in the expression for the galvanometer current is greater when B and G are interchanged, and therefore *the galvanometer current itself* is greater *before* they are interchanged, *i.e.* the arrangement shown in Fig. 392 is the more sensitive. *If G be greater than B*, then (a) is greater than (b) and G and B must be interchanged in Fig. 392 to obtain the more sensitive arrangement. Hence we have the following rule: *Whichever has the higher resistance—the battery or the galvanometer—must be put across from the junction of the two higher resistances to the junction of the two lower resistances.*

Note also, from the expression for the galvanometer current, that if $RS=PQ$, *i.e.* if $P/S=R/Q$, the galvanometer current is zero (Arts. 162, 224).

226. Measurement of Resistance by the Metre Bridge and Post Office Box.—In its simplest form the **Metre Bridge** (Fig. 393) consists of three thick bars of copper of negligible resistance fixed to a board and provided with two gaps for the insertion of the resistance to be measured and a standard resistance with which it is compared. A straight, hard uniform wire 1 metre in length joins the two copper end pieces and has behind it a scale divided into 1,000 equal parts. By means of a slider, contact can be made at any point on the wire, the exact position of which is indicated by a pointer attached to the slider and moving over the scale. The connections are shown in Fig. 393, in which *P* is the unknown resistance and *S* a standard known resistance of somewhat similar magnitude. The experiment consists in (1) closing *K* and (2) moving the slider along the wire until a point of contact *E* is reached at which the galvanometer is not deflected. With this condition realised,

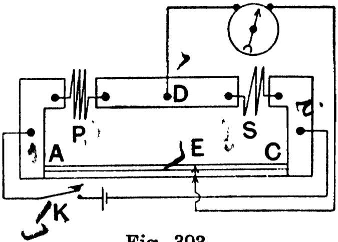


Fig. 393.

$$\frac{\text{Resistance } P}{\text{Resistance } S} = \frac{\text{Res. } AE}{\text{Res. } EC} = \frac{\text{Length } AE}{\text{Length } EC};$$

for, since the wire is uniform, resistance is proportional to length, hence

$$\text{Res. of } P = \frac{\text{Length } AE}{\text{Length } EC} \times \text{Res. } S.$$

Exp. 1. *To verify roughly the laws of Art.* 158.—Take two wires of the same material and cross-section but different lengths, measure their resistances and verify that resistance is proportional to length. Similarly, by using two wires of the same material and length but of different diameters, verify that resistance is inversely proportional to the cross-sectional area.

Measure the resistance (R), length (l), and diameter (d) of a wire and find the specific resistance (S) from the relation $S = Ra/l$, where $a = \cdot 7854d^2$; a may be found more accurately from the relation $a = \text{vol.}/l = w/\rho l$, where w is the mass in grammes and ρ the density in grammes per c.cm.; ρ may be found by weighing in air (w_1) and in water (w_2) and using the relation $\rho = w_1/(w_1 - w_2)$.

Exp. 2. *To find the temperature coefficient of copper.*—The copper wire is wound on a hollow perforated bobbin and placed in a vessel containing oil, a thermometer, and a stirrer. This vessel is placed in an outer vessel containing water which can be heated. The copper wire is connected to the gap of the bridge by a pair of thick copper leads. Before heating measure the resistance of the coil and note its temperature. Gradually raise the temperature, keeping the oil well stirred. When the temperature has risen about 10° C. remove the heater, continue stirring about a minute, and then measure the resistance, noting the temperature when the balance is obtained. Repeat, using temperature rises of about 10° C. up to 100° C. Take a similar set of measurements during cooling. Plot two curves, one for heating, the other for cooling, with temperature as abscissae and resistance as ordinates. Selecting the better curve, let R_{t_1} = resistance at any temperature t_1, and R_{t_2} = resistance at another temperature t_2; then

$$R_{t_1} = R_0(1 + at_1) \qquad R_{t_2} = R_0(1 + at_2),$$

$$\therefore \frac{R_{t_1}}{R_{t_2}} = \frac{1 + at_1}{1 + at_2}, \qquad i.e.\ a = \frac{R_{t_2} - R_{t_1}}{R_{t_1}t_2 - R_{t_2}t_1}.$$

Another form of Wheatstone Bridge is the **Post Office Box**; in dealing with this we shall for convenience write the relation of Art. 224, viz. $P/S = R/Q$, in the form $R/P = Q/S$, and shall assume S to be the unknown resistance.

The Post Office Box consists of a number of coils of known resistance arranged so as to form three arms of a Wheatstone Bridge, the fourth consisting of the coil whose resistance is to be determined. The method in which the coils are fixed and manipulated is shown in Fig. 394. Their ends are attached to solid brass blocks separated from each other by conical gaps, into which conical brass plugs can be inserted. By inserting a plug that particular resistance is cut out of circuit, for the current will pass from one block to the next through the plug;

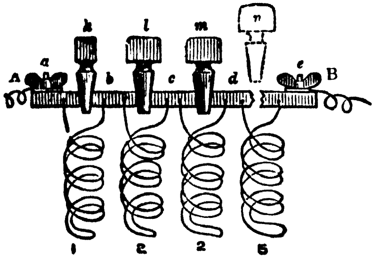


Fig. 394.

removing a plug necessitates the current going through the coil, thus adding that resistance to the circuit. The coils are doubled upon themselves in the manner indicated so as to eliminate the effects of inductance.

A plan of the box and its connections is shown in Fig. 395 The arms P and R are known as the *ratio* arms; each consists of three coils of 10, 100, and 1,000 ohms resistance. Q is called the *rheostat* arm, and consists of a series of coils whereby resistances ranging from 1 to 10,000 ohms can be obtained. S is the unknown resistance. The lettering of the box is identical with Fig. 392. The manipulations will be understood by considering the following experiment.

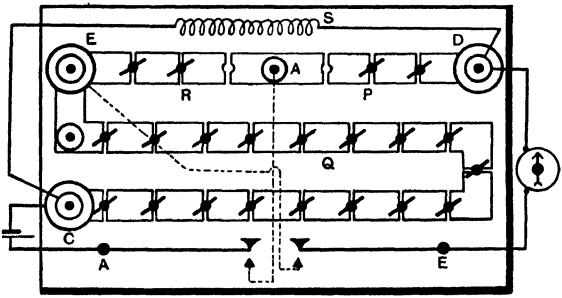


Fig. 395.

Exp. *To measure a resistance by the Post Office Box.*—Join up as indicated. *Take* 10 *ohms from R and* 10 *ohms from P.* By taking out different plugs in Q endeavour to find a resistance such that on closing first the battery key and then the galvanometer key the galvanometer is not deflected. Since R is equal to P, Q must be equal to S to secure a balance. In an actual test 2 ohms in Q at this stage gave a deflection to the right and 3 ohms gave a deflection to the left; hence the conclusion that *S was between* 2 *and* 3 *ohms.*

Make R equal to 100 *ohms, keeping P equal to* 10 *ohms,* and again endeavour to find a resistance in Q for no deflection Since R is now equal to $10P$, Q must be equal to $10S$ for a balance; hence in the test referred to it was only necessary at this stage to work with resistances between 20 and 30 ohms in Q. On taking 23 ohms the deflection was to the right and 24 ohms gave a deflection to the left; hence *S was between* 2·3 *and* 2·4 *ohms*

Make R equal to 1,000 *ohms, keeping P equal to* 10 *ohms*, and again try to find a value for Q for no deflection. In this case, since R is equal to $100P$, Q must be equal to $100S$ for a balance; in the test in question it was therefore only necessary to work with resistances between 230 and 240 ohms in Q. On taking 237 ohms the galvanometer was not deflected; hence *S was equal to* 2·37 *ohms.*

Should two consecutive resistances in the third step still produce deflections in opposite directions, the true value may be found by interpolation. To take an example: If 237 ohms gave a deflection of 30 divisions to the right, and 238 gave 40 divisions to the left, a balance would be obtained if Q could be made equal to $237 + \frac{30}{70}$, *i.e.* 237·428 ohms; in this case the value of S would be 2·37428 ohms.

If the unknown resistance is *very large*, R must retain its value, 10 ohms, throughout, and P must be made equal to (say) 1,000. R being $\frac{1}{100}$ of P, Q must be $\frac{1}{100}$ of the unknown to secure a balance. Thus, if 1,208 from Q gives no deflection, the value of S is 120,800 ohms.

227. Errors, Corrections, and Precautions in Metre Bridge Work.—The sources of error in Metre Bridge work may be briefly summarised as follows:—

(1) Lack of uniformity in the bridge wire. To avoid the errors arising from this the wire must be *calibrated* so that the ratio Res. AE/Res. EC is accurately known.

(2) Resistance of the end pieces too large and unequal to be neglected. These are determined experimentally as equivalent to so many divisions of bridge wire, and these are added to the respective sections AE and EC each time a test is made.

(3) The non-coincidence of the pointer (which moves along the scale) with the edge of the tapper (which makes contact with the wire). It is easily seen that the error is eliminated by interchanging the coils and taking the mean of the two results. (*Prove this.*)

(4) Variation of the resistance of the coils used owing to temperature changes during the experiment. To reduce this the current must be kept as small as possible, and

only allowed to flow for short intervals of time. Special precautions must be taken in special cases. If the temperature and temperature coefficients be known, corrections can be made.

(5) Errors due to thermo-electric effects. These are eliminated by using a reversing key in the battery circuit and balancing with current in opposite directions in the bridge. (*Explain this.*)

Exp. 1. *To calibrate a bridge wire.*—There are many methods in use; the following briefly outlines one method. In Fig. 396 B is an accumulator, R a rheostat, and G a galvanometer; the current indicated by G must be kept constant. P_1, P_2 are two contacts connected to a high resistance galvanometer HG. With a steady small current passing, P_1, P_2 are placed on the wire at 0–50 and the deflection of HG is noted; this is proportional to the PD and therefore to the resistance of this part. The contacts P_1, P_2 are then placed on 50 – 100, 100 – 150, etc., and the observations are repeated.

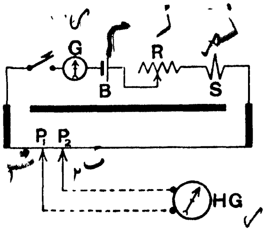


Fig. 396.

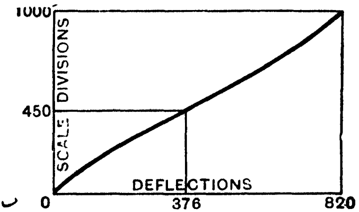


Fig. 397.

If d_1, d_2, d_3, etc., be the deflections, then d_1 is proportional to the resistance of the part 0 to 50, $(d_1 + d_2)$ to the part 0 to 100, $(d_1 + d_2 + d_3)$ to the part 0 to 150, and so on. A curve is now plotted with scale divisions as ordinates, and the sum of the deflections $d_1 + d_2$, etc., from zero to the scale divisions as abscissae. Such a curve, in which the sum of all the deflections for the whole wire 0–1000 is 820, is indicated in Fig. 397. Clearly if in a test a balance is obtained at division 450, the ratio of the resistances is not $450/550 = \cdot 82$, but $376/(820 - 376) = 376/444 = \cdot 846$.

By inserting a standard low resistance (S) and noting the deflection (D) when HG is joined to it, the actual resistance corresponding to the deflections may be found; thus the resistance of the first section is $d_1 S/D$. Hence the deflection curve may be marked to read the actual resistance of the wire.

Exp. 2. *To find the correction for the end pieces of a Metre Bridge.*—Join up as indicated in Fig. 393, P and S being in this case *known* resistances, so that the ratio $P/S = r$ (say) is known. Let the resistance of the end A be equal to a divisions of the wire and that of C equal to β divisions of the wire. Secure a balance and let $AE = d_1$ divisions and $AC = L$ divisions; then

$$\frac{P}{S} = \frac{d_1 + a}{(L - d_1) + \beta} = r \text{ .... } (1)$$

Now interchange P and S and again balance; let d_2 be the balancing distance from the end A; then

$$\frac{P}{S} = \frac{(L - d_2) + \beta}{d_2 + a} = r,$$

$$\therefore \frac{d_2 + a}{(L - d_2) + \beta} = \frac{1}{r} \text{ .... } (2)$$

From (1) $\quad (d_1 + a) = r(L - d_1) + r\beta$ .... (3)

From (2) $\quad r(d_2 + a) = (L - d_2) + \beta,$

i.e. $\quad r^2(d_2 + a) = r(L - d_2) + r\beta$ .... (4)

Eliminating β by subtracting (3) and (4)—

$$a = \frac{d_1 + r(d_1 - d_2) - r^2 d_2}{r^2 - 1},$$

β can similarly be found.

Exp. 3. *To measure with greater accuracy the resistance of a wire by the Metre Bridge.*—The form of bridge shown in Fig. 398 permits of modifications to secure greater accuracy.

The best arrangement of the apparatus to secure maximum sensitiveness depends on the conditions of the experiment (see Art. 225). The resistance of the standard S should be of similar magnitude to that of P, and in a general way the wire resistance should be about equal to their sum. As the wire resistance is frequently somewhat low compared with that of the coils, approximately equal resistances R_1 and R_2, each about equal to the unknown, are inserted as shown in Fig. 398. Assuming the wire uniform and the standards corrected for temperature, the *modus operandi* is as follows:—

(1) Determine roughly the value of P, and then select a standard S and resistances R_1 and R_2 of somewhat similar magnitude. Let R_1 be equivalent to n_1 divisions, and R_2 to n_2 divisions of the bridge wire. Let the end X have a resistance equal to a divisions, and Y a resistance equal to b divisions of the wire. On balancing as shown—

$$\frac{P}{S} = \frac{n_1 + a + l_1}{n_2 + b + (1000 - l_1)}.$$

(2) Reverse current, balance, and obtain the ratio $P : S$.

(3) Interchange P and S, and with the current going as in the first step, let l_2 be the balancing distance from the end X. In this case

$$\frac{P}{S} = \frac{n_2 + b + (1000 - l_2)}{n_1 + a + l_2}.$$

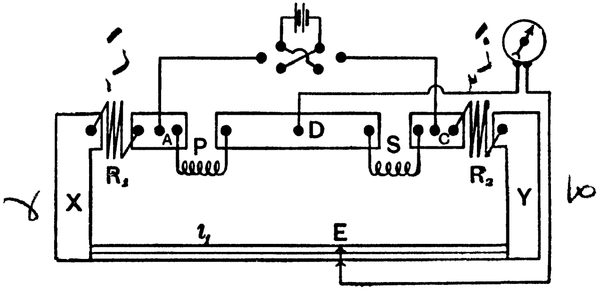


Fig. 398.

(4) Reverse current, balance, and obtain the ratio $P : S$. Adding numerators and denominators in (1) and (3)—

$$\frac{P}{S} = \frac{1000 + n_1 + n_2 + a + b + (l_1 - l_2)}{1000 + n_1 + n_2 + a + b - (l_1 - l_2)},$$

$$\therefore\ P = \frac{\alpha + (l_1 - l_2)}{\alpha - (l_1 - l_2)} \times S, \text{ where } \alpha = 1000 + n_1 + n_2 + a + b.$$

Similarly find P from (2) and (4) and take the mean.

228. The Carey Foster Bridge.—Fig. 398 will also serve to explain the Carey Foster method of measurement with the bridge. In this case R_1 and R_2, *which are generally of about the same value*, are the two resistances under examination. Let α be the *resistance* of the end X, β the *resistance* of the end Y, L the total length of bridge wire, and let ρ denote the resistance per unit length. Balancing as indicated in Fig. 398 we have

$$\frac{P}{S} = \frac{R_1 + \alpha + \rho l_1}{R_2 + \beta + \rho(L - l_1)}.$$

Now let R_1 and R_2 be interchanged and let l_2 be the balancing distance from the end X; hence

$$\frac{P}{S} = \frac{R_2 + \alpha + \rho l_2}{R_1 + \beta + \rho(L - l_2)}.$$

Thus—

$$\frac{R_1 + a + \rho l_1}{R_2 + \beta + \rho(L - l_1)} = \frac{R_2 + a + \rho l_2}{R_1 + \beta + \rho(L - l_2)}.$$

Adding 1 to each side we get

$$\frac{R_2 + \beta + \rho L + R_1 + a}{R_2 + \beta + \rho L - \rho l_1} = \frac{R_1 + \beta + \rho L + R_2 + a}{R_1 + \beta + \rho L - \rho l_2}.$$

Here the numerators are the same, hence the denominators are equal; equating the denominators we get

$$R_1 - R_2 = \rho(l_2 - l_1) = \text{Res. of the part } (l_2 - l_1).$$

Thus the difference between the two resistances R_1 and R_2 is equal to the resistance of the bridge wire between the two balancing points; the result, it will be noted, does not involve the end pieces or the values of P and S. The resistance of the length $(l_2 - l_1)$ may be taken from the calibration curve, and if R_2 be known R_1 is determined.

This method can be employed for "calibration" if $R_1 - R_2$ be accurately known, for $\rho = (R_1 - R_2)/(l_2 - l_1)$; thus if either P or S can be slightly altered at will, so as to bring the balancing points to various parts of the wire, ρ for the parts in question can be determined.

229. The Callendar and Griffiths Bridge.—This bridge is used with the platinum thermometer for the measurement of temperature. In Fig. 399 the arms P and S of the bridge are equal. Leads join the thermometer *pt* to the gap in the arm Q, and a similar pair of dummy leads close to the main ones is connected to a gap in the arm R; thus the resistance of the leads is eliminated. The wire *ab* is 50 cm. long and its resistance is ·25 ohm; at *r* there are eight coils, the resistances being ·1, ·2, ·4, ·8, 1·6, 3·2, 6·4, and 12·8 ohms. The tapper consists of a slider E which connects *ab* and *cd*. The thermometer itself consists of a platinum coil in a tube of glazed porcelain or glass.

In testing, the thermometer is subjected to the unknown temperature and the bridge balanced for no deflection; if this occurs with the slider x cm. from the centre of ab

$$\frac{P}{S}=\frac{R}{Q}=\frac{l+r+b+\rho x}{l+pt+b-\rho x},$$

where l is the resistance of the leads, b the resistance of half the wire ab, and ρ the resistance of one cm. of it; but P is equal to S, hence

$$l+pt+b-\rho x=l+r+b+\rho x,$$

$$\therefore\ pt=r+2\rho x=r+\frac{x}{100}.$$

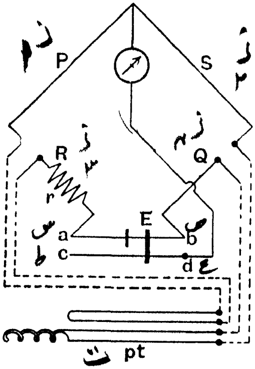


Fig. 399.

Knowing the resistance pt of the thermometer, the temperature to which it is exposed is known. The battery (not shown) joins the P, R and S, Q junctions, as usual.

230. Measurement of High Resistance. — The Wheatstone Bridge is unsuitable for the measurement of high resistances of the order of a megohm, since the conditions for sensitiveness must be violated. Of the many methods in use two will be briefly described here:—

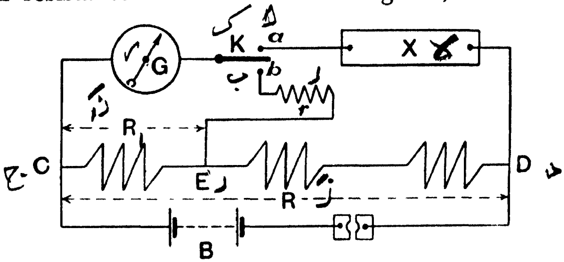


Fig. 400.

Exp. 1. *To measure a high resistance by the substitution method.* —Join up as in Fig. 400, where B is a convenient battery, R a high resistance (of the order 100,000 ohms), G a high resistance galvanometer, K a well insulated key, and X the high resistance to be measured. With the lever on the upper stud a the deflection D_1 of the galvanometer is obtained. A known resistance r is now introduced, X being cut out of circuit by moving the lever to the stud b. The resistance R_1 (also r if necessary) is adjusted, the total R between C and D being kept constant, until a deflection D_2 of somewhat similar magnitude to D_1 is obtained. If e be the P.D. between C and D, e_1 the P.D. between C and E, I_1 the current in the galvanometer in the first case, and I_2 the current in it in the second case,

$$I_1 = \frac{e}{G+X}, \qquad I_2 = \frac{e_1}{G+r},$$

$$\therefore \frac{I_1}{I_2} = \frac{e}{e_1} \cdot \frac{G+r}{G+X}, \qquad i.e. \quad \frac{D_1}{D_2} = \frac{R}{R_1} \cdot \frac{G+r}{G+X}$$

$$\text{or} \quad X = \frac{D_2 R(G+r) - D_1 R_1 G}{D_1 R_1}.$$

The **resistance of the insulation of a cable** may be found by the method outlined above. The cable is coiled up in a metal tank containing water, the ends only being outside, and these are well insulated to prevent leakage. One end of the metal core is joined *via* the key (stud a) to the galvanometer, the other end of the core being left "free," and the metal tank, which through the water is in contact with the outside of the insulation, is joined to the end D of the high resistance R; thus the insulation takes the place of X in Fig. 400.

The **specific resistance of the dielectric** may now be found by the relation of Art. 159, viz.—

$$\left\{\begin{matrix}\text{Insulation resistance} \\ \text{of length } l \text{ inches}\end{matrix}\right\} = \left\{ \cdot 366 \times \frac{s}{l} \times \log_{10} \frac{r_2}{r_1} \right\} \text{ ohms},$$

where s is the specific resistance in ohms per inch cube, r_2 and r_1 the external and internal diameters (or radii) of the insulation, and l the length of the cable in inches.

Exp. 2. *To measure a high resistance by the loss of charge method.*—The "loss of charge" method is suitable for the measurement of the dielectric resistance of a condenser or short length of cable, and is diagrammatically represented in Fig. 401, in which R is a condenser of which the dielectric resistance is required. The key K is closed and when the deflection of the electrometer E is quite steady K is opened, thus disconnecting the battery from E and R. The charge on the condenser gradually leaks away and the potential falls: this is indicated by the falling deflection of the electrometer. Time readings of the deflection are taken and a curve plotted with time as abscissae and deflections as ordinates. Selecting now any two points on the curve let V_1 denote the deflection corresponding to the first point and V_2 that corresponding to the second, and let t seconds be the time interval between the two; the dielectric resistance R is found from the relation

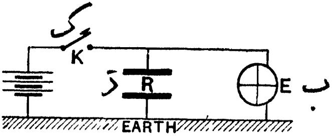


Fig. 401.

$$R = \frac{t}{2 \cdot 3026\, C \log_{10} \dfrac{V_1}{V_2}} \quad \text{(see below)},$$

where C is the capacity of the condenser. With C in microfarads R will be in megohms.

In the preceding it is assumed that the leakage only takes place at the condenser. To allow for any leakage at the electrometer we proceed thus:—The condenser R is cut out and the electrometer only charged. Time readings of the deflection are taken and a curve plotted showing the fall of potential due to leakage at E. The experiment is repeated with R in the circuit as explained above, and a curve plotted showing the fall due to E and R. From the two curves the fall due to R alone is obtained.

The expression for R in the preceding may be readily established. If Q be the charge on the condenser at any instant, V the potential, and C the capacity,

$$Q = CV, \qquad \therefore \frac{dQ}{dt} = C\frac{dV}{dt}.$$

But dQ/dt denotes the leakage current and is equal to V/R, hence

$$C\frac{dV}{dt} = \frac{V}{R}, \qquad \therefore \frac{1}{RC}\,dt = \frac{1}{V}\,dV.$$

Integrating, we arrive at the result

$$\frac{1}{RC}\,t = \log_e \frac{V_1}{V_2}, \qquad \therefore R = \frac{t}{C \log \dfrac{V_1}{V_2}},$$

$$\text{i.e.} \quad R = \frac{t}{2 \cdot 3026 \times C \log_{10} \dfrac{V_1}{V_2}}.$$

Time Constant of a Condenser.—Consider a condenser slowly discharging through a high resistance R which connects its terminals. Reasoning similar to the above may be applied, the leakage now being considered to take place through this resistance R. If V_1 be the initial and V_2 the final P.D. for an interval of t seconds,

$$\log_e \frac{V_1}{V_2} = \frac{t}{RC}, \quad \therefore \log_e \frac{V_2}{V_1} = -\frac{t}{RC},$$

$$i.e. \quad e^{-\frac{t}{RC}} = \frac{V_2}{V_1} \quad \text{or} \quad V_2 = V_1 e^{-\frac{t}{RC}}.$$

If $t = RC$, $V_2 = V_1 e^{-1} = \frac{1}{e} V_1$. Thus, if a charged condenser has its coats connected by a wire of resistance R ohms, the potential (and charge) will fall to $\frac{1}{e}$, *i.e.* $\frac{1}{2{\cdot}71828}$ of its initial value, in a time RC seconds—e or 2·71828 being the base of the Napierian logarithms. *RC seconds is called the time constant of a condenser of capacity C discharging through a resistance R.* Neglecting leakage through the condenser the method outlined above may evidently be used to find the value of the high resistance R joining the terminals and through which the condenser is slowly discharging.

231. Measurement of Low Resistance.—In the methods indicated below, the low resistance to be measured is put in series with a standard low resistance and the fall of potential in the two compared (1) by means of a calibrated wire, (2) by the galvanometer deflections.

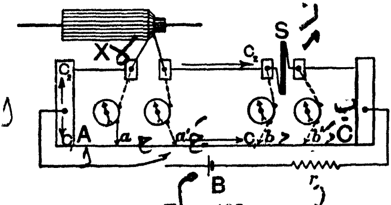


Fig. 402.

Exp. 1. *To measure a low resistance using a calibrated wire.*—In Fig. 402 AC is a standard (calibrated) low resistance wire joined to an accumulator B and an adjustable resistance r. The unknown resistance X and a standard low resistance of somewhat similar magnitude S are connected in series, and the two put in parallel with the standard wire. A sensitive galvanometer has one terminal joined to one end of the unknown, and by means of a movable contact a point a is found on the wire such that the galvanometer is not deflected. This is repeated at the other end of the unknown, and at both ends of S, as shown by the dotted lines in the figure; clearly then

$$\begin{aligned} \text{P.D. at the ends of } X &= \text{P.D. between } a \text{ and } a', \\ \text{and P.D. at the ends of } S &= \text{P.D. between } b \text{ and } b'; \end{aligned}$$

$$\therefore \frac{\text{P.D. at the ends of } X}{\text{P.D. at the ends of } S} = \frac{\text{P.D. between } a \text{ and } a'}{\text{P.D. between } b \text{ and } b'},$$

$$i.e. \quad \frac{\text{Res. } X}{\text{Res. } S} = \frac{\text{Res. } aa'}{\text{Res. } bb'},$$

and this may be written Length aa'/Length bb' if the wire be quite uniform; thus X is determined.

Exp. 2. *To measure a low resistance by comparison of deflections.*—In this case (Fig. 403) the ends of X and S and the high resistance galvanometer G are connected to a Pohl's commutator P. By means of the latter G is first put across X and the deflection d_1 is noted; it is then put across S and the deflection d_2 is noted; then

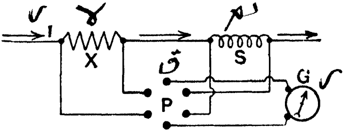


Fig. 403.

$$\frac{\text{Res. } X}{\text{Res. } S} = \frac{\text{P.D. at the ends of } X}{\text{P.D. at the ends of } S} = \frac{d_1}{d_2},$$

from which X is found. It is advisable after taking d_2 to again put G across X and note the deflection; if this is not the same as before the mean of the two is taken as the deflection for X.

232. Measurement of Battery Resistance.

—Battery tests are not a great success, for the resistance depends upon the current the battery is giving (decreasing as the current increases) and polarisation affects many resistance determinations; the following methods are typical and illustrate important principles:—

Exp. 1. *To measure the resistance of a cell by the condenser method.*—In Fig. 404 B is the cell, the resistance of which (b) is required, C is a condenser, BG a ballistic galvanometer, K a charge and discharge key, and r a known resistance.

With r disconnected from B, charge the condenser by depressing

K, and note the first swing of BG; let this be d_1. Discharge the condenser.

Connect the poles of B by the resistance r and again charge C, noting the first swing, d_2, of BG. Discharge the condenser.

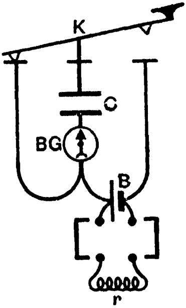


Fig. 404.

If E be the E.M.F. of the cell and V its terminal potential difference when joined by r, the charge given to the condenser in the first case is CE and in the second case CV, where C denotes the capacity of the condenser; hence $d_1 : d_2 = CE : CV = E : V$. Again, the current in r is given by $E/(b + r)$, and also by V/r (Art. 155); hence $E : V = (b+r) : r$. Clearly then

$$\frac{b+r}{r} = \frac{d_1}{d_2}, \quad \therefore b = r\frac{d_1 - d_2}{d_2}.$$

Exp. 2. *To measure the resistance of an accumulator by the ammeter and high resistance galvanometer method.*—Briefly this experiment is carried out as follows:—

(1) A reflecting galvanometer, with a large resistance (100,000 ohms) in series with it, is connected to the poles of the accumulator and the deflection d_1 observed.

(2) The poles of the cell are now joined to a second circuit consisting of an ammeter and variable resistance, and the latter is adjusted till a current of (say) 10 amperes is registered. While this current is flowing the galvanometer deflection d_2 is read.

(3) The accumulator and ammeter circuit are removed, a standard cell (E.M.F. = 1·434 volts) put in series with the galvanometer and its high resistance, and the deflection d_3 observed.

A deflection d_3 is produced by a pressure of 1·434 volts;

$\therefore$,, d_1 ,, ,, ,, $\frac{d_1}{d_3} \times 1{\cdot}434$ volts

and ,, d_2 ,, ,, ,, $\frac{d_2}{d_3} \times 1{\cdot}434$ volts.

Again, if E be the E.M.F. of a cell and V the terminal potential difference when a current passes, $E - V$ is the volts used in the cell, and if I be the current in the cell—

$$\text{Res. of cell} = \frac{E - V}{I}.$$

Now, in Case 1 the cell is practically on open circuit, so that $\left\{\frac{d_1}{d_3} \times 1{\cdot}434\right\}$ volts is its E.M.F. E; the terminal P.D. in Case 2 is

$\left\{\frac{d_2}{d_3} \times 1{\cdot}434\right\}$ volts, and neglecting the small portion taken by the galvanometer 10 amperes may be taken as the current in the accumulator ; hence

$$\text{Res. of cell} = \frac{\frac{d_1}{d_3}1{\cdot}434 - \frac{d_2}{d_3}1{\cdot}434}{10}$$

$$= \left\{{\cdot}1434\frac{d_1 - d_2}{d_3}\right\} \text{ ohms.}$$

Exp. 3. *To measure the resistance of a cell by Beetz's method.*—In Fig. 405 PQ is a calibrated wire, B the cell whose resistance b is required, C a second cell *of smaller E.M.F.*, and G a galvanometer. The positive pole of B is connected to P, and the negative pole to some point S; the positive pole of C is also connected to P, and the negative pole to some point T between P and S. The potential difference between P and T tends to drive a current through the lower branch in the direction $PCGT$, whilst the E.M.F. of the cell C tends to drive a current through the lower branch in the direction $TGCP$. Clearly it will be possible to find a point T such that G is not deflected, in which case *the P.D. between P and T will be equal to the E.M.F. of C*; hence the test is as follows:—With the connections as shown find the point T for no deflection. If E and E' be the E.M.F.'s of B and C, R and r the resistances of PS and PT, then

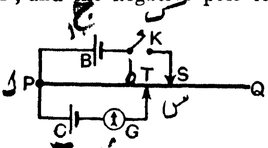


Fig 405.

$$\text{Current in } PS = \frac{E}{b + R} = \frac{E'}{r},$$

$$\therefore \frac{E}{E'} = \frac{b + R}{r}.$$

Alter the contact S to S' and find a point T' for no deflection ; if R' and r' be the resistances of PS' and PT'—

$$\frac{E}{E'} = \frac{b + R'}{r'},$$

$$\therefore \frac{b + R}{r} = \frac{b + R'}{r'}, \text{ i.e. } b = \frac{R'r - Rr'}{r' - r}.$$

The cell B must not be allowed to give a current for any length of time ; in practice K is closed and contact quickly made at T, and if G is deflected *K and T are immediately opened*; this is repeated until the required point T is found.

In the **electrometer method** the cell is connected to an electrometer and the deflection d_1 is observed; this is a measure of the E.M.F. (E). The poles are then joined by a wire of resistance r and the deflection d_2 is observed; this is a measure of the terminal potential difference (V). Here, as in Exp. (1) above,

$$\frac{b+r}{r} = \frac{E}{V} = \frac{d_1}{d_2}, \qquad \text{i.e. } b = r\frac{d_1 - d_2}{d_2}.$$

The same principle is utilised in the **potentiometer method** (Art. 235), d_1 and d_2 being in this case the balancing distances on the potentiometer wire.

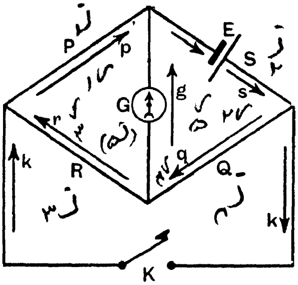


Fig. 406.

In **Mance's method** the cell is placed in the arm of the Wheatstone Bridge ordinarily occupied by the resistance to be measured (the S arm, say, of Fig. 406), and the usual battery branch contains a key K only (Fig. 406); the galvanometer will, of course, be deflected. Resistances are then adjusted *until the galvanometer deflection is the same whether the key K is open or closed.* When this condition is realised it can be shown that the usual calculation applies, viz.

$$\frac{\text{Res. } (S) \text{ of cell}}{Q} = \frac{P}{R},$$

and, since Q, P, and R are known, S can be determined.

The truth of the above is readily shown by an application of Kirchhoff's laws to Fig. 406, in which the small letters p, q, r, s, g, k denote the currents in the various paths; thus—

$$q = g + r \quad \dots\dots \quad (1)$$

$$s = g + p \quad \dots\dots \quad (2)$$

$$E = Ss + Qq + Gg \quad \dots\dots \quad (3)$$

$$O = Rr + Pp - Gg \quad \dots\dots \quad (4)$$

From (3)—

$$g = \frac{E - Ss - Qq}{G} \quad \dots\dots \quad (5)$$

From (4)—
$$g = \frac{Rr + Pp}{G} = \frac{R(q - g) + P(s - g)}{G}$$
$$\therefore\ Gg = Rq - Rg + Ps - Pg,$$

i.e.
$$g = \frac{Rq + Ps}{G + R + P} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

From (5)—
$$g = \frac{EP - SsP - QqP}{GP};$$

From (6)—
$$g = \frac{RqS + PsS}{S(G + R + P)},$$
$$\therefore\ g = \frac{EP + q(RS - QP)}{GP + S(G + R + P)}.$$

Now, if $RS - QP = 0$, the current g in G will not depend upon the current q, and therefore will not depend upon the current k (for any change in k will alter q); thus, if $S/Q = P/R$, the galvanometer deflection will be the same whatever the condition of the path K, and, therefore, whether the key K is open or closed.

233. Measurement of Galvanometer Resistance.—The most satisfactory method is to remove the needle and suspension in the case of a moving needle galvanometer, or clamp the coil in the case of a moving coil galvanometer, and measure the resistance in the usual way (Art. 226); this necessitates, of course, the use of a second galvanometer.

In **Kelvin's method** only the galvanometer under test is employed. This galvanometer is placed in the arm of the Wheatstone Bridge ordinarily occupied by the resistance to be measured (S in Fig. 392), and a key is placed in the usual galvanometer branch: the galvanometer will be deflected. Resistances are then adjusted *until the galvanometer deflection is the same whether the key referred to is open or closed*; it is clear that, when this is so, the points D and E (Fig. 392) are at the same potential, and therefore the usual calculation may be applied.

The principle underlying the **logarithmic decrement method** which is applicable to a reflecting galvanometer is interesting. The terminals of the galvanometer being left free, the log. dec. γ_1 is determined by starting the system oscillating, and noting the amplitudes of successive swings (Art. 190). The terminals are then joined as by known resistance R and the log. dec. γ_2 is determined. Finally the terminals are short circuited, *i.e.* joined by a short thick wire of negligible resistance, and the log. dec. γ_3 is determined. It is easy to show that

$$G = \frac{\gamma_2 - \gamma_1}{\gamma_3 - \gamma_2} R,$$

where G is the galvanometer resistance.

In the above we may write $\gamma = P + \frac{Q}{G + R}$, where γ is the logarithmic decrement, G the galvanometer resistance, R the resistance joining the terminals, and P and Q are constants. Now in case (1)

$$\gamma_1 = P + \frac{Q}{G + \alpha} = P + O = P,$$

and in case (2)

$$\gamma_2 = P + \frac{Q}{G + R} = \gamma_1 + \frac{Q}{G + R}, \quad \therefore Q = (G + R)(\gamma_2 - \gamma_1).$$

In case (3)

$$\gamma_3 = P + \frac{Q}{G} = \gamma_1 + \frac{Q}{G}, \qquad \therefore Q = (\gamma_3 - \gamma_1)G.$$

Hence $$(G + R)(\gamma_2 - \gamma_1) = G(\gamma_3 - \gamma_1),$$

$$\therefore G = \frac{\gamma_2 - \gamma_1}{\gamma_3 - \gamma_2} R.$$

234. Measurement of Electrolytic Resistance.—An obvious difficulty in these measurements is the fact that in most cases a back E.M.F. is set up, which, with ordinary methods of testing, would appear as a resistance, and, further, the back E.M.F. itself is not constant. In practice the electrodes are invariably of platinum coated with "platinum black." The following methods, amongst others, have been resorted to:—

(1) *Kohlrausch's Method.*—This is a metre bridge method, the electrolyte being in one gap and a standard resistance in the other. Current is supplied by the secondary of an induction coil, and as this is rapidly alternating in direction the opposite polarising effects at the electrodes neutralise each other. Balance is obtained by means of a telephone, the balancing point being that for which the sound in the receiver is a minimum. The condenser of

the induction coil should be removed, otherwise the currents in the two directions will not be equal, and polarisation will occur.

(2) *Fitzpatrick's Method.*—A better method, that of Fitzpatrick, is to supply continuous current from a battery as usual, having also the ordinary Wheatstone Bridge connections, including the galvanometer, but to pass the current first through a commutator, which is rapidly rotated by a motor and continually reverses the current flow, while at the same time the motor, by means of another commutator, continually reverses the galvanometer connections, so that the swings, if any, are all in one direction. The galvanometer circuit is closed a little after the battery circuit and opened a little earlier to avoid inductive effects.

(3) *Stroud-Henderson Method* —This is one of the best methods, and will be understood from Fig. 407. The resistances S and Q are made equal and very large. The resistance P is that of a tube of the liquid under test. The resistance R is partly that of another tube of the liquid exactly like the first, except that it is much shorter and partly made up by an adjustable resistance. This adjustable resistance, r, is altered until the galvanometer shows no deflection, in which case the resistance r equals that of a column of the liquid equal to the difference in length of the two tubes. Thus, although continuous current is employed, since the polarisation effects are the same in both arms of the bridge, their effect on the result is eliminated. Further, if l_1 be the length of the longer tube, l_2 that of the shorter tube, a the cross-sectional area of the tubes, and ρ the specific resistance of the electrolyte,

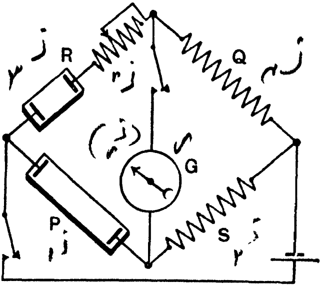


Fig. 407.

$$r = \rho\frac{l_1 - l_2}{a} = \rho\kappa, \quad \therefore \rho = \frac{r}{\kappa},$$

where κ *is a constant for the pair of tubes.*

In determining κ it is usual to allow for any difference in the cross-section of the tubes as follows:—Let M = mass of mercury required to fill the longer tube, m = the mass required to fill the shorter tube, a_1 = cross-sectional area of longer tube, a_2 = cross-sectional area of shorter tube, and d = density of mercury ; then

$$M = a_1 l_1 d, \ \therefore a_1 = \frac{M}{l_1 d}; \quad m = a_2 l_2 d, \ \therefore a_2 = \frac{m}{l_2 d}.$$

Now

$$r = \rho\frac{l_1}{a_1} - \rho\frac{l_2}{a_2} = \rho\left(\frac{l_1}{a_1} - \frac{l_2}{a_2}\right) = \rho\left[d\left(\frac{l_1^2}{M} - \frac{l_2^2}{m}\right)\right],$$

i.e.

$$r = \rho\kappa, \quad \therefore \rho = r/\kappa,$$

$$\text{where } \kappa = d\left(\frac{l_1^2}{M} - \frac{l_2^2}{m}\right).$$

235. Comparison and Determination of Electromotive Forces and Potential Differences.—One of the best methods of measuring and comparing E.M.F.'s and P.D.'s is that known as the **potentiometer method,** the principle of which has practically been referred to in Art. 232, Exp. 3. The potentiometer of Fig. 408 consists of seven uniform wires, each 1 metre in length, joined in series by thick copper bars, as indicated: scales graduated in millimetres are placed alongside the wires. The po-

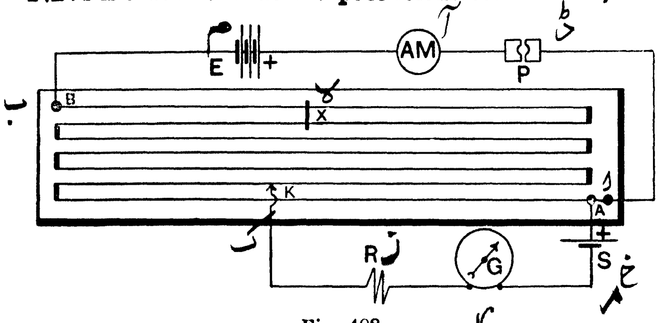


Fig. 408.

tentiometer wire is in series with an ammeter *AM* (for the detection of any current variation during the experiment), a plug key (*P*), and a battery (*E*) of three good accumulators, the positive pole of the latter being joined to the end *A* of the wire.

The positive terminal of a Latimer Clark Standard Cell *S* (E.M.F. = 1·434 volts at 15° C.) is connected to the end *A*, the negative terminal being joined through a resistance *R* and a sensitive galvanometer *G* to the sliding contact *K* placed 143·4 centimetres from the end *A*.

The P.D. between the points *A* and *K* produced by the battery *E* *tends* to send a current through the lower branch in the direction *ASGRK*, whilst the E.M.F. of the standard cell *tends* to drive a current in the opposite direction in this branch, viz. in the direction *KRGSA*. By altering the current in the potentiometer it can be arranged that the P.D. between *A* and *K* (given by the product of the current and the resistance of this portion of the wire) is equal to the E.M.F. of the cell *S*, in which case no current will flow in the lower branch and the galvanometer will not be deflected. This is attained by means of the slider *X*, consisting of a short piece of stout wire held firmly on the potentiometer wires; the position of *X* is altered, thus varying the total resistance of the wires till no deflection is indicated; finally *R* is removed for greater sensitiveness and *X* again slightly changed if necessary till there is absolutely no movement of the galvanometer. Clearly the P.D. between *A* and *K* is now 1·434 volts; and since the distance *AK* is 1,434 millimetres, each millimetre of the wire gives a P.D. of ·001 volt.

Exp. 1. *To measure the E.M.F. of a Leclanché cell by the potentiometer.*—Substitute for the standard cell *S* the Leclanché cell to be tested, its positive pole being connected to the end *A*. Adjust the position of the sliding contact *K* until the galvanometer is not deflected. If *K* be *d* mm. from *A* at the balance, the E.M.F. of the Leclanché is (·001 × *d*) volts. With the apparatus of Fig. 408 any E.M.F. up to about $5\frac{1}{2}$ volts can be measured in this way.

If the P.D. to be measured is a large one, say of the order 200 or 250 volts, a high resistance (10,000 ohms or more) is connected to the two points between which the P.D. is required, and the P.D. for a known fraction of this resistance is measured as above; from this the full P.D. is readily found, since fall of potential is proportional to resistance. Thus to measure the P.D. between the leads *CF* (Fig. 409) a resistance *CF* of 10,000 ohms is inserted as shown, and of this a part *CB* of 50 ohms is connected to the potentiometer, the high potential end *C* being joined to *A*. If the balancing distance from *A* be 1,250 mm. the P.D. between *C* and *B* is ·001 × 1250 = 1·25 volts, and the full P.D. between *C* and *F* is $1{\cdot}25 \times \frac{10000}{50} = 250$ volts.

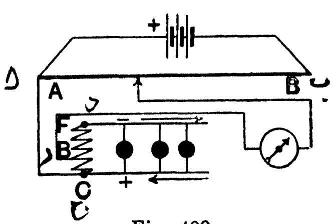


Fig. 409.

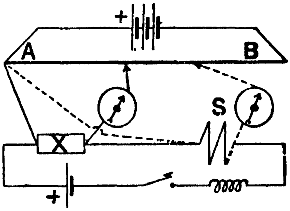


Fig. 410.

Exp. 2. *To measure a resistance by the potentiometer.*—The resistance *X* (Fig. 410) to be measured is put in series with a standard resistance *S*, an accumulator, and an adjustable resistance. The P.D. at the ends of the unknown is balanced on the potentiometer, as in the case above. This is repeated with the standard, and

$$\frac{\text{P.D. at ends of } X}{\text{P.D. at ends of } S} = \frac{(\text{Current in } X)(\text{Res. of } X)}{(\text{Current in } S)(\text{Res. of } S)} = \frac{\text{Res. of } X}{\text{Res. of } S},$$

i.e.

$$\frac{X}{S} = \frac{{\cdot}001 \times d_1}{{\cdot}001 \times d_2} = \frac{d_1}{d_2},$$

where d_1 and d_2 are the distances in millimetres from *A* to the sliding contact at the balances.

Exp. 3. *To measure a current by the potentiometer.*—The current to be measured is sent through a standard low resistance of ·1, ·01, or ·001 ohm according to circumstances, and the P.D. at the ends of this standard is measured in the usual manner; this P.D. divided by the resistance gives the current required. With the apparatus of Fig. 408, $5\frac{1}{2}$ volts can be measured, and this at the ends of a resistance of (say) ·001 ohm would mean a current of $5\frac{1}{2}/{\cdot}001$, *i.e.* 5,500 amperes; thus it is that large currents can be measured by the potentiometer method.

To measure the *resistance of a cell* by the potentiometer the cell is connected to the potentiometer as in Exp. 1, and its E.M.F. E is balanced. Its poles are then joined by a resistance r, and the terminal P.D. V is balanced. Clearly $E/V = d_1/d_2 = (b + r)/r$, *i.e.* $b = r\,(d_1 - d_2)/d_2$, where d_1 and d_2 are the balancing distances. ✓

In a thermo-electric circuit the electromotive force in the circuit under given conditions of temperature may generally be measured by a suitable modification of the potentiometer method. As the electromotive force in the circuit, usually expressed in *microvolts*, or millionths of a volt, is very small, the difference of potential along the potentiometer must be small and so subdivided that a difference of one-millionth of a volt may easily be read. One method of arranging this is indicated in Fig. 411. If the wire be divided into 1,000 divisions the difference of potential for each division will be about $1/10^7$ of the E.M.F. of the battery, and may therefore be small enough to admit of sufficiently accurate measurement.

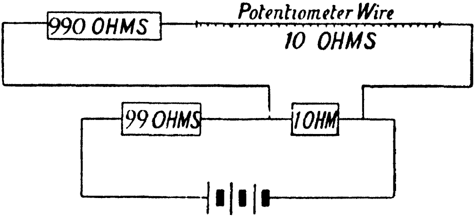


Fig. 411.

Of the many other methods for the measurement of the E.M.F. of a cell, three only will be briefly dealt with :—

Exp. 4. *To compare the E.M.F. of a Daniell's cell with that of a Leclanché cell by the sum and difference method.*—Place the two cells in series and complete the circuit through a tangent galvanometer and a resistance ; let θ_1 be the deflection. Now reverse one of the cells, keeping the total resistance as before, and let θ_2 be the deflection. If E_1 be the greater and E_2 the lesser E.M.F., and if K be the reduction factor of the galvanometer,

$$\frac{E_1 + E_2}{R} = K \tan \theta_1, \quad \frac{E_1 - E_2}{R} = K \tan \theta_2,$$

where R is the total resistance ; hence

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{\tan \theta_1}{\tan \theta_2},$$

$$\therefore \frac{E_1}{E_2} = \frac{\tan \theta_1 + \tan \theta_2}{\tan \theta_1 - \tan \theta_2}.$$

Exp. 5. *To compare the E.M.F.'s of two cells by the Lumsden method.*—Connections are made as shown in Fig. 412, and R_1 and R_2 adjusted for no deflection. The resistance R_1 is then altered to $R_1{}^1$ and R_2 adjusted to $R_2{}^1$, so that again there is no deflection. If E_1 and E_2 be the E.M.F.'s, B_1 and B_2 the resistances of the cells, and G the galvanometer resistance, then in the first case we have

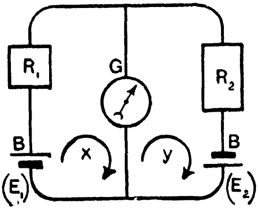


Fig. 412.

$$E_1 = (R_1 + B_1)x + G(x - y),$$
$$E_2 = (R_2 + B_2)y + G(y - x),$$

and, since the galvanometer is not deflected, $x = y$; hence

$$\frac{E_1}{E_2} = \frac{R_1 + B_1}{R_2 + B_2}.$$

Similarly, in the second case—

$$\frac{E_1}{E_2} = \frac{R_1{}^1 + B_1}{R_2{}^1 + B_2},$$

$$\therefore \frac{E_1}{E_2} = \frac{R_1{}^1 + B_1 - (R_1 + B_1)}{R_2{}^1 + B_2 - (R_2 + B_2)} = \frac{R_1{}^1 - R_1}{R_2{}^1 - R_2}.$$

Exp. 6. *To measure the E.M.F. of a cell by the condenser method.*—A condenser of capacity C is charged by the cell of E.M.F. E_1, then discharged through a ballistic galvanometer, and the first swing d_1 is noted. This is repeated with a cell of known E.M.F.. E_2, and the first swing d_2 is observed ; clearly—

$$d_1 \propto \text{Charge in Case 1} \propto CE_1,$$
$$d_2 \propto \text{Charge in Case 2} \propto CE_2,$$
$$\therefore \frac{E_1}{E_2} = \frac{d_1}{d_2}, \quad i.e.\ E_1 = \frac{d_1}{d_2}E_2.$$

236. The Crompton Potentiometer.—This is an excellent type of potentiometer for accurate work and is shown diagrammatically in Fig. 413. It consists of a wire divided into fifteen segments of equal resistance (about 2 ohms); fourteen of these are formed into coils, their ends being connected to the numbered studs shown at *E* on the right, the fifteenth being a stretched wire lying over a scale about 25 inches in length divided into 1,000 equal parts; thus the coils are permanently protected, and should the wire become worn or injured a new one may be substituted. In series with the potentiometer coils and wire is an adjustable resistance; this is shown at *G* and G_1 on the left, and is for the purpose of altering the P.D. at the ends of the fifteen segments so as to secure a balance with the standard cell in circuit; it corresponds to *X* and the back two wires of Fig. 408. *C* is the sliding contact, provided with a spring contact and micrometer adjustment for refined tests.

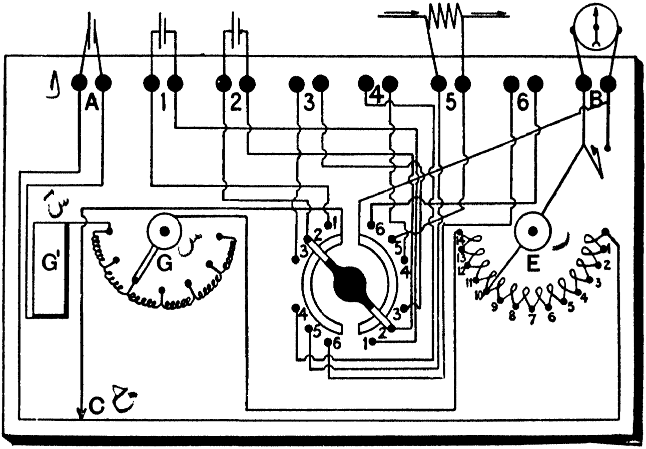


Fig 413.

The accumulator is connected to the terminals *A*, the galvanometer to terminals *B*, the standard cell to terminals 1, any unknown E.M.F.'s to the terminals 2, 3, 4, 5, and 6, and the double contact switch at the centre enables any of these E.M.F.'s to be put into the galvanometer circuit. The reader should carefully examine these connections and compare them with those of Fig. 408.

Working with one accumulator it is customary to standardise the instrument so that the total P.D. at the ends of the potentiometer coils and wire is 1·5 volts. To secure this, the revolving arm at E is placed on stud 14, and the sliding contact C at division 340 on the scale; the centre switch is now set on 1, thus bringing the standard cell into the galvanometer circuit, and the resistances in series with the coils and wire are adjusted till the galvanometer is not deflected. Clearly the P.D. for *each* coil and for the wire is now 1·434/14·34 or ·1 volt, and the P.D. for *each* of the 1,000 divisions is ·0001 volt.

The E.M.F. to be measured is now brought into the galvanometer circuit by moving the centre switch to the corresponding studs. With C on the extreme right, the revolving arm at E is adjusted till two successive contacts give deflections in opposite directions; E is now placed on the stud of lower value and C adjusted for no deflection. If this condition is realised with the arm E on stud 10 and the slider C on division 646, the E.M.F. under test is

$$\{(10 \times \cdot 1) + (646 \times \cdot 0001)\} \text{ volts,}$$

i.e.

$$1 \cdot 0646 \text{ volts.}$$

237. Determination of the Capacity of a Condenser.—Before proceeding with this section the student should again read the capacity and specific inductive capacity determinations dealt with in Electrostatics (Chapter VIII.). The capacity of a condenser *in electromagnetic units* may be measured by the following method:—

Exp. The condenser is charged to a definite difference of potential, V, then discharged through a ballistic galvanometer and the throw of the galvanometer needle noted. The same difference of potential, V, or, as this is generally too great, a known fraction of V is then applied so as to give a steady current through the galvanometer. The steady deflection thus produced is noted and compared with the throw due to the discharge. The capacity of the condenser can then be calculated from the constants involved by the conditions of the experiment.

In practice the condenser may be connected as shown at C in Fig. 414, so that by connecting the points a, b, it may be charged up to the difference of potential between the points A, B, on the external circuit of a constant low resistance battery, and by connecting a, c, it may be discharged through the galvanometer G. If α be the first angular throw of the needle, corrected for damping, we have

$$Q = \frac{HT}{\pi G} \cdot \frac{\alpha}{2}.$$

That is,

$$CV = \frac{H}{G} \cdot \frac{T\alpha}{2\pi}, \text{ or } C = \frac{H}{VG} \cdot \frac{T\alpha}{2\pi},$$

where V is the difference of potential between A and B.

To determine the value of H/VG here, let the points c, d, be connected so that a steady current passes from A to D through the galvanometer, and let δ denote the permanent angular deflection of the needle. Then the current i is given approximately by

$$i = \frac{H}{G} \cdot \delta.$$

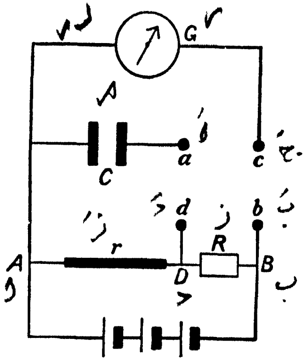


Fig.414.

But if S, the resistance of the galvanometer branch, is very large compared with r, the resistance between A and D, we have $i = v/S$, where v denotes the initial difference of potential between A and D. Also, if R, the resistance between D and B, is large compared with r, then

$$v = \frac{r}{R+r} V, \quad \text{and} \quad i = \frac{r}{R+r} \frac{V}{S}.$$

That is,
$$\frac{r}{R+r} \cdot \frac{V}{S} = \frac{H}{G}\delta, \quad \text{or} \quad \frac{H}{GV} = \frac{r}{(R+r)S\delta}.$$

Substituting this value in the result obtained above we get

$$C = \frac{rT\alpha}{2\pi(R+r)S\delta} = \frac{rTd_1}{2\pi(R+r)Sd_2},$$

where d_1 and d_2 are linear deflections on the galvanometer scale, d_1 being corrected for damping.

This result is sufficiently accurate when r is small compared with R and S, but for exact calculation we have

$$V = \frac{R+r}{R+r+B} E,$$

where E is the electromotive force and B the internal resistance of the battery, and

$$v = \frac{\rho E}{\rho + R + B}, \quad \text{where } \rho = \frac{rS}{r+S}.$$

The exact relation between v and V, assuming E to be constant, can thus be obtained, and the value of C deduced as above.

238. Comparison of Capacities.

—The capacities of two condensers may be readily compared by means of a ballistic galvanometer.

Exp. 1. *To compare two capacities by a ballistic galvanometer.*—Charge the first condenser by a battery of constant E.M.F. E, and then discharge through the galvanometer. Repeat with the second condenser. If C_1 and C_2 be the capacities and d_1 and d_2 the first swings corrected for damping, the quantities discharged are EC_1 and EC_2, and

$$\frac{EC_1}{EC_2} = \frac{C_1}{C_2} = \frac{d_1}{d_2}.$$

The accuracy of the result by the above method evidently depends on the accuracy of observation of the deflections—and experiment shows that the errors attending this observation may be considerable. It is found that much more accurate results may in all cases be obtained by *null* or *zero* methods, in which the measurement depends upon adjusting for no deflection of the galvanometer. Two of the best known null methods for comparing capacities are given below.

Exp. 2. *To compare two capacities by the Wheatstone Bridge or De Sautys method.*—In Fig. 415 K_1 is the condenser whose capacity C_1 is required, and K_2 a standard condenser of capacity C_2. The resistances R_1 and R_2 are adjusted until, on making contact at a and charging the condensers, the galvanometer is not deflected. When this is so, the potentials at B and C are equal, and since D is a common point, the P.D. on the condenser K_1 is the same as that on the condenser K_2. If V be this P.D., Q_1 the charge on K_1, and Q_2 the charge on K_2,

$$V = \frac{Q_1}{C_1} \quad \text{and} \quad V = \frac{Q_2}{C_2}; \quad \therefore \frac{C_1}{C_2} = \frac{Q_1}{Q_2}.$$

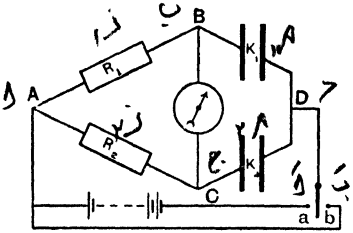


Fig. 415.

But Q_1 has passed through R_1 and Q_2 through R_2 in the same short interval of time, and since the current divides *inversely* as the resistance, $Q_1 : Q_2 = R_2 : R_1$;

$$\therefore \frac{C_1}{C_2} = \frac{R_2}{R_1} \quad \text{and} \quad C_1 = \frac{R_2}{R_1} C_2.$$

The condensers are discharged by moving the key to the contact *b*.

Exp. 3. *To compare two capacities by the method of mixtures or Kelvin's method.*—The principle of this method is shown in Fig. 416. The condenser C is charged to the difference of potential between A and B, and the condenser S to the difference of potential between B and C. By adjusting the resistances P and Q these two differences of potential can be made to have the inverse ratio of the capacities. When this adjustment is made we have

$$\frac{{}_{\mathrm{A}}V_{\mathrm{B}}}{{}_{\mathrm{B}}V_{\mathrm{C}}} = \frac{S}{C}$$

or

$${}_{\mathrm{A}}V_{\mathrm{B}} \cdot C = {}_{\mathrm{B}}V_{\mathrm{C}} \cdot S,$$

that is, *the condensers possess equal charges*, and since

$$\frac{{}_{\mathrm{A}}V_{\mathrm{B}}}{{}_{\mathrm{B}}V_{\mathrm{C}}} = \frac{P}{Q},$$

we evidently have for this adjustment

$$\frac{S}{C} = \frac{P}{Q}$$

or

$$C = \frac{Q}{P} S.$$

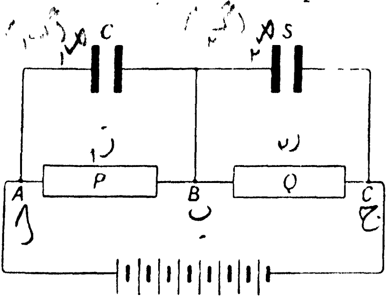


Fig. 416.

In order to test the equality of the charges the condensers after charging must first have their oppositely charged plates connected, so that the two charges are "mixed" and tend to neutralise. Then immediately this "mixing" is effected the combined condensers are discharged through a galvanometer. If the charges before "mixing" were equal the final charge after mixing will be zero, and there will then be no deflection of the galvanometer. The operations therefore consist of charging, mixing, and discharging through the galvanometer, and P and Q are adjusted until the galvanometer shows no deflection. When this adjustment is made we have

$$C = \frac{Q}{P} S,$$

where S is the capacity of the standard condenser.

Fig. 417 shows diagrammatically the connections for performing the three operations of the method in rapid sequence. By pressing the K_1 keys charging is effected; the K_2 keys, when closed, arrange the "mixing," and on pressing K_3 the condensers C and S as one combined condenser are discharged through the galvanometer G. By using a Pohl's commutator without the cross wires the K_1 and K_2 contacts shown in the diagram between the condensers can each be effected at one operation.

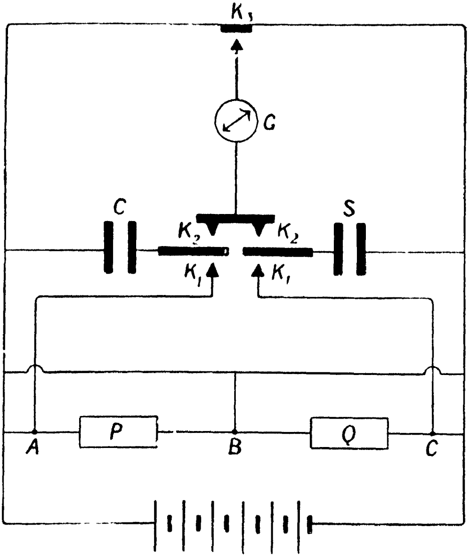


Fig. 417.

In these tests keys, etc., should be well insulated and resistances non-inductive. The determination of capacity by "oscillations" is referred to in Art. 313.

238a. Determination of Cable Faults.—Three cable faults will be briefly dealt with, viz. (1) a breakdown in the insulating covering only, the conductor remaining intact, (2) a complete breakdown in both conductor and insulating covering, (3) a breakdown in the conductor, the insulating covering remaining intact.

Test for No. 1. Let *BD* (Fig. 417*a*) represent the cable joining two distant stations *X* and *Y* and in which a partial earth exists at *Z*. At *Y* the cable is "looped" to a second one *AC* also running between *X* and *Y*, by a conductor *CD* of negligible resistance. The free ends of the cables are connected at the test station to two adjustable resistances *P* and *R*; a sensitive galvanometer *G* joins *A* and *B* through the key *L*, whilst the junction *J* is connected through *K* to the battery, the other pole of which is earthed.

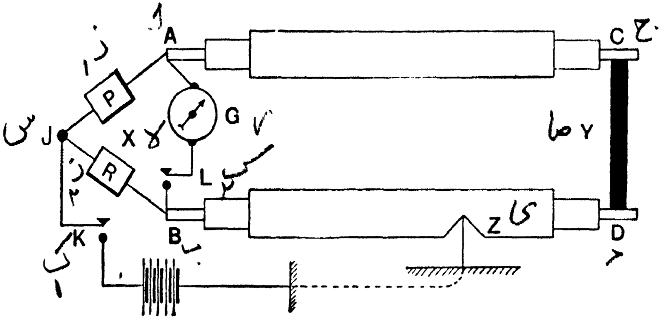


Fig. 417*a*.

The arrangement is thus equivalent to a Wheatstone Bridge, the arms being $P - R - ACDZ$ and ZB. If P and R be now adjusted, so that on closing K and then L the galvanometer is not deflected, we have—

$$\frac{P}{R} = \frac{\text{Res. } AC + \text{Res. } DZ}{\text{Res. } ZB},$$

hence $$\frac{P+R}{R} = \frac{\text{Res. } AC + \text{Res. } DZ + \text{Res. } ZB}{\text{Res. } ZB},$$

$$\therefore \frac{\text{Res. } ZB}{\text{Res. } AC + \text{Res. } DB} = \frac{R}{P + R},$$

$$i.e. \text{ Res. } ZB = \frac{R}{P + R} \times r,$$

where r = the resistance of the two cables in series. Thus, knowing the resistance per mile of the cable, the distance BZ corresponding to the resistance BZ is ascertained.

Test for No. 2. If the broken end of the cable makes a good "earth" the free end is joined to terminal D of the Post Office Box (Fig. 395) and terminal C is earthed. Since C and the broken end of the cable are both earthed (zero potential) they are, from an electrical point of view, virtually connected, so that the cable conductor from the free end to the break takes the part of S in Fig. 308. The resistance is therefore measured and the distance ascertained in the usual way.

Test for No. 3. The *insulation resistance* R from the free end to the break is measured (Art. 230) and the distance l found from the relation of Art. 159. The *capacity* C from the free end to the break is also measured (Art. 127, Exp. 3) and the distance l found from the relation of Art. 113. (For exact details see *Technical Electricity*).

It may be mentioned in passing that since resistance is defined as the ratio of P.D. to current, several *practical* resistance tests may be performed by the combined use of a suitable ammeter and voltmeter. The *hot resistance of a glow lamp*, for example, may be determined by inserting an ammeter in series with the lamp and a voltmeter across the lamp terminals. If I be the ammeter reading, V the voltmeter reading, and R the resistance of the voltmeter under these conditions, the current taken by the voltmeter is V/R, and, therefore, the current in the lamp is $I - V/R$; hence:—

$$\text{Res. of lamp} = \frac{\text{P.D. at its terminals}}{\text{Current in it}} = \frac{V}{I - \frac{V}{R}}.$$

For a description of various "commercial" testing instruments the student should refer to *Technical Electricity.*

Exercises XVI.

(1) Describe some method of determining accurately the specific resistance of an electrolyte. (B.E. Hons.)

(2) Explain and prove Mance's method for the determination of the internal resistance of a galvanic battery, and find an expression for the value of the current through the galvanometer. (Inter. B.Sc. Hons.)

(3) Describe carefully how you would use a potentiometer for measuring currents. How would you adapt it for use with large and small currents respectively? (B.Sc.)

(4) Describe the best method you know for determining the internal resistance of a battery. (B.Sc.)

(5) A Wheatstone Bridge is employed for the measurement of the resistance of a wire. Discuss the best arrangement of the conductors in order to secure the greatest sensibility, and show how to determine the current in the galvanometer in terms of the several resistances when the bridge is not balanced.
State Kirchhoff's laws for a system of linear conductors. (D.Sc.)

CHAPTER XVII.

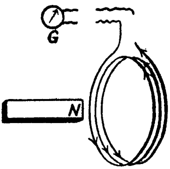
ELECTROMAGNETIC INDUCTION.

239. Fundamental Experiments. Laws of Electromagnetic Induction.—In 1831 Faraday described experiments whereby he clearly established the fact that **whenever the flow of induction or number of tubes of magnetic induction through a circuit is changed an E.M.F. is developed in the circuit,** such E.M.F. *lasting only while the change is taking place*; electromotive forces and currents produced in this way are spoken of as **induced** electromotive forces and currents respectively.

The magnetic induction or flux through a circuit may be changed by various means, *e.g.* by moving a magnet in the vicinity, by changing the current in a neighbouring circuit or by relative motion of the two circuits, by changing a current in the circuit itself, or by suitably moving the circuit in a magnetic field.

Exp. 1. *Motion of a magnet.*—Connect the coil (Fig. 418) to a sensitive galvanometer some distance away. Include a Leclanché cell and a resistance in the circuit and note the direction in which the galvanometer is deflected; let it be (say) *to the right.* While this current is flowing, test the polarity of the face of the coil towards the left; let it be (say) *a north face.* Hence we know that with the present connections a deflection to the right indicates that the current in the coil is making the face towards the left a north, and is, therefore, counter-clockwise viewed from this side. Evidently a deflection *to the left* will indicate that the face on the left is *a south face* and the current clockwise. Remove the cell and the resistance.

Quickly insert the north pole of a magnet (Fig. 418); the galvanometer will be deflected, showing that a current is induced in the coil: note the direction of deflection, which in this case will be to the right. Hold the magnet in the coil and the galvanometer will come to rest, *showing that the induced current is momentary, lasting only while the magnet is moving.* Quickly withdraw the north pole, and a momentary deflection to the left will be produced. Similarly, insert the south pole, and the momentary deflection will be to the left; withdraw it, and the momentary deflection will be to the right. Tabulate the results thus:—



Magnet approaching Coil

Fig. 418.

Motion of Magnet.	Direction of Deflection.	Induced Current makes near Face a
N. pole inserted	Right	North
N. pole withdrawn	Left	South
S. pole inserted	Left	South
S. pole withdrawn	Right	North

From the preceding we learn that when the north pole is inserted the induced current has such a direction that the face of the coil approached is a north face, *which, therefore, tends to oppose the motion of the magnet.* When the north pole is withdrawn the induced current has such a direction that this face is a south face, which, therefore, *tends to draw the magnet back again,* and so on. Hence we have the important law known as **Lenz's Law,** viz. **The direction of the induced current (and E.M.F.) is such that it tends to oppose the motion or change which produces it.** Further, it will be seen from Fig. 418 that when the north pole approaches the coil the number of tubes through the latter is being increased, and that the induced current has such a direction that it gives rise to tubes through the coil in the opposite direction to those due to the magnet, tending, therefore, to oppose and cancel the increase due to the magnet. When the pole is withdrawn it will be found that the tubes through the coil due to the induced current are in the same direction as those being removed, tending, therefore, to oppose and cancel the decrease due to the withdrawal of the magnet.✓ قرارداد شد

Again, by moving the magnet (1) quickly, (2) slowly, it will be found that the deflection is more pronounced in the first case; we may, therefore, infer that **the induced E.M.F. and current depend upon the rate of change of the flux, being greater the more rapid the change;** the induced current also depends, of course, upon the resistance (see below).

Exp. 2. *Action of a neighbouring circuit* (*mutual induction*).—In Fig. 419 *AB* diagrammatically represents a *coil* of wire joined to a galvanometer; it is referred to as the *secondary circuit. CD* represents a *coil* in series with a battery and key; it is referred to as the *primary circuit.*

Now start a current in the primary in the direction *C* to *D* and a momentary deflection of the galvanometer will ensue, showing that a current is induced in the secondary in the direction *B* to *A*, *i.e.* opposite or **inverse** to the primary current; it should be noted that starting a current in the primary means **increasing** the flux in the secondary, for the flux in the primary naturally reaches over to the secondary circuit.

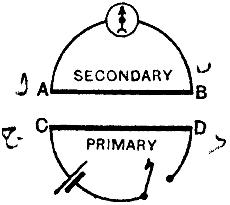


Fig. 419.

Switch off the primary current, and a momentary deflection of the galvanometer will follow, indicating a secondary current in the direction *A* to *B*, *i.e.* in the same direction as, or **direct** to, the primary current, and it should be noted that stopping the primary current means **decreasing** the flux in the secondary.

Similarly, *increasing* the primary current or *moving it nearer* to the secondary circuit results in an *inverse* current; *decreasing* the primary current or *moving it away from* the secondary circuit results in a *direct* current.

From the preceding we arrive at another important law, viz. **An increase in the flux results in an induced inverse current, whilst a decrease in the flux results in an induced direct current** (**I**ncrease — **I**nverse: **D**ecrease — **D**irect) This is quite in agreement with Lenz's Law, *e.g.* if the primary current flows from *C* to *D* and the primary is moved nearer the secondary, the induced secondary current flows from *B* to *A*; these two parallel currents, being in opposite directions, repel each other, so that the tendency is to drive the primary back again. Further, by considering the flux as before, it will be clear that the flux due to the induced current is always such as to oppose the change in the flux due to the primary. It should also be noted that not only does the primary act upon the secondary, but that the secondary reacts upon the primary; hence these effects are known as **mutual induction.**

Exp. 3. *Current changes in the circuit itself* (*self-induction*).—In Fig. 420 PQ is a solenoid, B a battery, G a galvanometer, and K a key arranged as indicated. Close K, and when the deflection is steady place a stop against the needle at n'. Open K. On again closing K the needle will be momentarily deflected beyond its fixed position. This indicates that at the moment of starting the current in the circuit the galvanometer current is greater than the normal, due to the fact that there is a momentary induced pressure in the solenoid in the direction Q to P, *i.e.* opposite to the current, which retards the growth of the current there, and therefore increases the portion through G.

Open K and place a stop against the needle in its normal position, *i.e.* at n. Close K. On opening K the needle will be momentarily deflected in the opposite direction. This indicates that when the current is stopped there is a momentary induced current in the solenoid in the same direction as the original current, *i.e.* from P to Q, which, therefore, passes through G in the opposite direction, Y to X. This induced current, at break, in the same direction as the original one, is called the "*extra current.*"

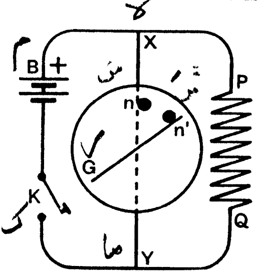


Fig. 420.

Thus from the above and modified experiments we learn that when a current is started in a circuit, or when an existing current is increased (flux *increasing*), an opposing or *inverse E.M.F.* is induced which retards the growth of the current, when a current in a circuit is stopped or reduced (flux *decreasing*) a *direct current* is induced, *i.e.* one in the same direction as the original. It is this latter "extra current" which frequently gives rise to a "spark at break." It will be further noted that these results are also quite in accord with Lenz's Law. These effects are known as "**self-induction.**"

From the preceding it will be clear that self-induction behaves in a circuit like *inertia*, e.g. when we try to produce a current in a circuit this self-induction or inertia tends to choke the current back, and when we try to stop the current this self-induction tries to make it keep on.

Further experiments indicate that the inductive effects in Exps. 2 and 3 are increased if the coils be wound upon an iron core, the effects being in fact proportional to the permeability; this indicates that *it is the change in the magnetic induction, not the change in the magnetic force*, which is the quantity involved in the experiments; of course in air (strictly in vacuo) B and H are numerically equal.

In Exp. 1 it is indicated that the induced E.M.F. depends upon the "rate of change"—strictly, *upon the rate of change of magnetic induction*; this may be shown more exactly as follows. Let PQ (Fig. 421) be a copper bar capable of sliding along the parallel copper rails SP and RQ, and let B be a battery joined to S and R. *Let the arrangement be in air*, and let H denote the intensity of the vertical field, supposed uniform. An application of Fleming's Left Hand Rule (Art. 170) to PQ will show that PQ will tend to move in the direction indicated; let it move through a small distance dx in time dt seconds.

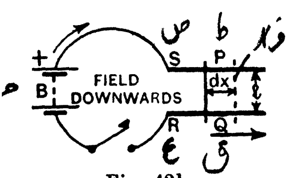


Fig. 421.

Then, since H is the number of unit tubes per square centimetre and ldx is the area swept through, l being the distance between the rails, $ldxH$ is the number of unit tubes cut during the motion, and if I be the current in the circuit the work done is $IldxH$ (Art. 171),

i.e. $$\text{work done} = IldxH = I\,dF,$$

where dF denotes the change in the number of unit tubes through the circuit.

The energy required to do this is derived from the battery. The total energy supplied by the battery in time dt is $EIdt$ and the energy spent in heat is I^2Rdt;

hence
$$EIdt = I^2Rdt + IdF,$$
$$\therefore\ IRdt = Edt - dF,$$
i.e.
$$I = \frac{E - dF/dt}{R},$$

and the E.M.F. of the circuit is therefore opposed by an E.M.F. equal to dF/dt, *i.e.* if e denote this induced E.M.F.—

$$e = -\frac{dF}{dt};$$

thus **the induced E.M.F. is equal to the rate of change of the number of unit tubes threading the circuit.**

As previously indicated, "rate of change of the number of unit tubes of induction" is really implied, *i.e.* B should replace H in the preceding, and dF should denote the change in the number of unit tubes of induction; the experiment, however, is *in air.*

Again, since $dF = lHdx$, $dF/dt = lH\,dx/dt$, and dx/dt is the velocity v of PQ, hence

$$e = -\frac{dF}{dt} = -lHv.$$

An examination of Fig. 421 will show that in such a case as the one considered the direction of the induced E.M.F. in the bar PQ (*viz.* Q to P) is readily given by the following rule known as **Fleming's Right Hand Rule**:—*Hold the thumb and the first two fingers of the right hand mutually at right angles. Place the forefinger in the direction of the lines of force and turn the hand so that the thumb points in the direction of motion. The second finger will point in the direction of the induced E.M.F. (and induced current).*

For calculation purposes it is well to remember that since $e = -dF/dt$ we may, neglecting sign, write—

$$\text{Induced E.M.F.} = \frac{\text{Change in flux}}{\text{Time in seconds}} \text{ (e.m. units)}$$
$$= \frac{\text{Change in flux}}{10^8 \times \text{Time in seconds}} \text{ (volts).}$$

Further, if there are n turns in a coil *each* acquires the above E.M.F., and since the several turns are in series the total E.M.F. is n (Change in flux)/Time in seconds. The product of the flux through a coil and the number of turns is called the *effective flux* or the *linkages*; hence

$$\text{Induced E.M.F.} = \frac{\text{Change in effective flux}}{\text{Time in seconds}}$$

$$= \frac{\text{Change in linkages}}{\text{Time in seconds}}.$$

Example. *A coil of wire is connected to a galvanometer, the resistance of the coil and galvanometer being* 200 *and* 400 *ohms respectively. The coil is moved in the field, and at a given instant there are* 20,000 *unit tubes through it, whilst* $\frac{3}{4}$ *of a second later the number is* 2,000. *If there are* 100 *turns in the coil find the average E.M.F. and current during this period.*

Effective flux or linkages at the beginning of the time in question $= 100 \times 20000 = 2{,}000{,}000$, and at the end of the time $= 100 \times 2000 = 200{,}000$; hence change in effective flux $= 1{,}800{,}000$, and

$$\text{Average induced E.M.F.} = \frac{18 \times 10^5}{10^8 \times \frac{3}{4}} = \cdot 024 \text{ volt,}$$

$$\textit{and } \text{Induced current} = \frac{\cdot 024}{200 + 400} = \cdot 00004 \text{ ampere.}$$

240. Quantity of Electricity set in Motion by Inductive Action.—Let the flow of induction through a circuit be denoted by F. If at any instant we have an infinitely small change dF in this flow in the infinitely small time dt, then, neglecting sign, $e = dF/dt$.

If R denote the resistance of the circuit, the current during the time dt will be given by

$$I = \frac{e}{R} = \frac{1}{R} \cdot \frac{dF}{dt},$$

and the quantity of electricity set in motion during this time will be

$$I \, . \, dt = \frac{dF}{R}.$$

It follows that, for any finite change of the flow of induction from F to F', the *quantity* of electricity inductively set in motion is given by

$$q = \frac{F - F'}{R}.$$

This shows that *the induced quantity is independent of the "rate of change," but varies inversely as the resistance of the circuit.*

The above explains why, in induction experiments with the ballistic galvanometer, the throw of the needle is the same whatever the time of variation of the flow of induction through the coil, provided this time is small compared with the time of swing of the needle. Also, as q varies inversely as R, it is evident that in order to get an appreciable throw of the galvanometer needle the resistance of the circuit, including the galvanometer, must be small, for the smaller it is the larger q will be. On the other hand, when a ballistic galvanometer is used to determine the quantity of electricity discharged through it by a condenser, the quantity of electricity is *fixed* and *small*, and the throw of the needle will be increased by the multiplying effect of a coil of many turns without being reduced by the resistance of the coil. *For these reasons a low resistance ballistic galvanometer is best to use for magnetic induction experiments and a high resistance instrument must be used for condenser work.*

241. The Case of the Rotating Coil.—Imagine a coil of wire capable of rotation about a vertical axis in (say) the earth's magnetic field; the number of tubes passing through the coil will vary as it rotates and currents will be induced in it. Consider the coil to start rotating from a position at right angles to the magnetic meridian, and the east side of the coil to rotate towards the south. When at right angles to the meridian, the greatest possible number of unit tubes cross the circuit of the coil, and when parallel to the meridian the number is diminished to zero. The effective flux across the coil when at right angles to the meridian is given by SAH, where S denotes the number of turns of the coil, A the area enclosed by the coil, and H the horizontal component of the earth's field. Hence, during the first 90° of a revolution, starting from a position at right angles to the meridian, the effective flux across the coil *decreases* from SAH to zero, and the direction of the induced current will be *from west to east* (*for the starting* *position) along the upper half of the coil.* The direction of the induced current is readily determined by the application of the rules of Art. 239.

During the next 90° of rotation the effective flux will evidently *increase* from zero to SAH, and the direction of the induced current will be from east to west (for the final position) along the upper half of the coil—that is, *the current will still be in the same direction in the coil*; for although its direction *in space* is reversed, the position of the coil is similarly reversed by revolution, so that the direction of the current in the coil is the same during the first 180° of revolution, starting from the position in which the flux through the coil is a maximum.

During the next 180° of revolution the same changes in the flux will take place as described above, and a current will be induced *in the coil* in the *opposite* direction to that induced during the first half-revolution, owing to the lateral reversal of the coil by that half-revolution. Hence *during each complete revolution the direction of the induced current changes as the coil passes through the position at right angles to the direction of the field.*

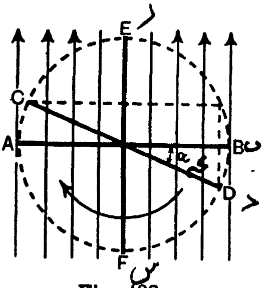


Fig. 422.

Now let Fig. 422 represent a plan of the preceding, AB being the original position of the coil, *viz.* at right angles to the field. The effective flux through the coil in the position AB is SAH and the effective flux F through the coil when it has rotated through an angle α into the position CD is given by

$$F = SAH \cos \alpha.$$

For the induced E.M.F. at this instant we have

$$e = -\frac{dF}{dt} = SAH \sin \alpha \frac{d\alpha}{dt},$$

and since $d\alpha/dt$ is the angular velocity ω—

$$e = SAH\omega \sin \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Again, if T be the time of one revolution, $\omega = 2\pi/T$, and if t be the time taken to rotate through the angle a, $T/t = 2\pi/a$, i.e. $a = 2\pi t/T$; hence

$$e = \frac{2\pi SAH}{T} \sin a = \frac{2\pi SAH}{T} \sin \left(\frac{t}{T} 2\pi\right) \quad \ldots\ldots (2)$$

Further, if the coil makes n revolutions per second, $T = 1/n$ and

$$e = 2\pi n SAH \sin a \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$ ✓

These expressions give the **instantaneous value of the induced E.M.F.** Clearly the E.M.F. will have its *maximum value* when $\sin a = 1$, *i.e.* when $a = \pi/2$ or $3\pi/2$, and the coil therefore in the position EF; the E.M.F. will have its *minimum value* (*viz.* zero) when $\sin a = 0$, *i.e.* when $a = 0$ or π or 2π, and the coil therefore in the position AB. Again, since $2\pi t/T = a$, the maximum value is reached when $2\pi t/T$ is equal to $\pi/2$ or $3\pi/2$, *i.e.* when $t = \frac{1}{4}T$ or $\frac{3}{4}T$; similarly the minimum value is reached when $t = 0$ or $\frac{1}{2}T$ or T. Further we have

$$\textbf{Maximum E.M.F.} = SAH\omega = \frac{2\pi SAH}{T}$$

$$= 2\pi n SAH \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The instantaneous E.M.F. is, from the above, proportional to the sine of the angle described from the starting position and the variation of this E.M.F. during one complete revolution is represented by the **sine curve** of Fig. 423, ordinates above the horizontal representing the induced E.M.F. in one direction, those below, the induced E.M.F. in the opposite direction. The induced current is obtained from the preceding expressions by dividing by R the resistance of the circuit; if the coil has no self-induction the current (dotted curve) will vary in the same way as the E.M.F.

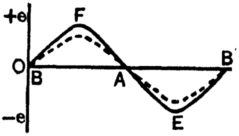


Fig. 423.

In the preceding we have dealt with the *instantaneous* and the *maximum* values of the induced E.M.F.; the **average E.M.F.** may be simply found as follows :—Imagine the coil to rotate through 90° from the position AB to the position EF. The flux through the coil changes from SAH to zero; hence the change in the flux in a quarter turn is SAH. The quantity of electricity set in motion is therefore SAH/R; hence in one complete revolution the quantity circulated will be $4SAH/R$, and if the coil makes n revolutions per second the quantity per second will be $4nSAH/R$. Since, however, quantity is equal to average current multiplied by time in seconds, we arrive at the result

$$\text{Average current} = \frac{4nSAH}{R}$$

$$\text{and } \textbf{Average E.M.F.} = \mathbf{4}nSAH \quad \ldots\ldots\ldots \quad (5)$$

From (4) and (5) it follows that the average E.M.F. and current are $2/\pi$, *i.e.* ·637 of the maximum E.M.F. and current.

Exp. *To determine the angle of dip by induced currents.*—Set up a coil as above, with its plane at right angles to the meridian, and put in series with it a ballistic galvanometer. Quickly rotate the coil *through* 180° about a vertical axis, and note the throw θ_1 of the galvanometer. If H be the horizontal component, the change in the effective flux is $2SAH$ and the quantity induced is $2SAH/R$; hence

$$\frac{2SAH}{R} \propto \theta_1.$$

Now arrange the coil in a horizontal position with the axis of rotation in the meridian, again turn through 180° and let θ_2 be the throw of the galvanometer; if V be the vertical component, it is clear that the quantity induced in this case is $2SAV/R$; hence

$$\frac{2SAV}{R} \propto \theta_2,$$

$$\therefore \quad \tan D = \frac{V}{H} = \frac{\theta_2}{\theta_1},$$

where D is the angle of dip. A coil properly mounted and arranged for this and similar work is called an **earth inductor.**

242. Measurement of Magnetic Fields. Standard Inductors.

—In measuring the intensity of a field of magnetic force or induction the usual practice is to determine the quantity of electricity set in motion in a suitable test coil or inductor, by suddenly removing it from the field or by rotating it through 180° in the field, as described in the case of the earth inductor. The test coil is connected in circuit with a ballistic galvanometer, and by noting the throw of the galvanometer needle under definitely arranged conditions both comparative and absolute measurements of induction may be made. The comparison of V and H for the determination of the dip, as given in Art. 241, is a simple example of this method of comparison.

If, however, we wish to compare two magnetic fields of very different intensities a different method has to be adopted. In comparing, for example, the horizontal component of the earth's field with the field between the poles of a strong electromagnet it is necessary to use a different inductor for each field, and the comparison therefore involves the constants of the inductors. The effective area of the inductor for the strong field may be small, but the effective area of the earth inductor must be large on account of the low intensity of the field. The general method is indicated in the experiments below.

It should be again noted, however, that these induction experiments relate to the *induction* in the field, and not to the magnetic force directly. If, however, the observations are made in the same medium, and if the permeability of the medium does not vary with the intensity of the magnetic field, then the induction at any point is directly proportional to the magnetic field at that point, and the ratio of any two induction values therefore determines the ratio of the corresponding field intensities. That is, if $B_1/B_2 = k$, then, since $B_1 = \mu H_1$ and $B_2 = \mu H_2$, *where* μ *is a constant*, we have $\mu H_1/\mu H_2 = k$ or $H_1/H_2 = k$. The value of μ for air is taken as unity and is practically constant.

Exp. *To determine the strength of the field between the poles of an electromagnet, using a standard earth inductor.*—Place a small test coil (consisting of s turns each of area a) in series with a ballistic galvanometer and an earth inductor (consisting of S turns each of area A). Set the earth inductor at right angles to the meridian, quickly rotate it through 180° about a vertical axis, and let d_1 be the throw of the galvanometer. As before, we have $2SAH/R \propto d_1$, where R is the resistance of the circuit and H the horizontal component of the earth's field.

Place the test coil in the field (F) to be measured, with its plane at right angles to the field. Quickly withdraw it from the field and let d_2 be the throw of the galvanometer. The effective flux is saF, and, as this is reduced to zero when the coil is withdrawn, the induced quantity is saF/R; hence $saF/R \propto d_2$. Thus we have

$$\frac{2SAH}{R} \bigg/ \frac{saF}{R} = \frac{d_1}{d_2}$$

or

$$\frac{2SAH}{saF} = \frac{d_1}{d_2},$$

$$\therefore F = \frac{2SA}{sa} \cdot \frac{d_2}{d_1} H.$$

As H is known, F is therefore determined: the earth inductor used in this experiment may be looked upon as a **standard earth inductor,** serving to standardise the observations taken with the small inductor acting as a test coil in the unknown field.

Another inductor, known as the **standard solenoidal inductor,** makes use of the uniform field in the interior of a long solenoid as a standard. If I denote the current in absolute units, the field in the interior of the coil is given by $4\pi nI$, where n is the number of turns *per unit length* of the coil. The inductor for use with this field usually consists of a few turns of thin and well insulated wire wound round the outside of the solenoid near its middle point. The induction throw is obtained by reversing the current in the solenoid. If n', a' denote the number of turns and area of the inductor respectively, and I the current in absolute units, then, on reversing the current, the change in the flow of induction through the inductor coil is $2(4\pi nI)n'a'$, i.e. $8\pi nIn'a'$, and the quantity of electricity set in motion is $8\pi nIn'a'/R$, where R is the resistance of the circuit in which the inductor is placed. If I is measured accurately by means of an ammeter in the solenoid circuit, this inductor may be used for standardising the observations of a test inductor placed in the same circuit with it.

Exp. *To determine the strength of the field between the poles of an electromagnet, using a standard solenoidal inductor.*—The arrangement is indicated in Fig. 424. Start the current in the circuit, and when the ballistic galvanometer (BG) is quite steady reverse the current by means of the key K and note the throw of the ballistic galvanometer. If this be d_1, then

$$\frac{8\pi n I n' a'}{R} \propto d_1.$$

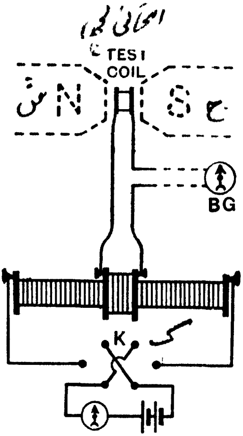


Fig. 424.

Place the test coil in the field as in the previous experiment, withdraw it, and let d_2 be the galvanometer throw ; then

$$\frac{saF}{R} \propto d_2.$$

Thus we have

$$\frac{saF}{R} \Big/ \frac{8\pi n I n' a'}{R} = \frac{d_2}{d_1},$$

$$\therefore\ F = \frac{8\pi n I n' a'}{sa} \cdot \frac{d_2}{d_1}.$$

An excellent standard for laboratory purposes is that known as **Hibbert's Magnetic Flux Standard.** It consists (Fig. 425) of a block of hard steel provided with a cylindrical groove, and magnetised as indicated. A brass cylinder B carries a coil C; it can be lowered into the groove, the coil thereby cutting the tubes due to the magnet, in consequence of which an induced charge circulates in the coil. The flux is determined at the outset by comparison with (say) a solenoidal standard.

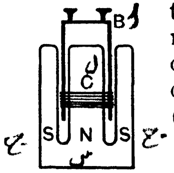


Fig. 425.

Professor Rowland's method of finding the distribution of magnetism along a bar magnet may now be briefly mentioned (see Exp. 7, p. 111). A coil embraces the magnet and is connected to a ballistic galvanometer; it is moved rapidly from point to point along the magnet and the first swings are noted. A curve with distances along the magnet as abscissae and first swings as ordinates is the distribution curve for the magnet.

243. Determination of the Constant of a Ballistic Galvanometer.—A standard inductor may be used for determining the constant of a ballistic galvanometer.

Exp. *To determine the constant of a ballistic galvanometer, using the standard solenoidal inductor.*—The arrangement is indicated in Fig. 426, where BG is the ballistic galvanometer under test. Switch on the current, and when BG is quite steady reverse the current by means of the key K: let a be the first angular swing. Assuming a galvanometer of the moving needle type, if Q be the quantity discharged—

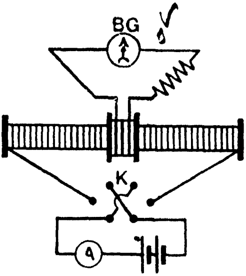


Fig. 426.

$$Q = \frac{HT}{\pi G} \sin\frac{a}{2}\left(1 + \frac{\gamma}{2}\right)$$
$$= k \sin\frac{a}{2}\left(1 + \frac{\gamma}{2}\right),$$

where k is the constant required. But (Art. 242)

$$Q = \frac{8\pi n I n' a'}{R},$$

$$\therefore\ k \sin\frac{a}{2}\left(1 + \frac{\gamma}{2}\right) = \frac{8\pi n I n' a'}{R},$$

i.e.

$$k = \frac{8\pi n I n' a'}{R \sin\frac{a}{2}\left(1 + \frac{\gamma}{2}\right)},$$

or, taking $\sin\frac{a}{2} = \frac{d}{4D}$, where d = scale deflection and D = distance between needle and scale, and neglecting damping—

$$k \doteq \frac{32\pi n I n' a' D}{Rd}.$$

If an earth inductor be used as the standard we get

$$Q = \frac{2SAH}{R} \quad \text{and} \quad Q = \frac{HT}{\pi G} \sin\frac{a}{2} = k\frac{d}{4D},$$

$$\therefore\ k\frac{d}{4D} = \frac{2SAH}{R},$$

i.e.

$$k = \frac{8SAD}{Rd}H.$$

Another method of finding the constant of a ballistic galvanometer consists in charging a condenser of known capacity (C) by means of a cell of known E.M.F. (E), and then discharging through the galvanometer. The quantity ($Q = CE$), the deflection (d), and the distance (D) being known—

$$Q = k\frac{d}{4D}, \quad \text{i.e. } k = \frac{4DQ}{d} = \frac{4DEC}{d};$$

hence k is determined. In this care must be taken with the units employed.

244. Miscellaneous Illustrations.—Amongst other important illustrations of inductive effects the following may be briefly mentioned:—

(1) *Faraday's Disc and Barlow's Wheel.*—A circular disc of copper is fixed so as to rotate round a central axis at right angles to its plane. If its plane be at right angles to the lines of force in a magnetic field, and a metal spring be made to press lightly on its edge, a continuous current will be found to flow between the spring and the centre of the wheel on making connection between the spring and the metal axis of the wheel. The actual *direction* of the E.M.F. depends on the direction of rotation and of the field, and is determined by the right hand rule (Art. 239).

Consider an infinitely thin radial strip of the disc passing from the centre to the spring. As this strip rotates through a small angle θ the area swept out is $\frac{1}{2}\theta r^2$, where r is the radius of the disc and θ is in circular measure; the number of tubes cut is, therefore, $\frac{1}{2}\theta r^2 H$, where H is the intensity of the field. If the disc makes n revolutions per second, the time taken to turn through θ is $\theta/2\pi n$ seconds, and the rate of change or rate of cutting tubes is $(\frac{1}{2}\theta r^2 H)/(\theta/2\pi n) = \pi n r^2 H$; this measures the P.D. between the centre and circumference.

(2) *Arago's Experiment.*—A disc of copper is made to rotate in a horizontal plane immediately below a delicately balanced magnetic needle, the axis of rotation of the disc being vertically below the pivot of the needle. As the disc rotates it is found that the needle is gradually deflected in the same direction as the rotation, and, if the rate of rotation is sufficiently high, finally takes up a motion of rotation in the same sense as the disc, but at a slower rate. This result is explained by the fact that currents are induced in the copper disc by its rotation relative to the magnet, and the reaction between the disc and the needle is (in accordance with Lenz's Law) such as to tend to stop the motion of the disc, but the needle being movable and not fixed the result of this reaction is that the needle is itself set in motion. The direction of the induced current in the disc is such that a current always flows along the diameter of the disc vertically below the needle in such a direction as to deflect the needle in the same direction as the rotation of the disc.

(3) If a magnetic needle be caused to oscillate and a sheet of copper be then placed beneath it, the needle quickly comes to rest; here, again, induced currents are developed in the copper which oppose the motion. The damping in the case of the moving coil galvanometer is explained in the same way.

245. Coefficient of Self-induction.—When a current flows through a coil it produces a flow of magnetic induction through the coil. This flow of induction is proportional to the current when the permeability of the surrounding medium is constant. In this case if F denotes the flow of induction through the coil and I the current in the coil, then

$$F = LI \text{ .... } (1)$$

where L is a constant. This constant is the *coefficient of self-induction of the coil.* If I be unity, F is numerically equal to L; thus **the coefficient of self-induction of a circuit is numerically equal to the flow of induction through the circuit when unit current passes.** Clearly *a circuit has a coefficient of self-induction of one C.G.S. unit if the flow of induction be unity when the unit electromagnetic current passes.* The practical unit is the **henry,** which is equal to 10^9 C.G.S. units; thus *a circuit has a coefficient of self-induction of one henry if the flow of induction be* 10^9 *when the unit electromagnetic current passes, and therefore* 1/10 *of* 10^9, i.e. 10^8, *when a current of one ampere passes.*

If the permeability of the surrounding medium varies with the intensity of magnetisation, then L is not a constant, but varies with the permeability of the medium and therefore varies with I.

When F varies on account of the variation of I, then we evidently have the relation

$$\frac{dF}{dt} = L\frac{dI}{dt}, \text{ that is } e = -L\frac{dI}{dt} \text{ .... } (2)$$

This relation shows that the induced electromotive force is proportional to the rate of change of current in the coil. The minus sign indicates that when dI/dt is positive the induced electromotive force, e, opposes the existing current. From (2), if dI/dt be unity, e numerically equals

L; hence **the coefficient of self-induction of a circuit is numerically equal to the E.M.F. round the circuit due to unit rate of change of the current in it.** Clearly also *a circuit has a coefficient of self-induction of one C.G.S. unit when a current increasing at the rate of one e.m. unit per second brings on an opposing E.M.F. of one e.m. unit*, and similarly *a circuit has a coefficient of self-induction of one henry when a current increasing at the rate of one ampere per second brings on an opposing E.M.F. of one volt.*

It will be seen later (Art. 254) that the work done in establishing a current I in a circuit of self-induction L is given by the expression

$$\text{Work } (W) = \tfrac{1}{2}LI^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

and this supplies another definition of L, for if I be unity L is numerically equal to $2W$; thus **the coefficient of self-induction of a circuit is numerically equal to twice the work done in establishing the magnetic induction accompanying unit current in the circuit.** The corresponding definitions of the C.G.S. unit and the henry may be readily derived.

The three definitions of L given above lead to constant and equal results provided the permeability of the medium is constant; if the permeability is not constant the three values are not identical.

The coefficient of self-induction of a circuit is frequently defined as *measured by the linkages of the circuit when absolute unit current is flowing through it.* This is merely another method of stating the first definition given above.

246. Coefficient of Self-induction of a Solenoid.—In the case of a solenoid having an air core we have

$$\text{Field inside} = \frac{4\pi SI}{l},$$

where S is the *total* number of turns, l the length, and I the current in e.m. units. If A be the area of cross-section—

$$\text{Magnetic flux} = \frac{4\pi SAI}{l},$$

and if I *be unity*—

$$\text{Magnetic flux} = \frac{4\pi SA}{l}.$$

But *each* turn of the solenoid embraces these tubes; hence

$$\text{Effective flux or Linkages} = \frac{4\pi S^2 A}{l},$$

$$\therefore\ L = \frac{4\pi S^2 A}{l}.$$

If a solenoid has 200 turns, is 20 cm. long, and has a cross-sectional area of 4 sq. cm.—

$$L = \frac{4\pi\,(200)^2 \times 4}{20}\ \text{C.G.S. units} = \frac{4\pi\,(200)^2 \times 4}{10^9 \times 20}\ \text{henry}$$

$$= \cdot 0001\ \text{henry}.$$

If instead of an air core we have one of permeability μ then

$$L = \mu\frac{4\pi S^2 A}{l}.$$

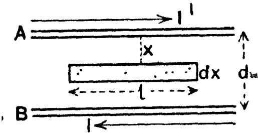


Fig. 427.

247. Coefficient of Self-induction for "Lead" and "Return" in the case of (1) Parallel Wires, (2) Coaxial Cylinders.—In Fig. 427 A and B are two *very long* parallel wires, in air, carrying equal currents I in opposite directions, constituting, in fact, a "lead" and "return"; let r denote the radii of the wires and d the distance apart (centre to centre). Considering a length l, the flux through the dotted area is $Hldx$ and H is equal to $2I/x + 2I/(d-x)$ (Art. 165); thus, if I be unity—

$$\text{Flux} = 2\left(\frac{1}{x} + \frac{1}{d-x}\right) ldx,$$

$$\therefore L = 2l\int_r^{d-r}\left(\frac{1}{x} + \frac{1}{d-x}\right)dx = 2l\left(\log_e\frac{d-r}{r} - \log_e\frac{r}{d-r}\right)$$

$$= 4l\log_e\frac{d-r}{r} = 4\log_e\frac{d-r}{r}\ \text{(per unit length)}.$$

If the wires are in contact $(d-r) = r$, and L is therefore zero.

The case of two concentric cylinders (represented in practice by a concentric cable) is shown in Fig. 428. At external points the fields due to the two carrying equal currents in opposite directions are equal and opposite, and the field inside (*i.e.* between them) is that due to the current I in the inner cylinder only. The field at a point distant r from the axis of the inner cylinder is therefore $2I/r$, and the flux through the dotted area, the length of which parallel to the cylinders is unity, is given by

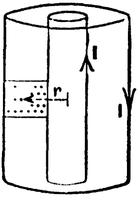


Fig. 428.

$$\text{Flux} = \int_a^b \frac{2I}{r}\,dr = 2I\Big[\log_e r\Big]_a^b$$
$$= 2I\log_e \frac{b}{a} \text{ (per unit length).}$$

Hence, if I be unity—

$$L = 2\log_e \frac{b}{a} \text{ (per unit length),}$$

the medium between the cylinders being air.

248. Coefficient of Mutual Induction.—If we have two separate circuits the variation of a current in one will set up an induced electromotive force in the other. Or, we may say that when a current exists in one there is a flow of induction through the other, and any variation of this flow of induction gives an induced electromotive force in that circuit. Let the two circuits be denoted by A and B and let I and I' be the currents in these circuits. Let the flow of induction *through* A due to the current I' *in* B be denoted by MI', where M is a constant involving the permeability of the medium. Then, as in Art. 36, for two magnetic shells the mutual energy of the two circuits is MII' Similarly, if the flow of induction *through* B due to the current I *in* A is denoted by $M'I$, the mutual energy of the circuits is given by $M'II'$. Hence we have $MII' = M'II'$ or $M = M'$. That is, the flow of induction through A for unit current in B is the same as the flow of induction through B for unit current in A. This constant M is *the coefficient of mutual induction for the two circuits.*

If, then, we have two circuits for which the coefficient of mutual induction is M, and if F be the flow of induction through one for current I in the other—

$$F = MI \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\therefore -\frac{dF}{dt} = -M\frac{dI}{dt},$$

i.e.

$$e = -M\frac{dI}{dt}, \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

e being the induced E.M.F.; the coefficient of mutual induction M is readily defined from either of these relations.

Thus from (1) **the coefficient of mutual induction of two circuits is numerically equal to the flow of induction through one when unit current passes in the other;** clearly *the coefficient of mutual induction of two circuits is one C.G.S. unit if the flow of induction through one is unity when the unit electromagnetic current passes in the other*; clearly also *the coefficient of mutual induction is one henry if the flow of induction through one is* 10^8 *when one ampere passes in the other.*

Again, from (2) **the coefficient of mutual induction of two circuits is numerically equal to the E.M.F. round one circuit due to unit rate of change of the current in the other;** clearly *it will be one C.G.S. unit when a current increasing in one at the rate of one e.m. unit per second results in an induced E.M.F. of one e.m. unit in the other; it will be one henry when a current increasing in one at the rate of one ampere per second results in an induced E.M.F. of one volt in the other.*

Further, since the mutual potential energy of two circuits is MII', *i.e.* M, if I and I' are unity we may say that **the coefficient of mutual induction of two circuits is numerically equal to the mutual potential energy of the two circuits when unit current flows in each.**

The coefficient M is sometimes defined *as measured by the linkages in one circuit due to unit current in the other*; this is merely another form of the first definition given above.

249. Coefficient of Mutual Induction of two Solenoids.—We shall only deal with the case of two solenoids so associated that there is no magnetic leakage, *i.e.* all the tubes pass through *both* circuits; the solenoidal inductor of Fig. 424 will furnish such an example.

Let S_1, A, l, and I denote the total number of turns, the cross-sectional area, the length, and the current in the case of the primary solenoid; then

$$\text{Magnetic flux} = \frac{4\pi S_1 A I}{l},$$

and if I be unity

$$\text{Magnetic flux} = \frac{4\pi S_1 A}{l}.$$

Further, if there are S_2 turns in the secondary,

$$\text{Effective flux or Linkages} = \frac{4\pi S_1 A}{l} \times S_2,$$

i.e.

$$M = \frac{4\pi S_1 S_2 A}{l}.$$

If instead of an air core we have one of permeability μ, then

$$M = \mu \frac{4\pi S_1 S_2 A}{l}.$$

250. Growth of the Current in a Circuit containing Resistance (R) and Inductance (L).—Let E denote the E.M.F. of the cell in the circuit. Then, during the variable state when the current is rising to its full value we have

$$E - L\frac{dI}{dt} = IR,$$

or, dividing by R and transposing, we get

$$\frac{E}{R} - I = \frac{L}{R} \cdot \frac{dI}{dt},$$

that is,

$$\frac{dI}{(E/R - I)} = \frac{R}{L} dt.$$

But

$$d(E/R - I) = - dI.$$

Hence we have

$$\frac{d(E/R - I)}{(E/R - I)} = - \frac{R}{L} dt.$$

Integrating this from the lower limits, where t and $I = 0$, we get

$$\log_e \frac{E/R - I}{E/R} = -\frac{R}{L}t,$$

I denoting the current at the end of the time t from the starting of the current.

This gives

$$e^{-\frac{R}{L}t} = \frac{E/R - I}{E/R},$$

or

$$e^{-\frac{R}{L}t} = 1 - \frac{IR}{E},$$

or

$$\frac{IR}{E} = \left(1 - e^{-\frac{R}{L}t}\right).$$

That is,

$$I_t = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right),$$

where I_t denotes the current at the end of the time t from the instant of closing the circuit.

In this relation E/R denotes the final value of the current. If we denote this by I we have

$$I_t = I\left(1 - e^{-\frac{R}{L}t}\right),$$

or

$$\frac{I_t}{I} = 1 - e^{-\frac{R}{L}t}.$$

From this it is evident that, when t is equal to L/R, $2L/R$, $3L/R$, etc., the ratio of the actual current to the maximum value attainable is given by $1 - 1/e$, $1 - 1/e^2$, $1 - 1/e^3$, etc. That is, at the ends of the time L/R, $2L/R$, $3L/R$, etc., the current value is ·6321, ·8647, ·9502, etc., of the final attainable value. The quantity L/R is called the *time constant* of the circuit

It is evident from the above that the current will require an infinite time to attain its full value, but that, as L/R is usually very small, it rapidly attains a value very nearly equal to its final value. This is shown in Fig. 429, which gives the curve of rise of a current in a circuit, where $R = 2$ ohms and $L = \frac{1}{50}$ henry, and for which, therefore, the time constant L/R is

$$\frac{\frac{1}{50} \times 10^9}{2 \times 10^9} = \frac{1}{100}.$$

In this circuit the current rises to ·6321 of its full value in ·01 sec., to ·8647 of its full value in ·02 sec., to ·9502 of its full value in ·03 sec., and so on, evidently attaining practically its full value in a small fraction of a second.

It will readily be understood that when L is great and R relatively small the time constant of the circuit will be large, and the current may take a considerable time to establish itself. On the other hand, when L is small compared with R the current rises very quickly. In all circuits where iron cores exist L will necessarily be large, and if these circuits are required to respond quickly to the make and break of a current it is evident that, in order to keep the ratio L/R small, R must be as large as possible consistently with obtaining a sufficiently strong working current.

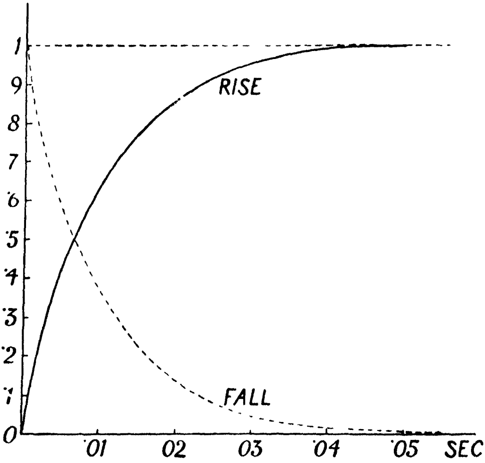


Fig. 429.

251. Decay of the Current in a Circuit containing Resistance (R) and Inductance (L).—When the circuit is broken the E.M.F. of the cell (E) disappears from the relation given above, and we get

$$IR = -L\frac{dI}{dt},$$

or

$$\frac{dI}{I} = -\frac{R}{L}dt.$$

Hence, if I be the value of the current at the instant of breaking circuit, then the current, I_t, at a time t afterwards is given by

$$\log_e \frac{I_t}{I} = -\frac{R}{L}t,$$

that is,

$$e^{-\frac{R}{L}t} = \frac{I_t}{I},$$

or

$$I_t = Ie^{-\frac{R}{L}t}.$$

Here it is evident that in times L/R, $2L/R$, $3L/R$, etc., from the break of the circuit, the current falls to $1/e$, $1/e^2$, $1/e^3$, etc., of its initial value, that is, to ·3679, ·1353, ·0498, etc., of the initial value.

The dotted curve of Fig. 429 shows the fall of the current in the circuit for which the full line gives the curve of rise of the current.

This investigation assumes that the circuit is broken instantaneously. In practice this is not the case. Usually the resistance of the circuit is increased, it may be rapidly, but not instantaneously to an infinite value, and in most cases the resistance of the circuit is varied even after the metallic circuit is broken by the reduced resistance of the air gap along the path of the extra current spark at the break of the circuit.

During the rise or fall of a current I the change in the flow of magnetic induction through the circuit is LI, and, by Art. 240, the quantity of electricity set in motion by this change is LI/R or EL/R^2. This is the quantity carried by the "extra currents"; at make the flow of electricity is diminished by this amount, and at break the additional flow takes place after the circuit is broken. It should be noticed that this quantity is equal to that carried by the steady current E/R in a time L/R, the time constant of the circuit. The field energy associated with the extra current is evidently equal to $\frac{1}{2}LI^2$, as explained in Art. 254.

252. The Case of a Circuit containing Resistance (R) and Capacity (C). Charging and Discharging a Condenser.—This may well be treated here for the sake of comparison with the two preceding sections. Let an E.M.F. (E) be applied to a circuit of resistance (R) containing a condenser of capacity C, and let E_t be the P.D. on the condenser at any instant. At the commencement of the charging process E_t is zero, and it gradually rises to the value E, its direction being, of course, *opposed* to that of the applied E.M.F.; *the charging current starts with its greatest value and decreases, becoming zero when* $E_t = E$. The case may be investigated after the manner of Art. 230, but the following modification is, perhaps, simpler.

If Q_t be the charge on the condenser at any instant, then $Q_t = E_tC$ and $dQ_t/dt = CdE_t/dt$. If I_t be the current at this instant $I_t = (E - E_t)/R = dQ_t/dt$: hence,

$$C\frac{dE_t}{dt} = \frac{E - E_t}{R},$$

$$\therefore \frac{dE}{E - E_t} = \frac{1}{CR}dt.$$

But $$d(E - E_t) = -dE_t,$$

$$\therefore \frac{d(E - E_t)}{E - E_t} = -\frac{1}{CR}dt.$$

Integrating this from the lower limits when t and $E_t = 0$,

$$\log_e \frac{E - E_t}{E} = -\frac{t}{CR},$$

$$\therefore 1 - \frac{E_t}{E} = e^{-\frac{t}{CR}},$$

i.e. $$E_t = E\left(1 - e^{-\frac{1}{CR}t}\right) \quad \text{.........} \quad (1)$$

And since $$I_t = \frac{E - E_t}{R} = \frac{E}{R}.e^{-\frac{1}{CR}t},$$

$$\therefore I_t = Ie^{-\frac{1}{CR}t}, \quad \text{...................} \quad (2)$$

where I is the starting value of the charging current. The expression (1) should be compared with that for

I_t in Art. 250, and (2) with that for I_t in Art. 251. CR is the *time constant*; it is the time the P.D. on the condenser takes to reach $(1 - 1/e)$, *i.e.* ·6321 of its *final* value, and the time the current takes to fall to $1/e$, *i.e.* ·3679 of its initial value.

The case of the condenser discharging has already been dealt with in Art. 230; using the notation of the present section it is there shown that

$$E_t = Ee^{-\frac{t}{CR}} \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (3)$$

$$\therefore \; I_t = \frac{E_t}{R} = \frac{E}{R} \cdot e^{-\frac{t}{CR}},$$

i.e.

$$I_t = Ie^{-\frac{1}{CR}t} \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (4)$$

These expressions should be compared with that for I_t in Art. 251. *It should be noted that both in the charge and discharge the current starts with its maximum value and decays exponentially; it should also be noted that the circuit does not contain inductance* (L).

SUMMARY.

Circuit with resistance (R) and Inductance (L).	Circuit with resistance (R) and Capacity (C).
Growth: $I_t = I\left(1 - e^{-\frac{R}{L}t}\right)$ Decay: $I_t = I.e^{-\frac{R}{L}t}$ Time Constant = $\frac{L}{R}$	Charge: $E_t = E\left(1 - e^{-\frac{1}{CR}t}\right)$ $I_t = I.e^{-\frac{1}{CR}t}$ Discharge: $E_t = E.e^{-\frac{1}{CR}t}$ $I_t = I.e^{-\frac{1}{CR}t}$ Time Constant = CR

252a. The case of a Circuit containing Resistance (*R*) Capacity (*C*) and Inductance (*L*). The *essential* points in the case of a circuit containing inductance (L) and capacity (C) *but no resistance* (R), and in the case of a circuit containing inductance (L) capacity (C) and resistance (R) are dealt with in Chapter XXII.

In the latter more general case if E be used for applied E.M.F., I for current and Q for charge the instantaneous electromotive force equation becomes

$$L\frac{dI}{dt} + IR + \frac{Q}{C} = E \quad \text{.................} \quad (1)$$

or, since $I = dQ/dt$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E \quad \text{.................} \quad (2)$$

The *full* solution to this would occupy too much space (see however Arts. 303, 304) and it may be found in any good work on the Calculus; the results however may be quoted :—

If $\frac{4L}{C} > R^2$ we get, for the "charging" process :—

$$Q_t = Q\left[1 - \frac{fe^{-gt}}{\sqrt{f^2 - g^2}}\cos\left(\sqrt{f^2 - g^2}\,t - a\right)\right] \text{.....} \quad (3)$$

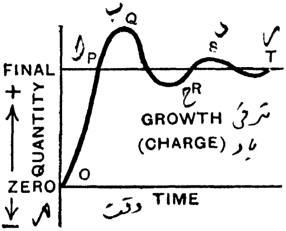


Fig. 429*a*.

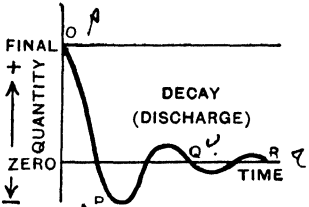


Fig. 429*b*.

where $f^2 = 1/LC$, $g = R/2L$, $\sin a = g/f$ and $Q =$ final charge. Further since $I = dQ/dt$, by differentiating (3) we get :—

$$I_t = Q\frac{f^2e^{-gt}}{\sqrt{f^2 - g^2}}\sin\sqrt{f^2 - g^2}\,.\,t \text{..............} (4)$$

From (3) it follows that the process is **oscillatory,** the charge being alternately greater and less than Q and finally settling down to this steady value: the curve $OPQRST$ in Fig. 429*a* shows the result in this case.

Again, from (4) it follows that at times $t = O, t = \frac{\pi}{\sqrt{f^2 - g^2}}$, $t = \frac{2\pi}{\sqrt{f^2 - g^2}}$, etc., $\sin \sqrt{f^2 - g^2}.t$ is zero and therefore I_t is zero. Further from (3) it follows that after times $\frac{\alpha + \frac{\pi}{2}}{\sqrt{f^2 - g^2}}$, $\frac{\alpha + \frac{3\pi}{2}}{\sqrt{f^2 - g^2}}$, etc., the value of Q_t, is the final steady value Q. This means that in both cases:—

$$\text{Time of half an oscillation} = \frac{\pi}{\sqrt{f^2 - g^2}}$$

$$\therefore \text{ Periodic Time} = T = \frac{2\pi}{\sqrt{f^2 - g^2}} = \frac{2\pi}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}$$

$$= 2\pi\sqrt{LC} \text{ if } R \text{ is negligible.}$$

The case of the discharge on removing the applied E.M.F. is dealt with by putting $E = O$ in (2) viz.,

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = O \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(5)}$$

and in this case if $\frac{4L}{C} > R^2$ the solution is:—

$$Q_t = Q\,\frac{fe^{-gt}}{\sqrt{f^2 - g^2}} \cos\left(\sqrt{f^2 - g^2}.t - \alpha\right) \quad \ldots\ldots \text{(6)}$$

Here also the discharge is **oscillatory** and is shown by the curve $OPQR$ of Fig. 429*b*. A little consideration will show that the periodic time is the same as in the preceding case.

If $\frac{4L}{C} < R^2$ it can be shown that the process in both cases is **non-oscillatory**: in the first case the charge merely rises gradually to the steady value Q, and in the second case it falls gradually from Q to zero. (See Chapter XXII.). Clearly $\frac{4L}{C} = R^2$ or $R = \sqrt{\frac{4L}{C}}$ gives the limiting value of R for oscillations. If R exceeds this value the process is continuous, if less than this it is oscillatory.

The above chief facts are merely summarised here; they are again dealt with in Chapter XXII.

In Arts. 250-252*a* the applied E.M.F. (E) is a "steady" E.M.F.: the cases of an *alternating* E.M.F. in (1) a circuit with resistance and inductance, (2) a circuit with resistance and capacity, and (3) a circuit with resistance inductance and capacity are treated in Chapter XX.

Incidentally it may be noted that in Fig. 429*a* the maximum charge (first "surge") is much greater than the final steady charge: hence it is that a condenser joined to supply mains (with small R and large L) may "break down" although its insulation can easily stand the final steady pressure. The remedy is to charge through a resistance which is afterwards cut out.

253. Inductive Resistances in Parallel.—It has been shown (Art. 157) that a *steady* current divides at A (Fig. 429*c*) in the inverse ratio of the resistances, and this is true whatever the value of L_1 and L_2, the inductances of the resistances, *but it does not hold for alternating currents or when the currents are varying in strength.*

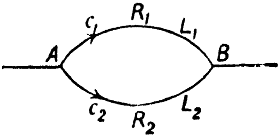


Fig. 429*c*.

Let x and y denote the currents in the two branches at any instant during a varying state. At that instant the difference of potential between A and B is given by

$$xR_1 + L_1\frac{dx}{dt} \quad \text{or} \quad yR_2 + L_2\frac{dy}{dt},$$

that is,
$$xR_1 + L_1\frac{dx}{dt} = yR_2 + L_2\frac{dy}{dt}.$$

If we apply this relation to the first instant of the variable state at the starting of the currents, where x and y have both zero value, we get

$$L_1\frac{dx}{dt} = L_2\frac{dy}{dt} \quad \text{or} \quad \frac{dy}{dt}\bigg/\frac{dx}{dt} = \frac{L_1}{L_2},$$

that is, *the rates of increase of the currents at the instant of starting are in the inverse ratio of the self-inductances of the branches.*

At the instant the full values of the currents are established $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are both of zero value, and the relation reduces to the usual simple form $i_1R_1 = i_2R_2$, where i_1 and i_2 are the final steady values of x and y, i.e. *the final steady currents are in the inverse ratio of the resistances of the branches.*

It must be noted, however, that if the current at A starts from a steady value and returns to the *same* value, after a period of variation, the total quantity of electricity that passes during the period of variation divides between the branches in the same way as a steady current. For example, the discharge current from a condenser or the currents due to induction which start and end at zero values come under this rule, so that *the quantity of electricity discharged from a condenser or set in motion by an inductive impulse divides among branching conductors in the inverse ratio of the resistances.*

The truth of this rule may be deduced from the relation

$$xR_1 + L_1\frac{dx}{dt} = yR_2 + L_2\frac{dy}{dt}.$$

For this gives

$$R_1xdt + L_1dx = R_2ydt + L_2dy,$$

and $x\,.\,dt$ and $y\,.\,dt$ evidently denote the quantity of electricity passing during the infinitely short interval of time dt, when the strengths of the currents are x and y respectively. Assuming the currents to vary from values i_1 and i_2 to I_1 and I_2, and integrating between these limits, we get

$$R_1\int_o xdt + L_1\int_{i_1}^{I_1} dx = R_2\int_o^t ydt + L_2\int_{i_2}^{I_2} dy.$$

Now, if $I_1 = i_1$ and $I_2 = i_2$, evidently $\int_{i_1}^{I_1} dx$ and $\int_{i_2}^{I_2} dy$ both vanish, and we have

$$R_1\int_o^t x\,.\,dt = R_2\int_o^t y\,.\,dt.$$

But $\int_o^t x\,.\,dt = q_1$, the quantity of electricity which has passed through R_1 in the time t, and $\int_o^t y\,.\,dt = q_2$, the quantity which has passed through R_2 in the same time. Hence $R_1q_1 = R_2q_2$, or

$$\frac{q_1}{q_2} = \frac{R_2}{R_1}.$$

254. Energy in the Magnetic Field of a Current. The Case of Two Circuits.

—In Art. 239 the energy equation in the case of a current circuit is given in the form

$$EIdt = I^2Rdt + IdF,$$

and since $F = LI$ this becomes

$$EIdt = I^2Rdt + LIdI.$$

This equation evidently means that the energy given out by the battery, $EI\,.\,dt$, is greater than the energy, $I^2R\,.\,dt$, dissipated as heat in the circuit by an amount $LI\,.\,dI$, which is associated with the establishment of the magnetic field due to the current, and must therefore be looked upon as the electromagnetic energy stored in the medium in which the magnetic field exists; as the current rises the amount of this energy increases until the steady state is attained.

It will be explained later that the energy from the cell really travels from the cell *through the medium* to the circuit, where in general it is dissipated, but at the starting of the current some of this energy, instead of passing to the circuit to be dissipated, remains in the medium as the energy of the magnetic field, and during this short period the energy dissipated is less than the energy given out by the cell by the amount which remains in the medium.

Similarly, when the circuit is broken, the current very rapidly decreases to zero value, and the energy of the magnetic field in the medium rushes into the circuit, where it is dissipated as heat, giving rise to what has already been described as the extra current at break.

From the equation given above the electromagnetic energy stored in the medium during a very small change dI in the current is $LI \,.\, dI$, where L is the coefficient of self-induction of the circuit. Hence, if a current increases from 0 to I in a circuit of self-inductance L, the total energy in the medium will be

$$\Sigma_0^I LI \,.\, dI \quad or \quad L\int_0^I I \,.\, dI,$$

i.e.

$$\textbf{Energy} = \tfrac{1}{2}LI^2$$
$$= \tfrac{1}{2}\frac{F}{I}I^2 = \tfrac{1}{2}FI$$
$$= \tfrac{1}{2}L\frac{F^2}{L^2} = \tfrac{1}{2}\frac{F^2}{L}.$$

These should be compared with the expressions developed in Art. 86 for the energy of a condenser and of a charged body, viz. Energy $= \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$. There is a real analogy between the two: one measures the electromagnetic energy in the magnetic field set up in the medium surrounding a circuit carrying a current, the other measures the electrostatic energy in the electric field set up in the dielectric of a condenser, or in the medium surrounding a charged body.

In the case of two circuits carrying currents I and I_1 let M be the coefficient of mutual induction and L and L^1 the coefficients of self-induction. The energy in the case of the first is $\frac{1}{2}LI^2$ and in the case of the second $\frac{1}{2}L^1I_1^2$, whilst the mutual energy is MII_1; thus, if E be the total energy—

$$E = \tfrac{1}{2}LI^2 + MII_1 + \tfrac{1}{2}L^1I_1^2$$
$$= \tfrac{1}{2}(LI^2 + 2MII_1 + L^1I_1^2).$$

As E must be positive, $LI^2 + 2MII_1 + L^1I_1^2$ must be positive. But L and L^1 are both positive, whilst the sign of M can be altered by reversing the connections of one of the circuits; hence in every case, *to be strictly exact*—

$$LL^1 > M^2.$$

In *engineering practice* the difference between LL^1 and M^2 is frequently small enough to be taken as zero, however.

255. Transfer of Energy from a Cell to its Circuit. Poynting's Theorem.—Some idea of the process of transfer of energy through the medium from a cell to its circuit may be obtained by considering the charging and discharging of a condenser.

Let the parallel plate condenser shown in Fig. 430 be charged by connecting its plates to the poles of a cell. When connection is made Faraday tubes 11, 22, 33, etc., each carrying unit positive charge at one end and unit negative charge at the other end, travel towards the condenser and quickly fill up the medium between the two plates.

When the charging *begins* the difference of potential between P and N (Fig. 431), the terminals of the cell, is equal to the E.M.F. of the cell, and no difference of potential has yet been established between A and B. It follows therefore that during the initial stages of the charging the difference of potential between two points, a and b, decreases as the points are taken farther from P and N, and nearer to A and B. The energy in a tube being proportional to the difference of potential between its ends, the tubes between P and N will tend, therefore, to move from PN towards AB in order to take up positions of minimum potential energy, and each tube, as it passes towards AB, undergoes loss of electrostatic energy, the amount lost being partly dissipated as heat in the conductors PA and NB, and partly transformed into electromagnetic energy of the magnetic field surrounding the conductors. This electromagnetic energy may be considered as of the nature of kinetic energy associated with the motion of the tubes of force, the electrostatic energy of which is assumed to be of the nature of potential energy.

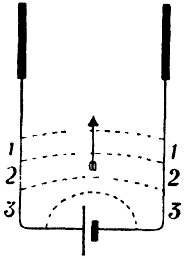


Fig. 430.

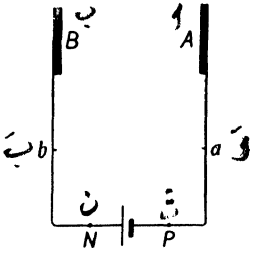


Fig. 431.

As the charging goes on the difference of potential between *A* and *B* rises until finally it becomes equal to the E.M.F. of the cell, and points on *PA* and on *NB* are at the same potentials as *P* and *N* respectively. When this state is attained the charging is complete and the tubes cease to travel through the medium from the cell to the condenser. The energy which has passed from the cell to the condenser through the medium between *PA* and *NB* is stored up in the electrostatic field between these conductors, all the electromagnetic energy produced being retransformed into electrostatic energy. During the transfer and attendant transformation a certain amount of energy has been dissipated as heat in the conductors *PA* and *NB*, so that during the charging the cell has given out more energy than is stored in the electrostatic field.

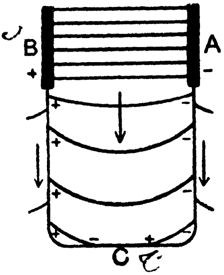


Fig. 432.

Let us now consider the discharge of the condenser when the plates *A* and *B* are disconnected from the cell and connected by a high resistance. ✓

Directly connection is made the Faraday tubes between *A* and *B* (Fig. 432) travel towards *C*, so as to reduce their energy by reducing the difference of potential between their ends. As each tube travels towards *C*, the energy it loses is partially dissipated as heat and partially transformed into electromagnetic energy in the medium surrounding the conductor *ACB*} and finally, when the ends of the tube meet, the tube disappears and all the energy in it is transformed into heat and electromagnetic energy, the latter being finally dissipated as heat in the circuit. As the discharge goes on, tube after tube disappears, and finally the whole energy of the condenser's field is dissipated and the condenser is discharged.

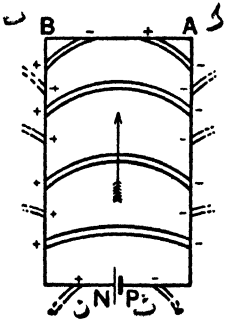


Fig. 433.

In the case of a simple circuit *PABN* (Fig. 433) made up of a cell and a conductor, the process of transfer and dissipation of energy by means of the medium is practically the same as described above.

When the current first starts some electrostatic energy is converted into electromagnetic energy as the magnetic field of the current is established, and when the circuit is broken this electromagnetic energy is dissipated as heat in the circuit, but when the current is steady there is a steady dissipation of electrostatic energy as heat in the circuit.

Faraday tubes pass through the medium from the cell out along the circuit, and each tube, as it shortens to nothing, as at *A* and *B*, gives up all its energy to be dissipated as heat in the circuit. There is therefore in the case of a steady current a steady flow of tubes of electrostatic energy from the cell out to the medium, the ends of each tube travel along the conductor of the circuit and the energy of each tube, diminishing as it travels along, is ultimately completely dissipated as heat in the circuit.

It must be remembered that Fig. 433 is only of a diagrammatic character. The tubes passing from the conductor *PA* to *NB* must be understood to start out from *PA* in all directions, and to curve round through the medium to *NB*, on which they terminate and on which they close in from all sides. The medium involved in the transfer of energy is not, therefore, confined to that directly between the conductors *PA* and *NB* as shown in the diagram, but includes the whole field in the neighbourhood of the circuit.

In an electric field such as we have described, where the tubes are not in equilibrium, but in motion, we have, as a result, a magnetic field—the magnetic field associated with the so-called currents in the conductors in the field. If we draw on the field a series of equipotential surfaces and also a corresponding series of surfaces of equal electric force, the lines of intersection of these surfaces can be shown to be lines of magnetic force. If the motion of the tubes in the field has attained a steady state these surfaces will be fixed, and the lines of intersection, that is, the lines of magnetic force, will be fixed. This is another way of saying that a steady current has a definite fixed magnetic field.

The above represents in a simple form one aspect of **Poynting's Theorem** on the transfer of energy. According to it the current from a cell consists of the positive ends of the Faraday tubes moving along the wire from the positive pole, and the negative ends moving along the wire from the negative pole, the two ends approaching as the tubes contract and disappear; the *energy* thus flows *through the medium.* Still more modern theory attributes the current to the motion of *negative electrons*; this is dealt with in a later chapter.

256. The Induction Coil. Modern Interrupters.—The induction coil is a practical application of the principles of mutual induction, consisting of an apparatus for the purpose of transforming a low P.D. between the terminals of a primary coil into a high P.D. between the terminals of a secondary coil. The two essential points are:—

(1) A given current in the primary coil shall make the linkages—flux × number of turns—in the secondary coil as large as possible.

(2) The current in the primary coil must be made and broken very rapidly.

The construction of the apparatus is shown in Fig. 434, and is, briefly, as follows: Round an inner core composed of a bundle of soft-iron wire is wound a coil consisting of a few turns of very stout wire, termed the *primary* of the coil; this is in series with a break consisting of a spring touching a contact screw, the spring carrying at its upper end a small piece of soft iron, which is attracted by the

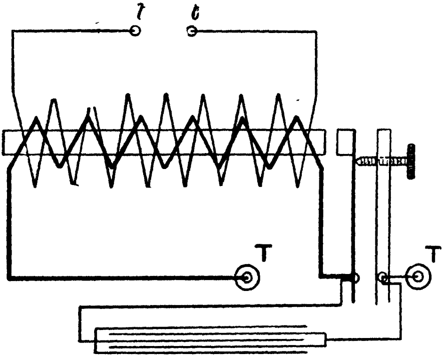


Fig. 434.

iron core of the coil when current flows through the primary, and falls back again when the primary current ceases and the iron core becomes demagnetised. Hence, on attaching the battery, an intermittent current passes through the coil, the frequency of the currents depending upon the strength and inertia of the spring. ✓

Surrounding the primary coil, but insulated from it with great care, is a coil consisting of a very great number of turns of very fine wire called the *secondary*. The ends of this wire are led out of the casing and are attached to terminals on the top of the coil, and usually when the coil is working the secondary circuit is complete, except for a gap, across which it is desired to cause sparks to leap.

Up to this stage in the construction the action may be briefly explained as follows. When the primary current passes (1) *an induced inverse current is developed in the secondary*, and (2) the core is magnetised, the soft iron head and spring are attracted and the primary circuit is broken; when the primary current ceases (1) *an induced direct current is developed in the secondary*, and (2) the core is demagnetised, the spring falls back against the screw, the primary current again starts and the actions are repeated. It follows from this that so long as the primary current is made and broken so long will currents alternating in opposite directions circulate in the secondary.

In practice matters are not so simple as indicated above, owing to the self-induction of the primary circuit itself. When the primary is "made" the resistance is small and the time constant L/R is therefore great; when the primary is "broken" the resistance is great and the time constant relatively smaller. Hence the decay of the primary is quicker than the growth, and therefore the inductive effect on the secondary at "break" is more pronounced than that at "make."

On the other hand, when the primary is broken the "extra current" developed in it in the *same* direction as the primary makes the break less sudden and definite than it would otherwise be (thereby reducing the inductive effect on the secondary), and, sparking across the interrupter, damages the surfaces of contact. To prevent these effects a condenser, made of sheets of tinfoil and paraffined paper, is connected with the primary circuit; one pole of it is connected to the pillar of the screw, and the other to the spring. The action of this condenser may be briefly explained thus. When the current is broken the self-induced current, having to charge the condenser is not able to spark across the break and thus the current is very suddenly broken. Again, the charged condenser at once discharges round the primary, but in the opposite direction to that of the primary current, and this tends to produce an induced current in the secondary in the same direction as that due to the break: in fact, in practice the flux in the core is *reversed* at each break, and the induced quantity in the secondary is almost twice that without a condenser. To be exact, the condenser current is really oscillatory, but these oscillations are quickly damped, dying out before the circuit is closed again, so that only the first discharge is practically important. Summarising, we may say that the secondary induced current at make is comparatively small, while that at break is intensified, so that in an ordinary induction coil with a condenser the secondary induced currents are those due to the breaks of the primary. The action of the condenser is further dealt with in Art. 256*a*.

In modern coils the vibrating hammer as a make and break is frequently replaced by other devices giving much more rapid interruptions. One form is the **Wehnelt Electrolytic Interrupter.** This is merely a cell containing as one electrode a large plate of lead immersed in dilute sulphuric acid, the other consisting of the end of a piece of thin platinum wire projecting from the end of a glass tube. This cell is placed in the primary circuit, so that the current leaves by the lead plate, and it is then found that if the pressure of supply exceed 24 volts or so, the current is rapidly broken and the secondary emits powerful discharges. Another form much used is the **Motor Mercury Interrupter** shown in Fig. 435. In this pattern a jet of mercury is discharged from a tube and caused to strike a toothed wheel which is rotated by a small motor at a high speed. The circuit of the primary coil is completed through the jet; hence when the jet strikes a tooth the primary is made, but when the jet misses the tooth the primary is broken, and the rate of this depends, obviously, upon the speed of rotation of the toothed wheel, which is under complete control. Also the teeth are tapered, and thus by either moving the jet or the wheel up or down relatively to each other the proportion of time during which the circuit is made and broken is under control too. This device gives excellent results. The mercury is pumped up by the action of the motor which rotates the wheel.

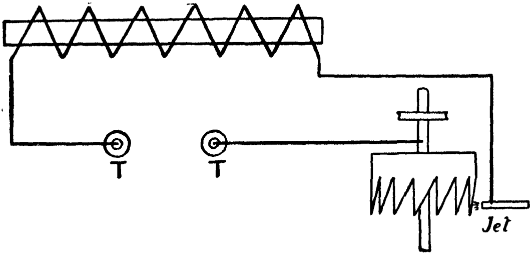


Fig. 435.

256a. The Effect of the Condenser of an Induction Coil upon the Secondary Current.

256a. The Effect of the Condenser of an Induction Coil upon the Secondary Current.—In the treatment which follows (a slight modification of Lord Rayleigh and Starling), the resistance of the secondary is neglected, its inductance being the important factor.

Let now L_1 and I_1 denote the inductance of and current in, the primary, L_2 and I_2 denoting the same for the secondary. Let M be the mutual inductance of the two circuits and C the capacity of the condenser. Finally let Q_1 and Q_2 denote charges circulating in the two circuits.

Clearly the electro-motive force equations (discharge starting) for the primary and secondary circuits are :—

$$L_1\frac{dI_1}{dt} + M\frac{dI_2}{dt} + \frac{Q_1}{C} = 0 \quad\dots\dots\dots\dots\dots\dots (1)$$

$$L_2\frac{dI_2}{dt} + M\frac{dI_1}{dt} = 0 \quad\dots\dots\dots\dots\dots\dots (2)$$

Again, since $I = dQ/dt$, and therefore $dI/dt = d^2Q/dt^2$, these become :—

$$L_1\frac{d^2Q_1}{dt^2} + M\frac{d^2Q_2}{dt^2} + \frac{Q_1}{C} = 0 \quad\dots\dots\dots\dots\dots\dots (3)$$

$$L_2\frac{d^2Q_2}{dt^2} + M\frac{d^2Q_1}{dt^2} = 0 \quad\dots\dots\dots\dots\dots\dots (4)$$

Multiply (3) by L_2 and (4) by M:—

$$L_1 L_2 \frac{d^2Q_1}{dt^2} + ML_2 \frac{d^2Q_2}{dt^2} + L_2 \frac{Q_1}{C} = O \text{ ....} (5)$$

$$ML_2 \frac{d^2Q_2}{dt^2} + M^2 \frac{d^2Q_1}{dt^2} = O \text{ ....} (6)$$

Subtract (6) from (5):—

$$(L_1 L_2 - M^2) \frac{d^2Q_1}{dt^2} + \frac{L_2}{C} Q_1 = O \text{ ....} (7)$$

This is an equation of the oscillatory type, viz. $\frac{d^2\theta}{dt^2} + z^2\theta = O$; *thus the charge and current in the primary are oscillatory.*

Again, from (2) we get on integrating:—

$$L_2 I_2 + MI_1 = A \text{ ....} (8)$$

When $I_2 = O$ let the value of I_1 be I then

$$A = MI.$$

Substituting this for A in (8) we get:—

$$I_2 = \frac{M}{L_2}(I - I_1) \text{ ....} (9)$$

Now I_1 we have seen is oscillating, and from the above it is oscillating between the limits $+ I$ and $- I$. From (9) I_2 evidently has its greatest value when I_1 has the value $- I$; hence:—

$$\textbf{Maximum value of } I_2 = \frac{2MI}{L_2} \text{ ....} (10)$$

If the condenser is absent, and R_1 denotes the resistance of the primary, the electro-motive force equations become:—

$$L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt} + I_1 R_1 = O \text{ ....} (11)$$

$$L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt} = O \text{ ....} (12)$$

Multiplying (12) by M and (11) by L_2, and subtracting as in the preceding case, we get:—

$$(L_1L_2 - M^2)\frac{dI_1}{dt} + L_2R_1I_1 = O.$$

This is an equation of the same type as that dealt with in Art. 251, *i.e.* it is of the form $a\frac{d\theta_1}{dt} + b\theta_1 = O$, and just as in this last case the solution is:—

$$\theta_1 = \theta e^{-\frac{b}{a}t},$$

so the solution to the above is:—

$$I_1 = Ie^{-\frac{L_2R_1}{L_1L_2 - M^2}t} = Ie^{-at} \text{ say.}$$

Again, from (12) as before:—

$$L_2I_2 + MI_1 = A_1$$

$$\therefore L_2I_2 + MIe^{-at} = A_1.$$

Putting $I_2 = O$ when $t = O$ we get $A_1 = MI$, and substituting this for A_1 in the preceding equation we get:—

$$L_2I_2 + MIe^{-at} = MI,$$

$$\therefore I_2 = \frac{MI}{L_2}\left(1 - e^{-at}\right).$$

As t increases more and more, e^{-at} becomes nearer and nearer zero, and is zero if t be infinite; hence:—

$$\textbf{Theoretical maximum value of } I_2 = \frac{MI}{L_2} \quad \ldots\ldots (13)$$

Comparing (10) and (13) we see that, *with the condenser, the maximum value of I_2 is double the maximum possible value of I_2 when the condenser is absent.* It should be noted that the resistance of the secondary has been neglected, so that the values of I_2 given above can never be actually reached in practice.

257. Principle of the Dynamo and Motor.—The dynamo is a commercial application of the principle of the rotating coil of Art. 241; in practice the rotating part consists of several coils suitably wound upon an iron core and known as the *armature*, whilst the magnetic field in which the armature rotates is produced by powerful electromagnets, known as the *field magnets*.

In Fig. 436 let *ABCD* be a coil capable of rotation about the axis *XX* in between the poles *N*, *S* of a magnet. If *AB* be rising and *CD* falling, an application of Fleming's right hand rule will show that the induced current is in the direction indicated, so that *P* is the positive and *Q* the negative end of the coil, and the current in the "external" circuit will be from *P* to *Q*. When the coil reaches the vertical the induced E.M.F. becomes zero (Art. 241). When *AB* begins to fall and *CD* to rise, the induced current will be as indicated in Fig. 437, so that *Q* is the positive and *P* the negative end of the coil, and the current in the external circuit is now from *Q* to *P*.

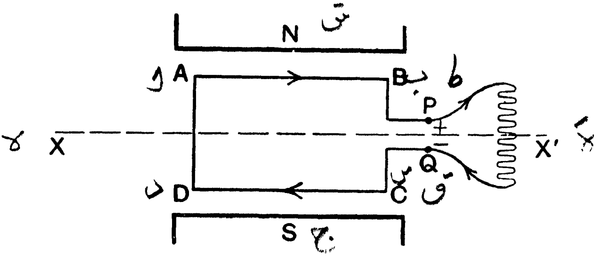


Fig. 436.

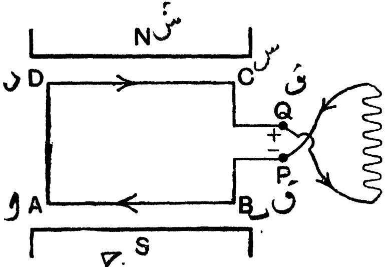


Fig. 437.

To make contact between the coil and the external circuit an arrangement known as slip rings (Fig.

438) is employed; a brush presses upon each ring and serves to lead the current, which is still alternating, to and from the external circuit. To obtain a direct instead of an alternating current in the external circuit a *split ring commutator* (Fig. 439) is used; if the gap in the ring passes under the brushes when the coil is vertical the external current will go up and down in value, but it will not reverse in direction.

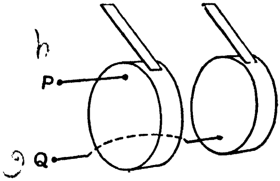


Fig. 438.

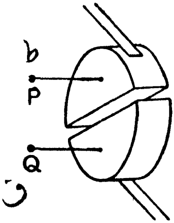


Fig. 439.

It is clear that by using a second coil at right angles to the above the induced E.M.F. in one coil will be a maximum when that in the other is zero, so that by suitably connecting the coils and using a *four-part commutator* the external current will be more constant in

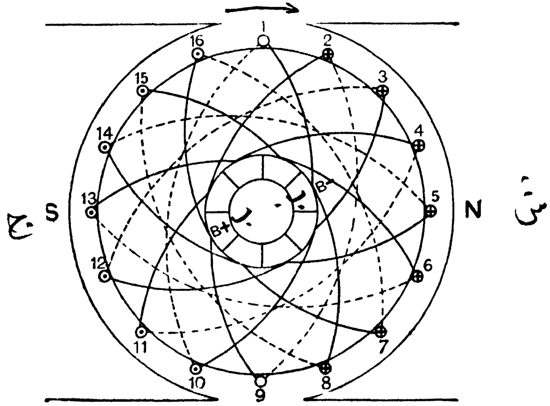


Fig. 440.

magnitude. This will be even better realised by having a large number of coils uniformly distributed along the outside of an iron core, suitably connecting them and using a commutator divided into a corresponding large number of parts or segments.

One method of connection is shown in Fig. 440. Here sixteen wires are spaced uniformly along the outside of an iron drum and cross connections are made as indicated, the continuous lines denoting connections at the front and the dotted lines those at the back of the armature.

From the front end of conductor 1 a connection is led via a commutator segment to the front end of conductor 8. Between 1 and 8 there are *six* conductors; adding one to this we get what is called the *pitch* (seven in this case), and this pitch must be adhered to for the whole winding. Thus the back end of conductor 8 is connected to the back end of conductor $8 + 7 = 15$, and the front end of 15 is joined via a commutator segment to the front end of a conductor which is seven ahead of it—viz. to the front end of conductor 6. The back end of 6 is joined to that of $6 + 7 = 13$, and the front of 13 to the front of 4 via a commutator segment. This is repeated, and finally the back end of 10 is joined to the back of 1, and a closed winding is obtained.

The brushes make contact as indicated with the commutator bars, to which conductors 1 and 8 and conductors 9 and 16 are connected. An application of the right hand rule will show that, with the armature revolving as indicated, the currents in the external conductors on the left are flowing from back to front, whilst those in the conductors on the right are flowing from front to back. Thus *in the armature circuit* the current has two paths—viz. (1) $B -$ along $1 - 10 - 3 - 12 - 5 - 14 - 7 - 16 - B +$, (2) $B -$ along $8 - 15 - 6 - 13 - 4 - 11 - 2 - 9 - B +$, whilst in the external circuit the current flows from $B +$ to $B -$.

A simple expression for the average E.M.F. of this machine may be readily established; thus—

Let F = total magnetic flux passing through the armature.

Z = total number of external wires connected in series on the armature,

n = number of revolutions per second.

Each external wire cuts F tubes *twice* in one revolution, *i.e.* each conductor cuts $2F$ tubes per revolution, and therefore $2Fn$ tubes per second; hence

$$\text{E.M.F. for each conductor} = \frac{2Fn}{10^8} \text{ volts.}$$

Again, since the brushes are placed diametrically opposite each other, the current has obviously two paths through the armature from brush to brush. Further, each path contains $Z/2$ conductors in series forming E.M.F., the two paths being arranged in parallel, and just as the combined E.M.F. of two equal batteries in parallel is the same as that of one battery, so the total E.M.F. in this case is that due to $Z/2$ conductors; hence

$$\begin{aligned}\text{Average E.M.F. of dynamo} &= \frac{2Fn}{10^8} \times \frac{Z}{2} \\ &= \frac{FnZ}{10^8} \text{ volts.}\end{aligned}$$

Most direct current machines are *self-excited*, *i.e.* the whole or a part of the current from the armature passes round the field magnets to further excite them, the residual magnetism of the fields being sufficient to start the action. Fig. 441 represents diagrammatically three methods known as *series*, *shunt*, and *compound* wound machines respectively; in the figure R_a is the armature, R_s and R_m coils on the field magnets. The whole principle of self-excitation is that, owing to the residual magnetism in the fields, the armature, on starting, has an E.M.F. and current developed in it, and this current passing round the fields strengthens them so that a larger E.M.F. and current are developed, and so on;

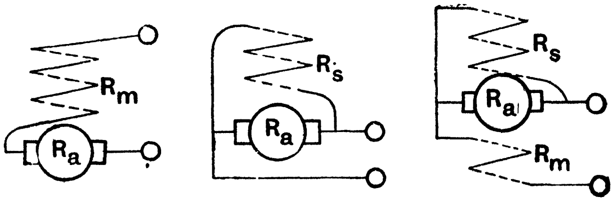


Fig. 441.

thus the machine quickly builds up. With a series machine the pressure at the terminals *increases* as the external current increases, whilst with a shunt machine it *decreases*; with a compound machine the pressure at the terminals can be made to be practically *constant* whatever the current.

The principle of the motor is that of the moving coil galvanometer; thus if a current from some external source be passed through the armature of Fig. 440 *it will begin to rotate*, the direction of rotation being determined by Fleming's left hand rule (Art. 170). Now when running it will of course act as a dynamo, and an application of Fleming's right hand rule (Art. 239) will show that the induced E.M F. e is opposite in direction to E, the pressure supplied by the external source; this back E.M.F. e is an important factor in practice. It is matter of easy proof that the watts transformed by this motor are eI, where e is the back E.M.F. in volts and I the current in amperes (cf. Art. 194), and that the torque (T) and horse-power are given by

$$T = \frac{FZI}{8{\cdot}52 \times 10^8} \text{ pound feet; H.P.} = T \times \frac{2\pi n}{550} = \frac{FnZI}{746 \times 10^8}.$$

The brake horse-power (B.H.P.) is less than this owing to various losses.

Motors may be series, shunt or compound wound. Series motors are used when a powerful starting torque is required, *e.g.* in electric trains and trams: shunt motors run at practically the same speed for all loads and are therefore useful for driving machine tools, etc.: in compound motors even more constant speed for all loads can be obtained but they are not largely used.

The current in a motor is evidently $(E-e)/R$ where E is the applied pressure and e the back E.M.F. If the motor be at rest and the full pressure E be suddenly applied the current will be E/R and as R is small this will be excessive and will burn out the armature. To avoid this a variable resistance (the "starter") is put in series with the armature and as the speed (and therefore e) increases, this resistance is cut out step by step until at full speed it is all out and the only resistance is R—that of the motor circuit.

For fuller details on Dynamos and Motors the student should read *Technical Electricity*, Chapters XIX. and XX.

257a. Telegraphy.—Only the simplest reference to one or two general principles can be given here.

In the *Morse System* the two signals which form words and numbers, etc. are currents of long and short duration. One method of receiving them is by means of a **Morse sounder** which consists of an electro-magnet fitted with an armature in such a position that the latter is drawn down upon a stop when the current flows and flies back against another stop when the current ceases; the operator thus listens to the clicks and notices whether the intervals between them are "short" or "long" ("dot" or "dash"). Another method of receiving the signals is by means of the **printing mechanism**; in this the armature is attached to a small inked wheel and under this is drawn by clockwork a long narrow strip of paper. When the armature is not attracted the wheel is clear of the paper but when attraction ensues the wheel comes into contact with the paper and prints a dot or a dash according as the duration of the current is "short" or "long." Combinations of dots and dashes form letters etc. according to a definite code, *e.g.* A · —, B — ..., C — · — · etc.

A simple circuit is shown in Fig. 441*a*. If the key on the left be depressed it will be lifted from the upper contact and current will flow from the battery on the left to the depressed key, thence to line, to the sounder on the right, to the key and to earth. The line galvanometers inserted at each end (Fig. 441*a*) show the operator, when "sending" if his apparatus is working correctly. If the distance between the stations is great the line current may be too weak to properly work the recorder; in this case a relay is employed, *i.e.* the line current actuates an electro-magnet so that it attracts an armature and thus brings a local battery into circuit to operate the recorder.

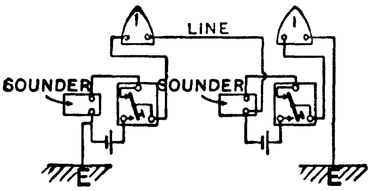


Fig. 441*a*.

In **Duplex Telegraphy** it is possible to send two messages simultaneously along the same wire, *i.e.* *A* can transmit to *B* at the same time as *B* is transmitting to *A*. The principle of one method—the **differential duplex**—will be gathered from Fig. 441*b*. The Morse magnets are each wound with two equal coils in opposite directions and arrangements are such that, for station *A* for example, the resistance of coil D_1 *plus* the resistance R_1 is equal to the resistance of coil C_1 *plus* the line resistance *plus* the resistance of coil C_2 and a similar relation holds for station B.

If A only is transmitting to B then on closing K_1 the current divides into two *equal* parts at X and since C_1 and D_1 are wound in opposition they cancel each others effect on M_1 which therefore does not respond, but the current passing through C_1 line C_2 to earth actuates M_2 so that the signal is recorded at B. A similar explanation holds *if B only transmits to A*.

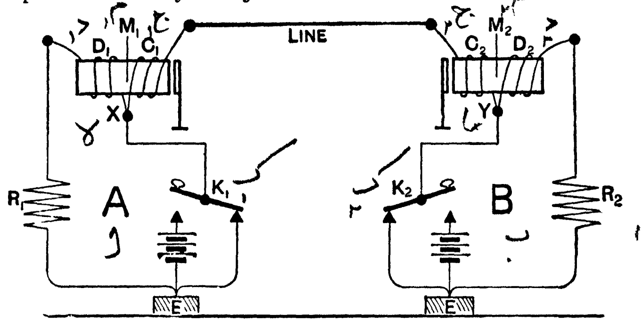


Fig. 441b.

Suppose now that *A and B are both "sending" together, say each sending a "long current."* As the two batteries are working "equal and opposite" through C_1, C_2 and line there will be (practically) no current in this part, but current continues to flow through D_1 and D_2 so that both M_1 and M_2 respond to a "long current," *i.e.* they respond as long as K_1 and K_2 are held down.

Now imagine that, simultaneously, *A sends a "long" and B a "short."* They start together say, then, when the key at B opens (key at A still down) the battery at B is cut out so that no current goes from it through D_2 but current now comes in from the line through C_2 so that M_2 continues to respond to the "long" from A. At A however, there are now equal currents in both C_1 and D_1 from the battery at A which cancel each others effect on M_1 so that it no longer responds. Thus *B's* signal to A ceases when K_2 is opened but *A's* signal to B continues as long as K_1 is closed. The student should think out other cases for himself.

As indicated, the above is only a brief glance at one or two general principles; for details and modern developments some good work on Telegraphy must be consulted.

257b. Telephony.—As in the preceding section, only the briefest reference to one or two general principles can be given here.

A section through the **Bell Magneto-telephone** is shown in Fig. 441c. Here M is a permanent magnet carrying at one end a piece of soft iron *S*. This forms the core for a coil *C* which has leads to the "line" and "return." In front of *S* is fixed a very thin soft iron disc *D D*. The main body of the instrument is of vulcanite.

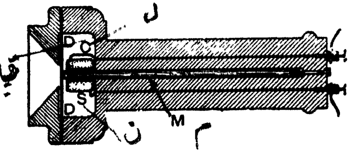


Fig. 441*c*.

The action is as follows: When a person speaks into the orifice the waves of sound cause the disc to vibrate, and as it moves to and from the soft iron core the distribution of the lines of force in the coil is altered. A current is therefore generated in *C*, the E.M.F. of which is determined by the rate at which the number of lines of force passing through *C* is changing. At the distant end of the line is a similar instrument, the terminals of which are directly connected to those at the transmitter, so that as the currents generated in *C* vary with the sound, so also do those received at the distant station. The reverse operation then takes place at the receiver. The current in its coils varying, the field in the neighbourhood varies, and the soft iron piece of the receiver attracts with varying strengths the vibrating disc immediately in front of it. Thus the disc at the receiver copies the movements of the disc at the transmitter. These movements being transferred to the air, the sound also is reproduced. No battery is needed to work the instrument, as the vibrating plate generates all the current required.

If sound waves are produced near a loose electrical contact, notably in the case of one or more sticks of carbon supported lightly by two fixed blocks of carbon, the vibrations cause the resistance of the points of contact to vary enormously, and thus a battery in series with the loose carbon will send a varying current to line as long as the vibrations continue; this varying current passing through a Bell magneto telephone will actuate it in the manner described above so that the sound is reproduced. Such an arrangement is shown diagrammatically in Fig. 441*d*. Modern **microphone transmitters** act on this principle; they contain granules of carbon between two plates of carbon, one of which is attached to the diaphragm of the transmitter and therefore receives the air vibrations.

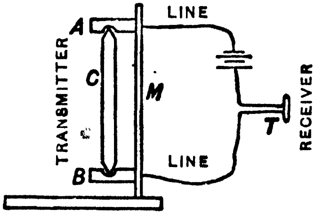


Fig. 441*d*.

In practice the microphone is used as the transmitter and the Bell magneto telephone is used as the receiver. Further, the microphone is not directly connected to the line as in Fig. 441*d*, but to the primary of an induction coil, the secondary of which is connected to the line; by this device the E.M.F. is magnified in the ratio of the secondary to the primary turns (Art. 250), and thus can overcome a high resistance in the line.

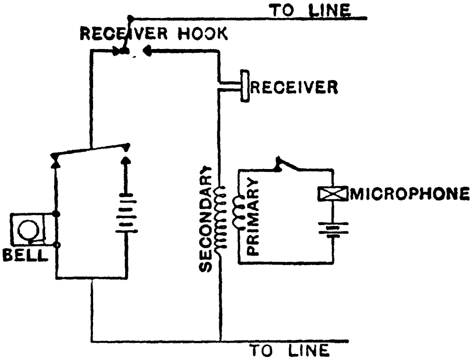


Fig. 441*e*.

A simple circuit is shown diagrammatically in Fig. 441*e*. When the receiver is *on the hook* the circuit is as shown and the calling bell can be rung by current coming in from the line. When the receiver is taken off the hook the latter moves to the other contact and the "speaking" circuit is then complete. A switch is included in the microphone circuit so that the battery is only sending current when the telephone is in use.

In telephone exchanges small electric lamps light up automatically when the instruments are lifted from the hooks and the exchange then rings up the other person with whom speech is desired connecting the two subscribers together. Other lamps indicate to the exchange operator when conversation is finished so that no ring off is necessary. For details of modern developments the student should refer to some standard work on Telephony.

Exercises XVII.

Section A.

(1) Describe experiments illustrating the chief laws of electro-magnetic induction.

(2) Prove that the induced E.M.F. is measured by the rate of change of the flow of induction.

(3) Develop expressions for the instantaneous value, the maximum value, and the average value of the induced E.M.F. in the case of a coil rotating about a vertical axis in the earth's field.

(4) Define coefficient of self-induction and coefficient of mutual induction.

(5) Describe the construction and explain the action of the induction coil.

Section B.

(1) A copper disc having a diameter of 40 centimetres is rotated about a horizontal axis perpendicular to the disc and parallel to the magnetic meridian. Two brushes make contact with the disc, one at the centre and the other at the edge. If the value of the horizontal component of the earth's field is 0·2 C.G.S., find the potential difference in volts between the two brushes when the disc makes 3,000 revolutions per minute. (B.E.)

(2) What are the magnitude and direction of the force acting on a straight conductor 10 centimetres long, placed at right angles to a magnetic field of 50 lines per square centimetre, the current through the conductor being 5 amperes? In what unit is your result expressed? (B.E.)

(3) A vertical hoop of wire, at right angles to the magnetic meridian, is quickly but with uniform speed turned through 180° about a vertical axis, its originally eastern half moving northward at first. State the direction in which the induced current passes round the wire, and determine the position of the hoop in which the induced E M.F is the greatest. (B.E).

Section C.

(1) What is a magnetic field? Describe a method of determining the magnetic dip by the revolution of a coil of wire about an axis in its own plane. (Inter. B.Sc.)

(2) A square conducting frame cut through at one place rotates in the earth's magnetic field about a vertical axis, passing through the middle points of opposite sides. Describe the variation in E.M.F. between the two sides of the break which consequently occurs, and calculate its maximum amount when there are 120 revolutions per minute, if the edge of the square is 25 cm. and the intensity of the earth's horizontal force is 0·18. (Inter. B.Sc.)

(3) Calculate the electromotive force generated, by virtue of the vertical component of the earth's field (which may be taken as $= 0{\cdot}41\ \text{cm.}^{-\frac{1}{2}}\ \text{gm.}^{\frac{1}{2}}\ \text{sec.}^{-1}$), in the axle of a railway carriage, of length 150 cm., travelling with a speed of 75 kilometres per hour. In what units is your answer given? How could you observe the existence of this electromotive force? (Inter. B.Sc. Hons.)

(4) When a conductor l centimetres in length carrying a current c (in C.G.S. electromagnetic units) is placed at right angles to a magnetic field of strength H, the force acting on the conductor is equal to Hlc dynes. Use this result to determine the value of the electromotive force generated when a conductor is moved with velocity v cm./sec. in a direction perpendicular both to its length and to a magnetic field of strength H. Deduce the result that the E.M.F. generated is equal to the rate at which the magnetic lines of force are cut by the conductor. (B.Sc.)

(5) A solenoidal coil 70 centimetres in length, wound with 30 turns of wire per centimetre, has a radius of 4·5 centimetres. A second coil of 750 turns is wound upon the middle part of the solenoid. Calculate the coefficient of self-induction of the solenoid, and the coefficient of mutual induction of the two coils. Will the inductance of the solenoid be affected by short-circuiting the ends of the secondary coil? (B.Sc.)

CHAPTER XVIII.

MEASUREMENT OF INDUCTANCE.

258. Introductory.—The Wheatstone Bridge arrangement for measuring resistances has been applied to the measurement of inductance. In the Wheatstone Bridge method for the comparison of resistances the balance for a steady current is made by pressing *first the battery key, and then, when the current is established, the galvanometer key.* If the galvanometer key were put down first and then the battery key the presence of inductive resistances in the arms of the bridge would, during the variable state of the currents, disturb the balance of the bridge, and the galvanometer would indicate this by a sudden throw of the needle at the instant of closing the battery key. If on closing the keys *in this latter order*, after first adjusting for exact balance with steady currents in the usual way, there is no throw of the needle, then either there is no inductive resistance in the arms or the induction effects in the arms of the bridge balance each other in the galvanometer.

The conditions for this balance of induction effects depend upon the inductances and resistances in the bridge arms, and may, therefore, be applied to compare inductances suitably placed in the bridge circuit. If there is only one inductive resistance in the circuit, the throw of the galvanometer needle due to the self-induction of this resistance may be compared with the throw produced by the discharge of a known quantity of electricity through the galvanometer, or ~~by~~ the permanent deflection caused by a known steady current when passed through the galvanometer.

259. Measurement of Self-inductance by the Rayleigh-Maxwell Method.—In Rayleigh's modification of Maxwell's original method of measuring self-induction the throw of the galvanometer needle in testing for the balance of transient currents, after obtaining exact balance for steady currents, is compared with the permanent deflection produced by the steady current obtained through the galvanometer by disturbing the balance for steady currents. This disturbance of balance is effected by making a small change in the resistance of one arm of the bridge.

For this method the coil whose inductance is to be measured is connected up, as shown at *R* in Fig. 442, in one arm of a Wheatstone Bridge arrangement of non-inductive resistances. If the resistance of the coil is very small it is advisable to insert a non-inductive resistance in the same arm with it.

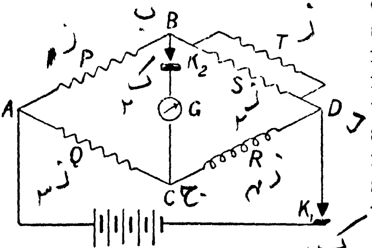


Fig. 442.

The simplest arrangement is to make $P = Q$, and to facilitate exact balancing for steady currents the resistance in the arm *BD* should be two resistance boxes arranged in parallel. Approximate balance for steady currents is obtained between the resistances *P*, *Q*, *R*, and *S*; then *S*, being adjusted to a value a little above the value necessary for exact balance, the resistance *T* is then adjusted until exact balance is obtained with a resistance $ST/(S + T)$, equal to *R*, in the arm *BD*. The keys are now worked in the order K_2, K_1, so as to test the balance for transient currents. The inductance in the arm *CD* will now cause the discharge of a quantity of electricity through the galvanometer, and there will be a sudden throw of the galvanometer needle. This throw is noted; let the scale deflection be d_1, indicating an angular deflection, δ_1, of the needle. The balance for steady currents is now disturbed by increasing T so that the value of $ST/(S+T)$ is increased by a small amount ρ. On testing for balance of steady currents there will now be a permanent deflection of the galvanometer. Let this be a scale deflection d_2, indicating an angular deflection, δ_2, of the needle.

If L denote the self-inductance of the coil and I the current established in the arm CD on testing for transient current balance by closing K_1 after K_2, then q, the quantity of electricity discharged through the galvanometer, is proportional to LI, that is $q = kLI$, where k is a constant.

But
$$q = \frac{HT}{\pi G} \sin \frac{\delta_1}{2},$$

and therefore
$$kLI = \frac{HT}{\pi G} \sin \frac{\delta_1}{2}.$$

In practice it is best to obtain the induction throw of the galvanometer needle by first balancing exactly for steady currents and then suddenly *reversing* the current. When this is done the quantity, q, is proportional to $2LI$ and we have
$$kLI = \frac{HT}{2\pi G} \sin \frac{\delta_1}{2}.$$

Also, when the resistance in the arm BD is increased by an amount ρ the potential difference for that arm is increased by an amount $I'\rho$, where I' is the current in the arm after the increase is effected and the balance for steady currents disturbed. Hence the permanent current determined through the galvanometer may be said to be due to this increment of potential difference in the arm BD and is therefore proportional to $I'\rho$. That is i, the current through the galvanometer, is given by $i = kI'\rho$, where k is, on account of the symmetry of the bridge, the same constant as for q above.

But
$$i = \frac{H}{G} \tan \delta_2,$$

and therefore
$$kI'\rho = \frac{H}{G} \tan \delta_2.$$

From the two results thus obtained we get

$$\frac{kLI}{k\rho I'} = \frac{T}{2\pi} \cdot \frac{\sin\frac{\delta_1}{2}}{\tan\delta_2}$$

and

$$L = \frac{T\rho}{2\pi} \cdot \frac{I'}{I} \cdot \frac{\sin\frac{\delta_1}{2}}{\tan\delta_2}.$$

Since δ_1 and δ_2 are small

$$\frac{\sin\frac{\delta_1}{2}}{\tan\delta_2} = \frac{d_1}{2d_2},$$

and, when ρ is very small, I and I' are approximately equal. Hence we get

$$\boldsymbol{L = \frac{T\rho}{2\pi} \cdot \frac{d_1}{2d_2}}.$$

When necessary the exact value of I'/I can be calculated in terms of the resistances involved.

260. Measurement of Self-inductance by the Rimington-Maxwell Method.—In this method the self-inductance is determined in terms of the capacity of a condenser by balancing the throw due to inductance in one arm of the Wheatstone Bridge against the throw due to the action of a condenser in another part of the bridge circuit.

The bridge circuit is arranged as shown in Fig. 443, which is the same as that given above, with the addition of a standard condenser of capacity C, having one terminal connected at A and the other terminal movable so that it can be connected at any point X on the resistance P in the arm AB.

Imagine the keys K_1 and K_2 to be closed, and that exact balance obtains for steady currents with the condenser connected between the points A and X. The presence of the condenser will not in any way affect the conditions of balance in the steady state, but if, while exact balance obtains, the key K_1 is suddenly opened the condenser will discharge round the circuit and through the galvanometer from C to B. The discharge through the galvanometer due to self-induction in R will, however, be from B to C. Hence by adjusting the position of the point X in the resistance P it is evidently possible under suitable conditions to charge the condenser to a difference of potential such that the portion of its charge discharged through the galvanometer shall be exactly equal to the quantity sent through the galvanometer by the induction effect in the coil in the arm CD.

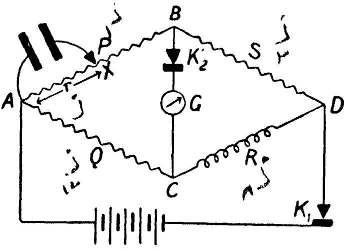


Fig. 443.

If C denote the capacity of the condenser, r the resistance of AX, and I_1 the steady current in AB, then the charge in the condenser is CI_1r. When discharge takes place the quantity passing through the circuit external to AX is

$$\frac{r}{Y}\cdot CI_1r \quad \text{or} \quad \frac{CIr^2}{Y},$$

where Y is the resistance of the circuit $\overset{\diagup D \diagdown}{AB \text{—} CA}$. Of this quantity

$$\frac{R+S}{R+S+G}\cdot\frac{CI_1r^2}{Y} = q_1$$

passes through the galvanometer. Similarly the quantity of electricity set in motion by self-induction in R is LI_2/Z, where Z is the resistance of the circuit $\overset{\diagup A \diagdown}{DB \text{—} CD}$, and of this quantity

$$\frac{P+Q}{P+Q+G}\cdot\frac{LI_2}{Z} = q_2$$

passes through the galvanometer. On working out the values of Y and Z and simplifying we get

$$\frac{q_1}{q_2} = \frac{CI_1r^2(R+S)}{LI_2(P+Q)}.$$

But
$$\frac{I_1}{I_2} = \frac{Q}{P} = \frac{R}{S}.$$

Hence
$$\frac{q_1}{q_2} = \frac{Cr^2}{L} \cdot \frac{R}{S} \cdot \frac{R+S}{P+Q}.$$

That is, since by the condition for steady balance $SQ = PR$,
$$\frac{q_1}{q_2} = \frac{Cr^2}{L} \cdot \frac{R(R+S)}{P(R+S)},$$

or
$$\frac{q_1}{q_2} = \frac{Cr^2}{L} \cdot \frac{R}{P}.$$

Therefore, when there is a balance for transient currents and $q_1 = q_2$, we have
$$Cr^2R = LP,$$

or
$$L = \frac{Cr^2R}{P}.$$

The balance for transient current should hold at make as well as break of the current. Hence the usual method of adjustment is to change the position of X until there is no movement of the needle on making and breaking or reversing current at K_1 while K_2 is closed. It will be clear from the relation
$$L = \frac{Cr^2R}{P}$$

that, as the maximum value of r is P, the minimum value of C with which the adjustment for balance is possible is L/PR.

261. Measurement of Self-inductance by the Anderson Method.—By a modification of the preceding due to Professor Anderson it is possible to avoid the moving contact at X. Fig. 445 gives Anderson's method, and Fig. 444 a slight modification; the latter will be considered first.

An extra non-inductive resistance, N (Fig. 444), is introduced at AE and the condenser is permanently connected between the points E and B. The value of N does not affect the condition for steady balance of the bridge, so that by altering it after the steady current balance is obtained this balance is not disturbed and the charge in the condenser can be adjusted so as to give the balance for transient currents. The operations of this method consist therefore of first obtaining exact balance for steady currents and then adjusting N until there is also exact balance for transient currents.

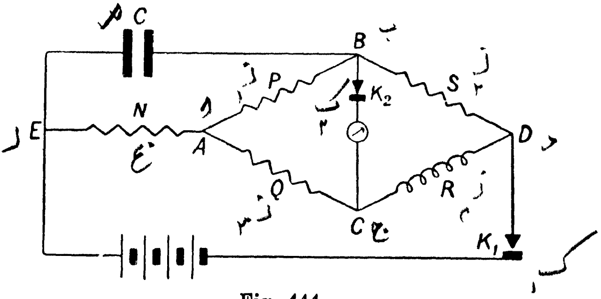


Fig. 444.

Adopting the same notation as above it is evident that F, the charge in the condenser, is $[N(I_1 + I_2) + PI_1]C$, and the portion of this tending to pass through the galvanometer is evidently

$$\frac{R+S}{R+S+G} \cdot \frac{P}{Y} \cdot F = q_1.$$

Also, as above,

$$q_2 = \frac{P+Q}{P+Q+G} \cdot \frac{LI_2}{Z},$$

and after substitution for Y and Z and simplification we get

$$\frac{q_1}{q_2} = \frac{FP(R+S)}{LI_2(P+Q)},$$

and on substituting for F and reducing as before we get

$$\frac{q_1}{q_2} = \frac{C}{L}(NR + NS + PR),$$

so that when $q_1 = q_2$

$$L = C(NR + NS + PR).$$

In Anderson's method (Fig. 445) the condenser is connected between A and the galvanometer branch at N, and r is the adjustable non-inductive resistance.

The experiment consists in (1) adjusting in the usual way until there is no deflection on closing first K_1 and then K_2, and (2) adjusting r until there is no throw on closing first K_2 and then K_1.

Let x, y, and z denote the *quantities* passing through P, through Q, and into C respectively in time t. Now

P.D. between A and N = P.D. between A and C,

$$\therefore \frac{z}{C} = Q\frac{dy}{dt}, \quad \therefore \frac{dy}{dt} = \frac{1}{Q} \cdot \frac{z}{C} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(1}$$

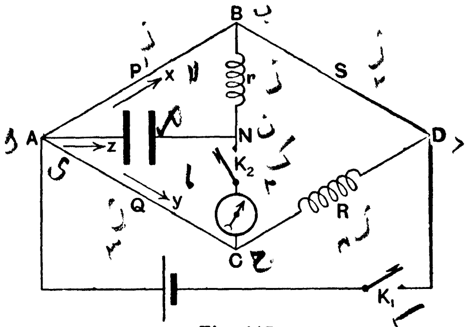


Fig. 445.

Again—

P.D. between A and B = P.D. between A and N + P.D. on r,

$$\therefore P\frac{dx}{dt} = \frac{z}{C} + r\frac{dz}{dt}, \quad \therefore \frac{dx}{dt} = \frac{1}{P}\left(\frac{z}{C} + r\frac{dz}{dt}\right) \quad \ldots\ldots\ \text{(2}$$

Further—

P.D. between C and D = P.D. on r + P.D. between B and D,

$$\therefore R\frac{dy}{dt} + L\frac{d}{dt}\frac{dy}{dt} = r\frac{dz}{dt} + S\left(\frac{dx}{dt} + \frac{dz}{dt}\right) \quad \ldots\ldots\ldots \text{(3)}$$

i.e. $$\frac{R}{Q}\frac{z}{C} + \frac{L}{QC}\frac{dz}{dt} = r\frac{dz}{dt} + S\frac{dz}{dt} + S\left\{\frac{1}{P}\left(r\frac{dz}{dt} + \frac{z}{C}\right)\right\},$$

$$\therefore \frac{dz}{dt}\left(r + S + \frac{rS}{P} - \frac{L}{QC}\right) = \frac{z}{C}\left(\frac{R}{Q} - \frac{S}{P}\right) = 0 \text{ since } \frac{R}{Q} = \frac{S}{P},$$

i.e. $$r + S + \frac{rS}{P} - \frac{L}{QC} = 0$$

or $$L = QC\left(r + S + \frac{rS}{P}\right),$$

$$\therefore \boldsymbol{L = C(rQ + rR + SQ)}.$$

262. Comparison of Two Self-inductances by Maxwell's Method.—The method proposed by Maxwell is troublesome in practice, but simple in theory.

The two coils whose self-inductances are to be compared are placed each in series with a non-inductive resistance in the adjacent arms of a Wheatstone Bridge arranged as shown in the diagram of Fig. 446. The method of adjustment consists in obtaining a balance for steady currents with different values of R and S, the total resistances in the arms CD and BD respectively, until the ratio of these values is so adjusted that there is also exact balance for transient currents.

If L_1 and L_2 denote the self-inductances of the coils in BD and CD, and i_1 and i_2 the currents in ABD and ACD, then the electromotive impulses in these coils are $L_1 i_1$ and $L_2 i_2$ respectively, and, from the position of the coils relative to the galvanometer, the electromotive impulses through the galvanometer are in opposite directions and in the ratio $L_1 i_1 / L_2 i_2$. But $i_1/i_2 = R/S$ and therefore $L_1 i_1 / L_2 i_2 = L_1 R / L_2 S$, so that when there is balance for transient currents and these impulses are equal we have $L_1 R = L_2 S$, or $L_1/L_2 = S/R$. Hence, although balance for steady currents is obtained when $P/Q = S/R$, whatever the values of S and R may be, the balance for transient currents is obtained only when $S/R = L_1/L_2$. It is necessary therefore to vary the ratio of S to R, adjusting P and Q for balance for steady currents for every value of the ratio until finally, on testing for balance of the transient currents, exact balance is obtained. When this troublesome double adjustment is made we have

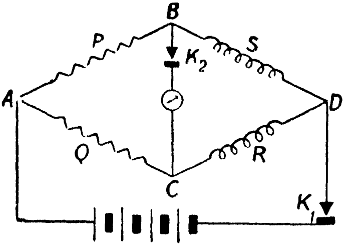


Fig. 446.

$$\frac{L_1}{L_2} = \frac{S}{R} = \frac{P}{Q}.$$

263. Comparison of Two Self-inductances by the Niven-Maxwell Method.

—A modification of this method, proposed by Niven, allows the comparison to be made without repeated adjustment of balance for steady currents.

The bridge resistances are arranged as shown in Fig. 447, and an additional adjustable non-inductive resistance N is connected between the points b and c, the point b being the junction of s and σ, the inductive and non-inductive resistances in the arm BD, and the point c, the junction of the resistances r and ρ in the arm CD. The coil r in Cc is also shunted with a plug key K, so that, when the plug is in, the resistance in Cc is practically nothing.

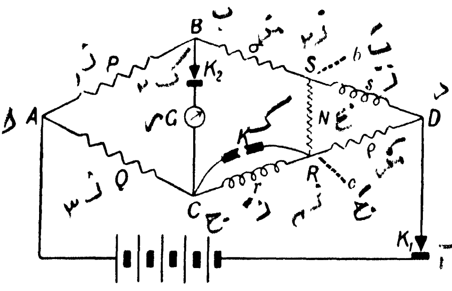


Fig. 447.

The first operation of this method consists in obtaining a balance for steady currents with the plug in the key K, the resistance σ of zero value, and the value of N infinite. This adjustment gives $P/Q = s/\rho$. Then K is unplugged and the resistance σ adjusted until balance for steady currents is again obtained. This gives $P/Q = (\sigma + s)/(r + \rho)$, and we therefore have $P/Q = s/\rho = \sigma/r$, that is b and c are for steady currents at the same potential. For the third operation the balance for transient currents is tested and the resistance N adjusted until exact balance is obtained. When this balance is obtained it can be shown, by the method worked out for similar cases above, that

$$\frac{L_1}{L_2}=\frac{P(N+\rho+s)}{NQ},$$

L_1 and L_2 being the inductances of s and r respectively.

264. Measurement of Mutual Inductance by Carey Foster's Method.

—The mutual induction for two coils may be measured in terms of the capacity of a condenser by the following method due to Professor Carey Foster.

The two coils P and S (Fig. 448), P the primary and S the secondary, are connected, as shown in the diagram, in circuit with a condenser of capacity C, the non-inductive resistances R and Q, and the galvanometer G. By means of a key at K the current in P can be made, broken, or reversed, and an inductive impulse equal to MI or $2MI$ thereby set up in S, where M is the coefficient of mutual induction for the coils and I is the current in R and P. The condenser also, with its terminals connected to the points A and B, becomes charged or discharged or has its charge reversed according as the current is made, broken, or reversed. By the arrangement of the circuit the two discharges through the galvanometer due to the inductance of S and the capacity of C are in opposite directions, and may be adjusted to equality by adjusting R or Q until the galvanometer shows no deflection on working the key at K. When this adjustment is made we have

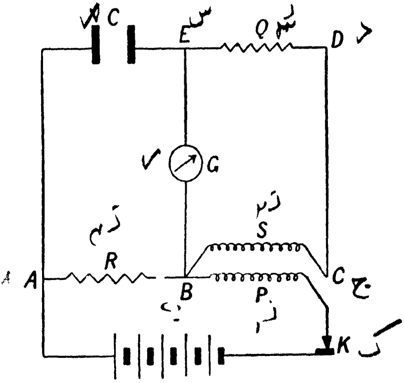


Fig. 448.

$$M = CR(Q + S).$$

Proof.—The charge in the condenser is CIR, and the portion of this which passes through the galvanometer is

$$\frac{Q+S}{Q+S+G} \cdot CIR.$$

The electromotive impulse set up in S is MI, and the quantity of electricity set in motion by it is

$$\frac{MI}{G+S+Q}.$$

This quantity all passes through the galvanometer. Hence we have

$$\frac{Q+S}{Q+S+G} \cdot CIR = \frac{MI}{Q+S+G},$$

or

$$(Q+S)CR = M.$$

265. Comparison of Mutual Inductances by Maxwell's Method.—This arrangement (Fig. 449) is exactly that of Lumsden's method for comparing E.M.F.'s (Chapter XVI.), the electromotive impulses M_1I and M_2I produced in the two secondary coils by making or breaking the current I in the primary coils taking the part of the E.M.F.'s in Lumsden's experiment.

The apparatus is arranged as in Fig. 449, and R_1 and R_2 are adjusted until the galvanometer is not deflected when the primary current is made or broken. If x and y denote the induced quantities circulating in S_1 and S_2 and t the time, then, applying Kirchhoff's law to each secondary compartment—

$$(S_1 + R_1)\frac{x}{t} + G\frac{x-y}{t} = M_1\frac{I}{t},$$

$$(S_2 + R_2)\frac{y}{t} + G\frac{y-x}{t} = M_2\frac{I}{t},$$

$$\therefore\ (S_1 + R_1)x + G(x-y) = M_1I,$$

$$(S_2 + R_2)y - G(x-y) = M_2I;$$

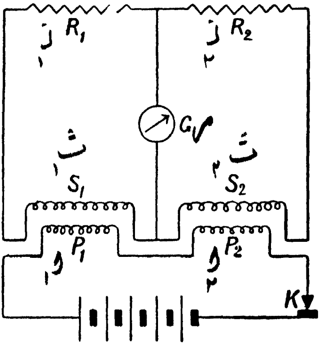


Fig. 449.

and, since $x = y$,

$$\frac{M_1}{M_2} = \frac{S_1 + R_1}{S_2 + R_2}.$$

If R_1 and R_2 be altered to $R_1{}^1$ and $R_2{}^1$, so as again to secure a balance—

$$\frac{M_1}{M_2} = \frac{S_1 + R_1{}^1}{S_2 + R_2{}^1},$$

$$\therefore\ \frac{M_1}{M_2} = \frac{R_1 - R_1{}^1}{R_2 - R_2{}^1}.$$

266. Comparison of the Self-inductance of a Coil with the Mutual Inductance between it and another Coil by Maxwell's Method.—In this method the first coil is put into one arm of the bridge, and the second is put in the battery branch, the connections being such that the E.M.F. due to self-induction in L is opposite to the E.M.F. in L due to the inductive effect of the other coil.

The arrangement is shown in Fig. 450. On balancing for steady currents

$$P/S = Q/R.$$

During the varying period (K_2 closed before K_1) the P.D. between A and B is

$$L\frac{dp}{dt} - M\frac{di}{dt} + Pp$$

and the P.D. between A and C is Qq. For balance, therefore,

$$L\frac{dp}{dt} - M\frac{di}{dt} + Pp = Qq.$$

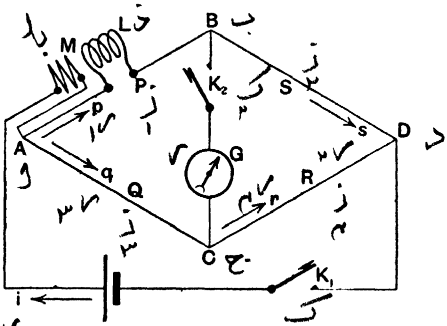


Fig. 450.

But $i = p + q$,

$$\therefore\ L\frac{dp}{dt} - M\frac{dp}{dt} - M\frac{dq}{dt} + Pp = Qq,$$

i.e.

$$(L - M)\frac{dp}{dt} - M\frac{dq}{dt} = Qq - Pp.$$

As no current passes through G, $p = s$ and $q = r$, and, since $Ss = Rr$,

$$\therefore\ Sp = Rq, \quad \text{i.e. } q = \frac{Sp}{R}$$

and

$$S\frac{dp}{dt} = R\frac{dq}{dt}, \quad \text{i.e. } \frac{dq}{dt} = \frac{S}{R}\frac{dp}{dt}.$$

Substituting in the preceding equation—

$$(L - M)\frac{dp}{dt} - \frac{MS}{R}\frac{dp}{dt} = \frac{QS}{R}p - Pp.$$

But $QS = RP$, i.e. $QS/R = P$; hence

$$(L - M)\frac{dp}{dt} - \frac{MS}{R}\frac{dp}{dt} = 0,$$

$$\therefore\ L - M = \frac{MS}{R},$$

$$\therefore\ \frac{L}{M} = \frac{R + S}{R}.$$

267. Conclusion.—The student who works through the preceding tests will come to the conclusion either that he is a poor experimenter, or that "the apparatus is wrong"—probably the latter. Both conclusions may possibly be correct, but the methods are nevertheless troublesome and unsatisfactory at the best. Newer methods depending on the employment of alternating currents are being developed; the difficulty, however, has been the lack of a convenient galvanometer for alternating current work. This difficulty has, in a measure, been overcome by the introduction of the **vibration galvanometer** of Mr. Albert Campbell. It consists of a light coil with a bifilar suspension in between the poles of a magnet; the vibration frequency can be altered and made to correspond with the frequency of the alternating current. When "tuned" in this way the spot of light on the scale becomes a band whose length is proportional to the current strength. Campbell describes, in the *Proceedings of the Physical Society*, several methods of comparing inductances and capacities with this galvanometer. Thus, with a capacity C, self-inductance L, and mutual inductance M, arranged as indicated in Fig. 451, and resistances adjusted for no current in the galvanometer, it can be shown that

$$\frac{M}{C} = R_1R_3;\ \frac{L}{M} = \frac{R_1 + R_2}{R_1};\ \frac{L}{C} = R_1R_3 + R_2R_3.$$

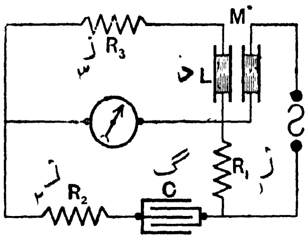


Fig. 451.

Exercise. Establish the above relationships.

CHAPTER XIX.

INDUCED MAGNETISATION AND MEASUREMENT OF PERMEABILITY AND OTHER QUALITIES.

268. Revision.—Before proceeding with this chapter the student must revise thoroughly Arts. 9-15 of Chapter I.

The truth of the relation $B = H + 4\pi I$ may again be shown. Consider a long thin rod of iron placed in a uniform magnetic field in air with its length parallel to the field. The rod will be magnetised by induction, and the ends will exhibit polarity; let m denote the strength of the poles. The number of unit tubes emanating from the north pole is $4\pi m$; these may be assumed to be continuous *throughout the bar* from the south pole to the north pole, so that, if a be the cross-sectional area of the bar, the number of tubes *per unit area* due to the magnetisation of the bar is $4\pi \frac{m}{a}$, i.e. $4\pi I$ (Art. 20), where I is the intensity of magnetisation. In addition, there are H unit tubes per unit area due to the field; hence the total number of unit tubes per unit area is $H + 4\pi I$, and as this gives the magnetic induction or flux density B,

$$B = H + 4\pi I.$$

To be exact the value of H to be used in the above is its value *in the material*, and, as previously indicated, this is less than the value H of the original field, owing to the demagnetising effect of the poles (Arts. 4, 9). From the above, still neglecting this disturbance, we have

$$\frac{B}{H} = 1 + 4\pi \frac{I}{H}, \quad \text{i.e.} \quad \mu = 1 + 4\pi\kappa,$$

The above is the usual "practical" method of proving the relation $B = H + 4\pi I$: the *strict* proof however is that given in Art. 270.

This is the usual way of presenting these relations, but it will be seen that they imply that $\boldsymbol{B}$, $\boldsymbol{H}$, and $\boldsymbol{I}$ are quantities of the same dimensions, and that μ and κ are mere numbers. A more complete way of presenting the matter is as follows. If μ be defined from the relation $\boldsymbol{B} = \mu \boldsymbol{H}$ it is evident that μ can be unity only for an unmagnetisable medium. Hence, if we take air as a magnetisable medium of permeability μ_0, and consider for a field of intensity $\boldsymbol{H}$ the flow of induction across a crevasse, filled with air, in a medium of permeability μ, we have μH as the induction in the medium, and $\mu_0 \left[\boldsymbol{H} + \frac{4\pi}{\mu_0}(\boldsymbol{I} - \boldsymbol{i})\right]$ as the induction in the air-gap, $\boldsymbol{i}$ being the intensity of magnetisation of the air. Hence we have

$$\mu \boldsymbol{H} = \mu_0 \left[\boldsymbol{H} + \frac{4\pi}{\mu_0}(\boldsymbol{I} - \boldsymbol{i}) \right] \quad \text{or} \quad \boldsymbol{B} = \mu_0 \boldsymbol{H} + 4\pi(\boldsymbol{I} - \boldsymbol{i}),$$

that is $\mu = \mu_0 + 4\pi(\boldsymbol{I} - \boldsymbol{i})/\boldsymbol{H}$. Here, if μ_0 be taken as unity and $(\boldsymbol{I} - \boldsymbol{i})$ as the intensity of magnetisation of the medium, we get $\boldsymbol{B} = \boldsymbol{H} + 4\pi \boldsymbol{I}$ and $\mu = 1 + 4\pi\kappa$, as above.

269. Magnetising Force in a Magnetisable Body.—We have frequently referred to the fact that the magnetic or magnetising force to be used in the present investigations is that *in the material* and not that in the original field before the material is placed there, and that the former (H_1) is less than the latter (H) owing to the demagnetising reaction of the poles of the specimen. It is stated in Art. 9 that $H_1 = H - NI$, where I is the intensity of magnetisation of the specimen and N is a factor depending on its shape and dimensions. If the specimen is a long thin rod (300 to 500 diameters) the poles are at a considerable distance from the middle portions of the rod, and therefore do not produce any marked weakening of the original field along the middle portions; in this case H_1 may be taken identical with H. Further, in the case of a ring magnetised by a coil of wire wrapped closely round it there are no free poles, and again H_1 may be taken equal to H.

To apply to the present investigation the exact and fundamental definition of the intensity of a magnetic field, viz. *that it is measured by the force in dynes on a unit pole,* we must imagine a cavity in the material in which to place the unit pole. A cavity of any shape will not do, however, for the walls of the cavity will exhibit magnetism and exert an influence on the unit pole, so that the force on the latter will not be H_1, *i.e.* the force due merely to the combined action of the original field and the end poles of the specimen.

Consider, however, a cavity such as is shown at *a* (Fig. 452), *viz.* a long indefinitely thin tunnel in the direction of magnetisation. The sides of the tunnel will exhibit no magnetisation, the poles at the ends of the tunnel will be weak and far away from the centre, and will not appreciably affect a unit pole put there; thus the field intensity at the centre of the tunnel will give the actual value of the magnetising force H_1 in the specimen. Hence **the magnetic or magnetising force H_1 inside the specimen is measured by the force in dynes on a unit north pole placed at the centre of a long and indefinitely narrow tunnel in the direction of magnetisation.**

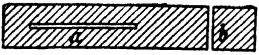


Fig. 452.

In Art. 32 it is shown that in the case of a uniformly magnetised sphere the intensity of the field inside due to the sphere itself is $\frac{4}{3}\pi I$, where I is the intensity of magnetisation, and an examination of Fig. 78 will show that this is in the opposite direction to I. If this magnetisation be produced by the sphere being in a field of intensity H, and if H_1 denote the magnetising force in the sphere—

$$H_1 = H - \tfrac{4}{3}\pi I,$$

and comparing this with the expression

$$H_1 = H - NI,$$

we see that for a sphere $N = \frac{4}{3}\pi$.

For an ellipsoid Maxwell gives the formula

$$N = 4\pi \left(\frac{1}{e^2} - 1\right)\left(\frac{1}{2e}\log\frac{1+e}{1-e} - 1\right),$$

where a, b, and c are the semi-axes, c being the long one in the direction of the field and $a = b = c\sqrt{1-e^2}$. The "dimension ratio" c/a is equal to $1/\sqrt{1-e^2}$; thus from the above the values of N for various dimension ratios may be determined. For dimension ratios 300, 400, and 500, N has the values ·00075, ·00045, and ·0003.

270. Induction in a Magnetisable Body.—We can also define the induction, B, in a manner similar to the above. Consider, for example, an indefinitely thin crevasse cut across the specimen (Fig. 452, *b*), its plane faces being perpendicular to the direction of magnetisation, and of course infinitely large compared with the distance between them, and imagine a unit north pole in the crevasse. Just as the force on unit charge between two oppositely electrified infinite planes is $4\pi\rho$, where ρ is the surface density of electrification, so the force on the unit pole due to the walls of the cavity is $4\pi I$. In addition, there is the force H_1 defined above. Thus the total force on the unit pole is $H_1 + 4\pi I$ and this measures the induction B. Hence **the induction or flux density B is measured by the force in dynes on a unit north pole placed in an indefinitely narrow crevasse with its parallel faces at right angles to the direction of magnetisation.**

The relations of Art. 268 may now be more precisely written

$$B = H_1 + 4\pi I,$$

$$\mu = \frac{B}{H_1} = 1 + 4\pi\frac{I}{\mathrm{H}_1} = 1 + 4\pi\kappa.$$

The + sign denotes addition when the two terms are in the same direction and vector addition in other cases.

271. Magnetisation induced in a Sphere of Permeability μ in a Uniform Field in Air.—Let H be the strength of the inducing field and I the intensity of magnetisation of the sphere, both in the same direction.

If H_1 be the magnetising force in the sphere, then, from Arts. 9, 32 and 269,

$$H_1 = H - \tfrac{4}{3}\pi I.$$

Now $$B = H_1 + 4\pi I,$$

$$\therefore\ B = H - \tfrac{4}{3}\pi I + 4\pi I = H + \tfrac{8}{3}\pi I.$$

But $$B = \mu H_1 = \mu\,(H - \tfrac{4}{3}\pi I),$$

$$\therefore\ H + \tfrac{8}{3}\pi I = \mu\,(H - \tfrac{4}{3}\pi I).$$

From this we obtain the following :—

$$I = \frac{3}{4\pi}\cdot\frac{\mu - 1}{\mu + 2}\cdot H,$$

$$B = \frac{3\mu}{\mu + 2}H,$$

$$H_1 = \frac{3}{\mu + 2}H.$$

These expressions indicate the marked demagnetisation effect in such a case as the one under consideration. Thus, if μ be 900, H_1 is only $\frac{3}{902}H$, *i.e.* practically $\frac{1}{300}$ of H, and B is $\frac{2700}{902}H$, *i.e.* practically $3H$; if there were no demagnetisation B would be $900H$.

If κ be used instead of μ, then, since $\kappa = (\mu - 1)/4\pi$, we obtain

$$I = \frac{3\kappa}{3 + 4\pi\kappa} H,$$

and corresponding expressions can be obtained for B and H_1.

If the sphere of permeability μ be in a medium of permeability μ_1 it is matter of simple proof that

$$H_1 = \frac{3\mu_1}{\mu + 2\mu_1} H,$$

which of course becomes $H_1 = 3H/(\mu + 2)$ if μ_1 be unity.

The reader should compare these results with the corresponding results in electrostatics (Art. 104).

272. Movement of Paramagnetics and Diamagnetics in a Magnetic Field.

—It has been indicated (Art. 11) that a paramagnetic body tends to move into the strongest part, and a diamagnetic body into the weakest part, of a magnetic field. This has been shown experimentally, but the fact may also be deduced from theoretical considerations.

The potential energy of a magnet of moment M placed at a point in a magnetic field where the intensity is H is (Art. 27) given by the expression

$$\text{Potential Energy} = -MH,$$

the axis of the magnet being along the field.

If I be the intensity of magnetisation and v the volume $M = Iv$, the above becomes

$$\text{Potential Energy} = -IvH.$$

If the body be originally neutral, then, neglecting demagnetisation effects and assuming $I = \kappa H$, we may write—

$$\text{Potential Energy} \propto -\kappa H^2.$$

Now the body in question will tend to move into such a position that the potential energy is a minimum. For a paramagnetic κ is positive, and $-\kappa H^2$ is a minimum when H^2 has its greatest value; *thus a paramagnetic will tend to* *move to where the field is strongest.* For a diamagnetic κ is negative, and the expression on the right becomes κH^2, which is evidently least when H^2 has its lowest value; *thus a diamagnetic will tend to move to where the field is weakest.*

273. Force on a Small Magnetic Body in a Magnetic Field.—For simplicity we will consider the case of a small sphere of susceptibility κ. With the usual notation

$$\text{Potential Energy} = -IvH.$$

Now for a small displacement dx in the direction of a force F producing the displacement the work done is $F.dx$. But the work done is also given by the difference between the potential energy at the beginning and that at the end of the displacement, viz. $d(-IvH)$; hence

$$F.dx = d(-IvH),$$

$$\therefore\ F = -Iv\,\frac{dH}{dx}.$$

For the sphere in question it is shown in Art. 271 that

$$I = \frac{3\kappa}{3+4\pi\kappa}\,H.$$

Hence
$$F = -\frac{3\kappa}{3+4\pi\kappa}\,v\,\frac{H.dH}{dx},$$

$$i.e.\quad F = -\frac{v}{2}\cdot\frac{3\kappa}{3+4\pi\kappa}\cdot\frac{dH^2}{dx}.$$

Thus—(*a*) if the field is uniform H^2 is constant and the force on the sphere is zero, (*b*) if κ be positive (paramagnetic) F is in one direction, if negative (diamagnetic) it is in the opposite direction, and we may draw the same conclusion as that at the end of Art. 272. It should be noted that κ for bismuth, the most powerful diamagnetic, is only $(-\cdot 0000025)$.

274. Energy in a Magnetic Field.—Consider a closely wound ring solenoid such as is referred to in Art. 269. If S denotes the number of turns, l the mean circumference, a the cross-sectional area, *and* i *the current*, the intensity (H) of the field inside is given by

$$H = \frac{4\pi S i}{l},$$

and the flux density B is given by

$$B = \mu H = \frac{4\pi \mu S i}{l},$$

where μ is the permeability of the medium. The flow of induction through each turn is therefore Ba, and as this passes through S turns we have for the *effective* flux F_1 (Chapter XVIII.)—

$$F_1 = BaS = \frac{4\pi \mu a S^2 i}{l}.$$

Putting i equal to unity, the coefficient of self-induction L becomes

$$L = \frac{4\pi \mu a S^2}{l},$$

and the total energy in the medium is, therefore (Art. 254),

$$\text{Total energy} = \tfrac{1}{2} L i^2 = \frac{2\pi \mu a S^2 i^2}{l}.$$

Now the volume of the medium is, in this case, al, hence

$$\begin{aligned}\text{Energy per unit volume} &= \frac{2\pi \mu a S^2 i^2}{l} \Big/ al \\ &= \frac{2\pi \mu S^2 i^2}{l^2} \\ &= \left(\frac{4\pi S i}{l} \cdot \frac{4\pi \mu S i}{l}\right) \Big/ 8\pi \\ &= \frac{HB}{8\pi} = \frac{\mu H^2}{8\pi} = \frac{1}{\mu} \frac{B^2}{8\pi}.\end{aligned}$$

These expressions should be compared with the corresponding ones in Electrostatics (Chapter VI.).

275. Boundary Conditions.—Similarly, boundary conditions analogous to those referred to in Electrostatics must be satisfied when tubes pass from one medium to another of different permeability. These are—

(1) The tangential components of the field must be the same in the two media.

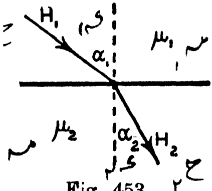


Fig. 453.

(2) The normal components of the induction must be the same in the two media.

Thus, in Fig. 453, from the first condition we have

$$H_1 \sin \alpha_1 = H_2 \sin \alpha_2;$$

and from the second condition

$$B_1 \cos \alpha_1 = B_2 \cos \alpha_2,$$

$$\text{i.e.}\quad \mu_1 H_1 \cos \alpha_1 = \mu_2 H_2 \cos \alpha_2.$$

Dividing one of these equations by the other—

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\mu_1}{\mu_2}.$$

It follows from this that when tubes pass from one medium to another of smaller permeability they are bent *towards* the normal, and when they pass into one of greater permeability they are bent *away from* the normal. The paramagnetic sphere in air (Fig. 50 (*a*)) illustrates the latter case, and the diamagnetic sphere in air (Fig. 50 (*b*)) illustrates the former.

276. Theoretical Lifting Power of a Magnet.—This may be determined as follows. Let I denote the intensity of magnetisation of the iron. Then, assuming the magnetisation to be uniform in the magnet and its attached keeper, we may take the surface densities of the charges of magnetism on the opposing surfaces of the magnet and keeper as I and $-I$, and the force of attraction of one surface on the other per unit area of surface is, as in Art. 118, given by $2\pi I^2$. Hence, if A denote the surface area of the poles, and the intensity of magnetisation at every point on the surface be the same, the lifting power of the magnet in dynes is given by $2\pi I^2 A$, where I is expressed in C.G.S. units. Since B, the magnetic induction in the iron, is equal to $4\pi I$ (H being zero), we have $I = B/4\pi$, and the lifting power $2\pi I^2 A$ can be expressed in the form

$$\left.\begin{array}{c}\text{Lifting power or Attraction between pole}\\ \text{and armature}\end{array}\right\} = \frac{B^2 A}{8\pi}\ \text{dynes}.$$

277. Magnetometer Method of Measuring Permeability, etc.—The method outlined below is applicable to specimens in the form of wires or rods, the length being at least 300 or 400 diameters, in which case the effective magnetising force H_1 in the specimen may be taken as identical with H, the magnetising force as calculated from the dimensions of the magnetising solenoid and the current strength.

Consider first a long magnet placed vertically (Fig. 454), with its upper pole N on a level with, and at distance d due magnetic east of, a magnetometer needle situated at O. The field F at O, in the direction NO, due to this magnet is

$$F = \frac{m}{d^2} - \frac{m}{d^2 + l^2} \cos \theta$$

$$= \frac{m}{d^2} - \frac{m}{d^2 + l^2} \cdot \frac{d}{\sqrt{d^2 + l^2}}$$

$$= m \left(\frac{1}{d^2} - \frac{d}{(d^2 + l^2)^{\frac{3}{2}}} \right), \quad \dots(1)$$

where m is the pole strength of the magnet; if the magnet is very long the term $d/(d^2 + l^2)^{\frac{3}{2}}$ may be neglected, and we get

$$F = \frac{m}{d^2} \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (2)$$

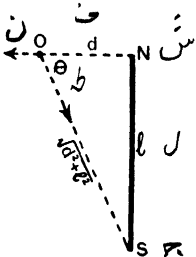


Fig. 454.

Now consider the magnetometer needle at O deflected (say) through an angle $\theta°$. If h denotes, in this case, the horizontal component of the earth's field, then from the relation $F = h \tan \theta$ (Art. 33) we have

$$m \left(\frac{1}{d^2} - \frac{d}{(d^2 + l^2)^{\frac{3}{2}}} \right) = h \tan \theta,$$

$$\therefore m = \frac{h \tan \theta}{\dfrac{1}{d^2} - \dfrac{d}{(d^2 + l^2)^{\frac{3}{2}}}} = \frac{d^2 (d^2 + l^2)^{\frac{3}{2}}}{(d^2 + l^2)^{\frac{3}{2}} - d^3} h \tan \theta \dots (3)$$

or, using the approximate relation $F = m/d^2$, we have

$$m = d^2 h \tan \theta \quad \dots\dots\dots\dots\dots\dots \quad (4)$$

But if I be the intensity of magnetisation and a the cross-sectional area, $I = m/a$, i.e. $m = Ia$; thus

$$I = \frac{d^2 (d^2 + l^2)^{\frac{3}{2}}}{a\{(d^2 + l^2)^{\frac{3}{2}} - d^3\}} h \tan \theta \quad \text{...........} \quad (5)$$

or

$$I = \frac{d^2}{a} h \tan \theta \quad \text{......................} \quad (6)$$

and, since all the terms on the right-hand side are known ($h = \cdot 18$), the intensity of magnetisation I is determined. The relation (6) is the one invariably used in practice.

So far we have considered a permanent magnet NS, but the student will now be in a position to understand the actual experiment. The specimen of iron or steel is placed vertically inside the magnetising solenoid SS (Fig. 455), its upper end being on a level with the magnetometer needle. The solenoid is in series with a battery B, galvanometer G, and variable resistance R. Since the current in the solenoid affects the needle, the circuit also includes a compensating coil C; with the specimen removed and a current passing the position of C is adjusted until the magnetometer needle is unaffected, in which case C is cancelling the effect of the solenoid. The specimen is now inserted, a current passed, and the intensity of magnetisation I calculated from the relations (5) or (6) above.

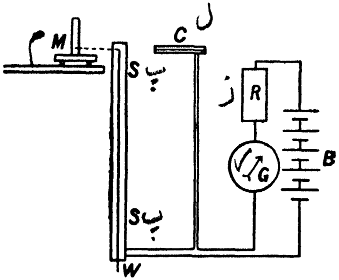


Fig. 455.

The effective magnetising force H_1 may be taken as equal to H, that due to the solenoid, and the latter is given by the expression

$$H = \frac{4\pi S i}{l_1},$$

where S is the total number of turns of the solenoid, l_1

its length, and i the current in absolute units given by the galvanometer G. Thus we have for the specimen in question (using the relation (6) for simplicity)—

$$I = \frac{d^2h \tan\theta}{a},$$

$$\kappa = \frac{I}{H} = \frac{d^2hl_1 \tan\theta}{4\pi Sia},$$

$$B = H + 4\pi I = \frac{4\pi\,(Sia + d^2hl_1 \tan\theta)}{l_1 a},$$

$$\mu = \frac{B}{H} = \frac{Sia + d^2hl_1 \tan\theta}{Sia}.$$

As the rod is vertical it is subject to the inductive action of V, the vertical component of the earth's field ($V = \cdot 43$); for this reason the magnetising field is, to be exact, $\frac{4\pi Si}{l_1} \pm V$; this should replace H in the above. A better method is to eliminate the effect of V by winding over the first solenoid a second solenoid connected to a separate battery; with no current in the solenoid SS, the unmagnetised rod is inserted, and the direction and strength of the current in this second solenoid so arranged that the needle remains in its true zero position; this earth neutralising current is then kept constant throughout the experiment.

In practice a series of values of θ and i are obtained, starting with i small and increasing to a maximum; this is done by varying the resistance R. Readings are also taken as i is reduced from the maximum to zero. By having a reversing key in the circuit i can be reversed and increased to a maximum, then reduced to zero, and again increased to a maximum in the original direction. Thus all the details are obtained for the plotting of *B.H.*, *I.H.*, and hysteresis curves.

278. Ballistic Method of Measuring Permeability, etc.—In this method the specimen is in the form of a ring, in which case, as there are no free poles, the effective magnetising force in the specimen is identical with that calculated from the dimensions of the solenoid and the strength of the current.

The apparatus and connections are shown in Fig. 456. The magnetising solenoid is closely wound over the ring and is connected to a reversing key S, a battery F, an adjustable resistance R, and an ammeter or galvanometer A. An additional coil D, consisting of a few turns is wound over a part of the ring and connected to a ballistic galvanometer BG, and a standard inductor—say an earth coil EC.

The earth coil or inductor is first placed with its plane horizontal, and rapidly rotated through 180° about a horizontal axis in the magnetic meridian, and the throw θ of the ballistic galvanometer is noted. If V be the vertical component of the earth's field, n the number of turns and a the face area of EC, and R the resistance of the circuit made up of D, BG, and EC, the quantity induced is $2Vna/R$; but the quantity induced is also given by $k\theta$, where k is a constant for the galvanometer; hence

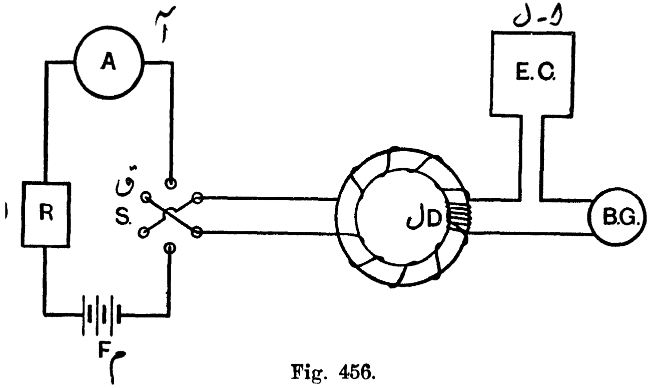


Fig. 456.

$$k\theta = \frac{2Vna}{R}, \qquad \therefore\ k = \frac{2Vna}{R\theta}.$$

A small current i_1 is now started in the magnetising solenoid; this produces a flux density (say) b_1 in the specimen, and a momentary induced quantity circulates through the ballistic galvanometer circuit, producing a throw θ_1. If a_1 be the cross-sectional area of the ring the total flow of induction is $b_1 a_1$, and if there are n_1

turns in the coil D the quantity induced is $b_1a_1n_1/R$; but the induced quantity is also $k\theta_1$, hence

$$\frac{b_1a_1n_1}{R} = k\theta_1,$$

$$\therefore\ b_1 = \frac{kR\theta_1}{a_1n_1},$$

and, substituting the value of k previously obtained,

$$b_1 = \frac{2Vna}{n_1a_1}\frac{\theta_1}{\theta};$$

thus b_1 is known.

The current is now increased by a *further* step i_2 producing an *additional* flux density b_2; if θ_2 be the galvanometer throw—

$$b_2 = \frac{2Vna}{n_1a_1}\frac{\theta_2}{\theta}.$$

This step by step process is repeated. If B be the final flux density and i the *final total current*—

$$B = b_1 + b_2 + b_3 + \quad = \Sigma b$$

and $$H = \frac{4\pi Si}{l},$$

where S is the number of turns in the magnetising solenoid and l the mean circumference of the ring. Knowing B and H, μ, κ, and I can be obtained from the relationships previously established, and the various curves may be plotted.

A better method is to *reverse* the field by means of the key S and observe the throw θ, which, of course, is then due to a change $2B$ instead of B for any value of the magnetising current. Further, a standard solenoidal inductor is much better than the earth inductor for the calibration part of the experiment.

279. Energy Dissipation due to Hysteresis.—In Art. 10 it is stated that the loss of energy per unit volume for a cycle of magnetisation is represented by the area of the IH hysteresis loop, or by the area of the BH hysteresis loop divided by 4π; these facts can now be readily established.

Let the moment (magnetic) of one of the molecular magnets of the specimen be M and let α be the angle between its magnetic axis and the direction of the magnetising field. The component moment in the direction of the field is $M \cos \alpha$, and the sum of these, *i.e.* $\Sigma M \cos \alpha$, for all the molecular magnets in unit volume will be the intensity of magnetisation I since the sum of the other components perpendicular to the field, viz. $\Sigma M \sin \alpha$ is zero.

Thus:—

$$\Sigma M \cos \alpha = I,$$

$$\therefore\ d\Sigma M \cos \alpha = dI,$$

$$i.e.\quad -\Sigma M \sin \alpha . d\alpha = dI.$$

Again, the couple due to the field acting on the molecular magnet of moment M when it is inclined at an angle α to the field is $MH \sin \alpha$ (Art. 20), and the work done when it moves through a small angle $d\alpha$ is couple × angle, *i.e.* $MH \sin \alpha . d\alpha$ or $-MH \sin \alpha . d\alpha$ if $d\alpha$ be a small *reduction* in the angle α. Hence *considering again the unit volume* we have:—

$$\text{Work per unit volume} = -\Sigma MH \sin \alpha . d\alpha$$

$$= H(-\Sigma M \sin \alpha . d\alpha) \text{ if } H \text{ be assumed constant,}$$

$$= HdI.$$

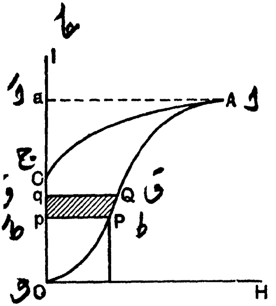


Fig. 457.

The truth of the above may also be seen as follows:—Consider a cube of the material of one centimetre side and let I be the intensity of magnetisation and H the field. This cube may be viewed as a magnet 1 cm. long and of pole strength $+I$ and $-I$. Let the intensity change from I to $I + dI$. This is equivalent to a pole of strength dI being carried from one face to the opposite face through a distance of 1 cm. Hence since H is the field, HdI is the force, and the work done for the 1 cm. path or the *change of energy per unit volume* is therefore given by the expression:—

$$\text{Work per unit volume} = HdI.$$

Now let OA (Fig. 457) be the magnetisation curve ($I.H.$) of a specimen of iron and consider a small step in the process represented by PQ. The step is supposed to be so short that the magnetising force may be assumed constant during the step and of magnitude Pp or Qq which for a very small step are equal. Let this be denoted by H. Let the intensity Op be I and let Oq be $I + dI$ so that pq represents dI. Since the work done *on* the material per unit volume in magnetising is HdI, this work for this short step is evidently represented by the shaded area $PQqp$. Clearly then *the work done on the material per unit volume* for the whole magnetisation represented by the path OA will be the sum of all the small areas such as $PQqp$ for all the short steps into which the magnetisation may be supposed to be divided, *i.e.* it will be represented by the area $OPQAaO$. Similarly if the field be reduced to zero (in which case the curve AC is obtained) the work *restored*, i.e. *the work done by the material per unit volume* will be represented by the area $AaCA$.

Hence :—

$$\left.\begin{array}{l}\text{Excess of work done } \textit{on} \text{ unit volume}\\ \quad\text{over work done } \textit{by} \text{ unit volume}\end{array}\right\} = \text{area } OPQACO.$$

Consider now the complete hysteresis loop shown in Fig. 458. From what has been said, the horizontally shaded areas represent work done *on* the material per unit volume, and the vertically shaded areas represent work done *by* the material per unit volume, and, as appears from the figure, *the excess of the work done on the material per unit volume over that done by it is represented by the area of the hysteresis loop $DAB'CD$.*

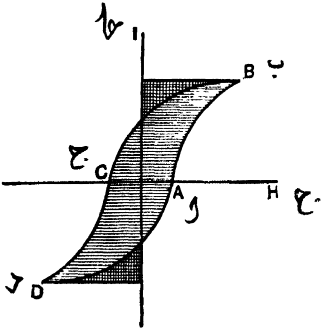


Fig. 458.

If in the hysteresis diagram the ordinates represent B

and not I, then, since $B = 4\pi I + H$, it is evident that the length of each ordinate *intercepted by the loop* will be 4π times its length on a diagram where the ordinates represent I. Hence, if we suppose the loop to be divided into an infinite number of vertical strips, the *area* of each strip in the B,H diagram will be 4π times the area of the same strip in the I,H diagram. But we have shown that the area of the loop on the I,H diagram represents the energy dissipated as heat in unit volume of the material, therefore on a B,H diagram the area of the loop divided by 4π will represent this energy.

The latter fact is also seen from the relation

$$HdB = 4\pi HdI + HdH,$$

$$\therefore \int_0 HdB = 4\pi \int_0 HdI + \int_0 HdH,$$

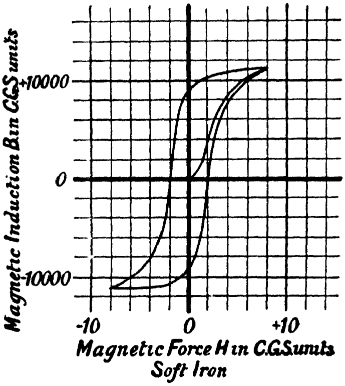


Fig. 459 (A).

where $\int_0$ is the integral round the whole path. The term $\int_0 HdH$ is zero, for if H be plotted against H the result is a straight line, for which the enclosed area is zero; hence the BH area $\int_0 HdB$ is 4π times the IH area $\int_0 HdI$.

The energy in question is dissipated as heat in the material. If d be the density of the material in grammes per c.cm., s the specific heat, and t the rise in temperature,

$$\text{Heat produced in unit volume} = dst$$

$$\text{and} \quad ,, \quad ,, \quad ,, \quad = \frac{\text{Area of } IH \text{ cycle}}{4{\cdot}2 \times 10^7},$$

$$\therefore t = \frac{\text{Area of } IH \text{ cycle}}{d \times s \times 4{\cdot}2 \times 10^7}.$$

This gives the rise in temperature per cycle of magnetisation. The work per unit volume per cycle is of the order 10,000 ergs for soft iron (annealed) and 118,000 ergs for hard steel. Fig. 459 gives hysteresis loops for soft iron wire (A) and annealed steel wire (B), and clearly brings out the marked loss of energy in the latter case.

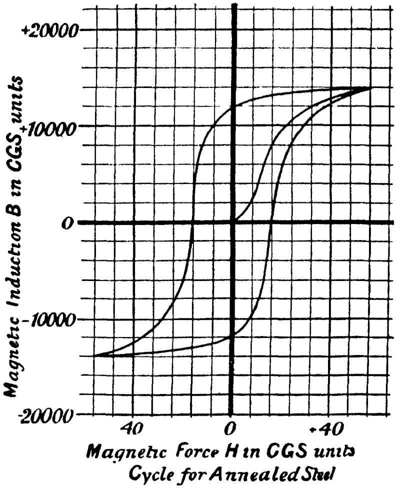


Fig. 459 (B).

280. The Magnetic Circuit. Magnetic Circuit Units. — Consider the case of an anchor ring of iron magnetised by a current passing in a wire wound closely round it as shown in Fig. 460. As previously indicated, the intensity, H, of the field inside is $4\pi SI/l$, where I is the current strength and the flux density $B = \mu H$ is $4\pi\mu SI/l$, where μ is the permeability of the iron. If a denotes the cross-sectional area of the iron, the total induction F across any section of the iron ring is given by

$$F = Ba = \frac{4\pi\mu a SI}{l},$$

$$\therefore\ 4\pi SI = \frac{l}{a\mu}F.$$

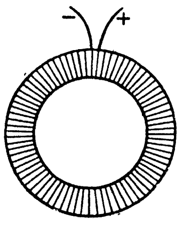


Fig. 460

Now $4\pi SI$ being equal to $4\pi SI/l$ multiplied by l evidently denotes the work done in carrying unit pole round the axis of the coil against the magnetic force $4\pi SI/l$, that is, $4\pi SI$ gives the difference of magnetic potential for the circuit of the ring. Also F

denotes the total flow of induction round the circuit of the iron ring. Comparing this magnetic circuit with an electric current circuit, $4\pi SI$ corresponds to the electromotive force and F to the current or flow of electricity. Further, comparing the above expression with the corresponding one for an electric circuit, viz.

$$E = RI,$$

it will be seen that $l/a\mu$ corresponds to R, the resistance of the circuit, and since $R = \rho l/a$, where ρ is the specific resistance, $1/\mu$ corresponds to ρ. The analogy here indicated has suggested the name **magnetomotive force** for $4\pi SI$, and magnetic resistance or **reluctance** for $l/a\mu$. Just as in the current circuit we have

$$\text{Current} = \frac{\text{Electromotive Force}}{\text{Resistance}}, \quad i.e. \ I = \frac{E.M.F.}{R},$$

so in the magnetic current we have

$$\text{Induction or Flux} = \frac{\text{Magnetomotive Force}}{\text{Reluctance}},$$

$$i.e. \quad F = \frac{M.M.F.}{Z},$$

where M.M.F. is $4\pi SI$ and Z is $l/a\mu$; it is well to remember the more detailed expression

$$F = \frac{4\pi SI}{\frac{l}{a\mu}}.$$

The term *reluctance* is used in preference to resistance, for $l/a\mu$ is not the true analogue of electrical resistance. Thus taking $1/\mu$ to correspond to specific resistance, μ should correspond to specific conductivity. But μ, the permeability of the medium, really corresponds to specific inductive capacity, and *the real analogue of a piece of magnetised material is a polarised dielectric*, the positive and negative charges at each end of a tube of induction in the dielectric corresponding to the positive and negative charges at each end of the tubes of induction passing through the magnetised material. Again, whilst the resistance of a conductor at a given temperature is independent of the current strength, the reluctance in the case of the magnetic circuit varies considerably with the induction or flux. Further, the electric circuit involves the expenditure of energy *as long as the current lasts*; in the magnetic circuit energy is only required to establish the flux.

From the preceding we may define "**magnetomotive force**" as being *numerically equal to the work done on a unit north pole in carrying it once round a closed magnetic path against the magnetic force*; "**reluctance**" may be defined as *the magnetic resistance offered by the substance to the passage of magnetic flux*, and hence "**reluctivity**" may be defined as *the magnetic resistance offered to the passage of magnetic flux between two opposite faces of a centimetre cube of the substance.*

The student must carefully distinguish between magnetic or magnetising force and magnetomotive force; thus in the preceding the magnetic or magnetising force is $4\pi SI/l$, but the magnetomotive force is $4\pi SI$.

The idea of a magnetic circuit in which *magnetomotive force* and *reluctance* are related by a law corresponding to Ohm's law has been found extremely useful in the design of dynamos, motors, and other electromagnetic machines. Thus in a magnetic circuit made up of soft iron, cast iron, and air gaps, if μ_1, μ_2, μ_3 denote the permeabilities, l_1, l_2, l_3 the lengths, and a_1, a_2, a_3 the cross-sections of these materials, we have for the circuit

$$\text{Total reluctance} = \frac{l_1}{a_1\mu_1} + \frac{l_2}{a_2\mu_2} + \frac{l_3}{a_3\mu_3}.$$

If the magnetising coil for the circuit has S turns and carries a current I, then

$$\text{Magnetomotive Force} = 4\pi SI,$$

and the flow of induction round the circuit is given by

$$\mathbf{F} = \frac{4\pi SI}{\dfrac{l_1}{a_1\mu_1} + \dfrac{l_2}{a_2\mu_2} + \dfrac{l_3}{a_3\mu_3}},$$

or generally

$$\mathbf{F} = \frac{4\pi SI}{\Sigma \dfrac{l}{a\mu}}.$$

This result brings out very clearly how a very narrow air gap in a magnetic circuit may, because of the comparatively small value of μ for air (unity), cause a large increase in the reluctance, and therefore seriously diminish the total flow of induction round the circuit.

If the current be in amperes, $M.M.F. = 4\pi SI/10 = 1{\cdot}257SI$, and the above expression becomes

$$F = \frac{1{\cdot}257SI}{\Sigma \dfrac{l}{a\mu}},$$

$$\therefore\ SI = {\cdot}8F\Sigma\frac{l}{a\mu}.$$

The product SI of the number of turns and the current in amperes is called the "ampere turns."

Example. *Find the number of ampere turns needed to send a flux of* 40,000 *unit tubes of induction through a closed soft iron ring formed of square iron of* 2 *cm. side bent into a circle of outside diameter* 22 *cm. Find also the ampere turns for the same flux if the ring be cut into two halves, the two halves being separated by two air gaps each of* 1 *mm.*

CASE 1.

$$a = 2 \times 2 = 4 \text{ sq. cm.}$$

$$l = \pi \times \text{mean diameter} = 20\pi \text{ cm.}$$

$$B = \frac{F}{a} = \frac{40000}{4} = 10000.$$

From curves for soft iron, when $B = 10000$ $\mu = 1900$,

$$\therefore\ SI = {\cdot}8 \times 40000 \times \frac{20\pi}{4 \times 1900},$$

i.e. Ampere turns = **264.**

CASE 2.

$$a \text{ (for air gaps)} = 4 \text{ sq. cm.}$$

$$l \quad ,, \quad ,, \quad = {\cdot}2 \text{ cm.}$$

$$\mu \quad ,, \quad ,, \quad = 1.$$

$$\therefore\ SI = {\cdot}8 \times 40000\left(\frac{20\pi}{4 \times 1900} + \frac{{\cdot}2}{4 \times 1}\right),$$

i.e. Ampere turns = **1864.**

The following **magnetic circuit units** have been suggested, but they are not extensively used:—

Magnetic Field .... **Gauss** (Art. 18)

Magnetic Induction or Flux (**F**) ...... **Maxwell** (formerly the **Weber**)

Magnetomotive Force	**Gilbert**
Reluctance	**Oersted**

$$\text{Maxwells} = \frac{\text{Gilberts}}{\text{Oersteds}}.$$

281. Work on the Engineering Side.—The importance of the determination of the magnetic properties of iron, etc., in electrical engineering practice has led to the introduction of several commercial instruments and methods for the purpose; thus we have Ewing's Hysteresis Tester, Hopkinson's Bar and Yoke, and Ewing's Double Bar and Yoke, Ewing's Permeability Bridge, Drysdale and Thompson's Permeameters, etc. Space will permit only of a brief reference to one or two here.

(1) EWING'S HYSTERESIS TESTER.—This is shown diagrammatically in Fig. 461. *M* is a permanent magnet pivoted at *Y* so that it can move in a plane at right angles to the plane of the paper; it carries a pointer *P* which moves over the scale *S'*, a control weight *W*, and a vane *V* which, moving in oil, renders the instrument dead beat. The specimen *S* is pivoted at *X* and can be rotated between the poles of *M* by means of the hand wheel *H*. As the specimen rotates it is magnetised by the field of *M*, the lag in magnetisation causing it to drag the magnet *M* after it, thereby producing a certain deflection of *P* over the scale *S'*; the greater the lag the greater the deflection, so that the latter is proportional to the hysteresis loss in the specimen. The instrument is calibrated by means of a specimen of known hysteresis supplied with it.

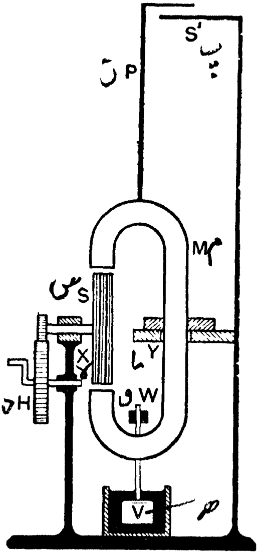


Fig. 461.

(2) THOMPSON'S PERMEAMETER.—This is shown in Fig. 462. The rod under test is a good fit at *A*, where it passes through the wrought-iron yoke, and it also makes good contact with the yoke at *C*. The rod carries a magnetising coil, so that the magnetising force *H* is known. With the current passing the contact at *C* is broken by a pull on the spring balance, in which case it can be shown that

$$B = 1317\sqrt{\frac{P}{S}} + H,$$

where P is the pull in pounds and S the area in square inches of the surface of contact at C; thus B is determined.

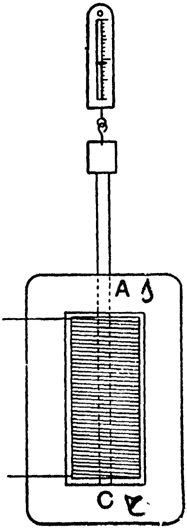


Fig. 462.

(3) HOPKINSON'S BAR AND YOKE.—The bar to be tested carries a magnetising coil and is fixed into a heavy soft iron yoke (Fig. 463). A secondary coil S encircles the middle of the rod and is connected to a ballistic galvanometer, and the throw θ is observed when the magnetising current is reversed. If l_1, a_1, and μ_1 be the length, cross-sectional area, and permeability of the rod and l_2, a_2, and μ_2 the corresponding values for the yoke, we have

$$\text{Reluctance} = \frac{l_1}{a_1\mu_1} + \frac{l_2}{a_2\mu_2},$$

$$\text{Magnetomotive Force} = 4\pi Si/10,$$

where S and i are the turns and current in amperes in the magnetising coil; thus

$$\mathbf{F} = \frac{4\pi Si}{10\left(\frac{l_1}{a_1\mu_1} + \frac{l_2}{a_2\mu_2}\right)}.$$

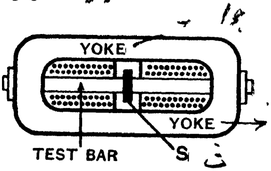


Fig. 463.

But $\mathbf{F} = Ba_1$; hence

$$B = \frac{4\pi Si}{10\left(\frac{l_1}{\mu_1} + \frac{l_2 a_1}{a_2\mu_2}\right)},$$

and, as B is known from the throw of the ballistic, μ_1 is determined.

282. Work on the Experimental Side.—Much experimental work has been done on the determination of κ and μ, especially for bodies other than the ferromagnetics.

(1) ROWLAND'S EXPERIMENTS.—Rowland determined the susceptibility of crystals of bismuth and calc spar by finding the time of vibration of suitable specimens in a magnetic field.

Theory shows that

$$t = 2\pi\sqrt{\frac{d}{-\kappa(P+Q)}},$$

where t is the period, d the density, and P and Q factors depending on the variation of the field along and perpendicular to the axis. The variation was found by means of a small inductor and a ballistic galvanometer; thus κ was determined.

(2) CURIE'S EXPERIMENTS.—Curie has carried out a series of elaborate experiments on the determination of κ and the effect of temperature, the basis of the method being to attach the body under test to a torsion arm and to measure the torsion necessary to keep it at a given point on the field of an electromagnet. The ratio of the susceptibility to the density was called by Curie the "**Specific coefficient of magnetisation (γ).**"

For most diamagnetics it was found that γ does not vary with the field, nor does it vary with the temperature except in the cases of bismuth and antimony, which show a decrease in γ with rise in temperature. For bismuth the following has been found to hold between the temperatures 20° C. and 273° C.—

$$10^6\gamma = 1{\cdot}35\,[1 - {\cdot}00115\,(t - 20)].$$

In the case of paramagnetics Curie found that "*the product of the specific coefficient of magnetisation and the absolute temperature is constant,*" which is another form of **Curie's Law** referred to in Art. 11. Glass if paramagnetic at ordinary temperatures was found to become diamagnetic at high temperatures. For iron the value of γ decreases with the temperature, and at 1000° C. it was found to be nearly the same as for air at 20° C., but the decrease is by no means regular.

(3) FLEMING AND DEWAR'S EXPERIMENTS.—These experimenters investigated the susceptibility of oxygen at very low temperatures; their results indicate that at $-$ 182° C. the value of $10^6\kappa$ is $+$ 324, which gives a value for μ of 1·004.

(4) WEISS' EXPERIMENTS.—Weiss determined the value of I for iron, nickel, and magnetite at the temperature of liquid hydrogen and found that the moment per gramme molecule is, in each case, a small integral multiple of 1123·5. Weiss calls this the "**magneton gramme,**" and concludes that in each atom of these substances there is at least one fundamental magnet having a constant moment ($M = 16{\cdot}4 \times 10^{-22}$) the same for each of them, and this fundamental magnet he calls the "**magneton.**" Calculation shows that iron, nickel, and magnetite contain 11, 3, and 7 of these magnetons per molecule respectively.

Exercises XVIII.

Section B.

(1) Describe the ballistic method of determining the relation between the magnetisation and the magnetic force of iron in the form of a ring. (B.E. Hons.)

(2) Explain in detail some method of measuring the energy lost through magnetic hysteresis, and describe some of the principal results of experiment. (B.E. Hons.)

Section C.

(1) Define magnetic force, H, and magnetic induction, B. Show that the energy per unit volume of the magnetic field between two plane poles is given by $\frac{BH}{8\pi}$. (B.Sc.)

(2) Prove that the area of the H,B cycle denotes 4π times the energy dissipated per c.c. of metal during each magnetic cycle. (B.Sc.)

(3) A long solenoid of ten turns to the centimetre contains an iron rod 2 cm. diameter, cut in two. Find the force necessary to separate the two halves of the rod when a current of 3 amperes is flowing in the solenoid; given that on reversing this primary circuit 60 micro-coulombs flow through a secondary circuit of ten turns wound round the iron rod, and having a total resistance of 100 ohms. (B.Sc.)

(4) Define magnetic induction, B, and magnetising force, H, and give an account of an experimental method of determining their relation for a specimen of soft iron. (B.Sc.)

(5) Show in what features a magnetic circuit is analogous to an electric circuit. In what respects does the analogy fail? (B.Sc.)

(6) In what respects do the magnetic properties of iron and steel differ? Define the terms intensity of magnetisation (I), induction (B), and magnetic force (H). How do you obtain the relation $B = H + 4\pi I$? (B.Sc.)

(7) Discuss the effects of and the methods of dealing experimentally with free magnetism in the measurement of magnetic permeability. Find an expression for the effect of a thin radial crevasse upon the magnetisation of an anchor ring. (B.Sc. Hons.)

(8) Show that the work per cubic centimetre performed in taking a specimen of iron through a cycle of magnetisation is represented by the area of the cycle upon the $H - I$ diagram. Describe how the energy loss due to hysteresis may be determined for a given material. (B.Sc. Hons.)

CHAPTER XX.

ALTERNATING CURRENTS AND TRANSFORMERS.

283. Circuit with Resistance but no Inductance or Capacity.—In the case of the rotating coil of Art. 241 the following relationships have been established :—

Induced E.M.F.	
Instantaneous Value (E).	Maximum Value (E_0).
$E = SAH\omega \sin \alpha$ $= SAH\omega \sin \omega t$ $E = 2\pi nSAH \sin \omega t$ $E = \dfrac{2\pi SAH}{T} \sin \dfrac{2\pi t}{T}$	$E_0 = SAH\omega$ $E_0 = 2\pi nSAH$ $E_0 = \dfrac{2\pi SAH}{T}$

$$\text{i.e. } E = E_0 \sin \omega t.$$

The current I at any instant is given by

$$I = \frac{E_0}{R} \sin \omega t = I_0 \sin \omega t,$$

where I_0 denotes the maximum value of the current, viz. E_0/R. Thus both the current and the E.M.F. are harmonic with the same period and phase, but of different amplitudes.

The meaning of the terms used above and in succeeding sections may again be noted. If at a particular moment an alternating E.M.F. has a certain value and is just going to commence a certain set of variations, then the time which elapses between this instant and the moment when the E.M.F. has the same value and is going to commence an identical set of variations is called the **period** (T), and the number of periods in one second is called the **frequency** (n), the maximum value of the E.M.F. being called the **amplitude.** In certain cases the maximum current occurs *after* the maximum E.M.F. and the current is then said to **lag,** whilst in certain other cases it occurs *before* and is then said to **lead**; the lag and lead are spoken of as the **phase difference.**

284. Circuit with Resistance and Inductance.—

Perhaps the simplest method of investigation is to suppose a simple harmonically varying or alternating current exists in a circuit, and to find what must be the nature of the electromotive force in the circuit in order to produce such a current. Let the current in the circuit be denoted by

$$I = I_0 \sin \omega t.$$

Hence dI/dt is $I_0\omega \cos \omega t$, and the induced electromotive force due to self-induction, given by LdI/dt, is $LI_0\omega \cos \omega t$. Now by applying Ohm's Law to the circuit we get

$$E = IR + L\frac{dI}{dt},$$

$$\therefore\ E = I_0 R \sin \omega t + LI_0\omega \cos \omega t,$$

$$\text{i.e. } E = I_0 [R \sin \omega t + L\omega \cos \omega t].$$

This may be written

$$E = I_0 \sqrt{R^2 + L^2\omega^2} \sin (\omega t + \phi),$$

if $\dfrac{R}{\sqrt{R^2 + L^2\omega^2}} = \cos \phi$ and $\dfrac{L\omega}{\sqrt{R^2 + L^2\omega^2}} = \sin \phi$,

that is, if

$$\tan \phi = \frac{L\omega}{R} = \frac{2\pi n L}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Here the maximum value of E is $I_0 \sqrt{R^2 + L^2\omega^2}$, and if this be denoted by E_0 we have

$$E = E_0 \sin (\omega t + \phi).$$

Hence when the current varies harmonically in a circuit the electromotive force in the circuit also varies harmonically, with the same frequency, and the maximum value of the electromotive force is $\sqrt{R^2 + L^2\omega^2}$ times the maximum value of the current. The phase of the electromotive force is, however, in advance of that of the current, or *the phase of the current lags behind that of the electromotive force by $\phi/2\pi$ of a complete period, the angle of lag ϕ being such that tan $\phi = L\omega/R$,* where R is the resistance, L the self-inductance of the circuit, and $\omega/2\pi$ the frequency (n) of the alternations of the current and the electromotive force in the circuit. Hence, if the electromotive force in the circuit be given by

$$E = E_0 \sin \omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

then for the current we have, from the above,

$$I = I_0 \sin (\omega t - \phi) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$\text{i.e. } I = \frac{E_0}{\sqrt{R^2 + L^2\omega^2}} \sin (\omega t - \phi) \quad \ldots\ldots\ldots\ldots \quad (4)$$

since I_0, the maximum current, is given by

$$I_0 = \frac{E_0}{\sqrt{R^2 + L^2\omega^2}} = \frac{E_0}{\sqrt{R^2 + 4\pi^2 n^2 L^2}} \quad \ldots\ldots\ldots \quad (5)$$

The quantity $\sqrt{R^2 + L^2\omega^2}$ is called the **impedance** of the circuit (it may be defined as the effective resistance encountered by an alternating current) and the quantity $L\omega$ is called the **reactance** of the circuit. These quantities and their relations are best remembered by the triangle shown in Fig. 464; the hypotenuse is the impedance, the base the resistance, the vertical the reactance, and the angle ϕ between the base and the hypotenuse is such that tan $\phi = 2\pi nL/R$, *i.e.* it is the angle of lag.

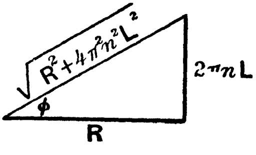


Fig. 464.

The student may prefer the following treatment of the preceding :—

In a circuit containing inductance and resistance, if E follows the simple harmonic law, $E = E_0 \sin \omega t$—

$$IR + L\frac{dI}{dt} = E_0 \sin \omega t.$$

Now assume as a trial solution that

$$I = Z \sin (\omega t - \phi),$$

where Z and ϕ are to be determined. Substituting—

$$ZR \sin (\omega t - \phi) + ZL\omega \cos (\omega t - \phi) = E_0 \sin \omega t \text{ .... } (6)$$

$$\therefore ZR (\sin \omega t \,.\, \cos \phi - \cos \omega t \,.\, \sin \phi)$$
$$+ ZL\omega (\cos \omega t \,.\, \cos \phi + \sin \omega t \,.\, \sin \phi) = E_0 \sin \omega t,$$

i.e. $(ZR \cos \phi \,.\, \sin \omega t + ZL\omega \sin \phi \,.\, \sin \omega t)$

$$- (ZR \sin \phi \cos \omega t - ZL\omega \cos \phi \,.\, \cos \omega t) = E_0 \sin \omega t.$$

Hence, equating coefficients—

$$ZR \cos \phi + ZL\omega \sin \phi = E_0 \text{ .... } (7)$$

$$- ZR \sin \phi + ZL\omega \cos \phi = 0 \text{ .... } (8)$$

Squaring and adding—

$$Z^2R^2 + Z^2L^2\omega^2 = E_0{}^2, \quad \text{i.e. } Z^2 (R^2 + L^2\omega^2) = E_0{}^2,$$

$$\therefore Z = \frac{E_0}{\sqrt{R^2 + L^2\omega^2}},$$

and $$I = \frac{E_0}{\sqrt{R^2 + L^2\omega^2}} \sin (\omega t - \phi) \text{ .... } (9)$$

From (8) $$\tan \phi = \frac{L\omega}{R} = \frac{2\pi nL}{R} \text{ .... } (10)$$

Further $$I_0 = \frac{E_0}{\sqrt{R^2 + L^2\omega^2}} = \frac{E_0}{\sqrt{R^2 + 4\pi^2n^2L^2}} \text{ .... } (11)$$

285. Graphic Representation.—The relation between the *impressed* (E), the *effective* (IR), and the *self-induced* (LdI/dt) electromotive forces expressed by the equation

$$E = IR + L\frac{dI}{dt}$$

can be represented graphically. Each of the quantities varies harmonically and may therefore be represented by the projection of the radius of a circle revolving with a period equal to that of the electromotive force alternations. The self-induced electromotive force $- L\frac{dI}{dt}$ lags a quarter of a period behind the effective electromotive force IR, for when one is at its maximum the other is at zero value.

If in Fig. 465, therefore, OE represents the maximum value of the effective electromotive force and ON the maximum value of the induced electromotive force, then OP drawn parallel to NE will represent the impressed electromotive force. For, since $OPEN$ is a parallelogram, the projection of OE on any straight line through O is equal to the sum of the projections of OP and ON. Hence, if

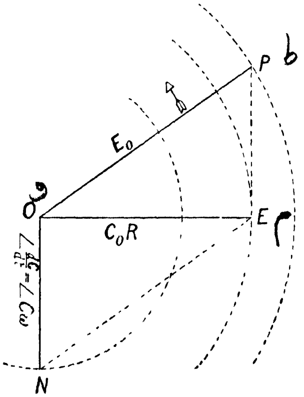


Fig. 465.

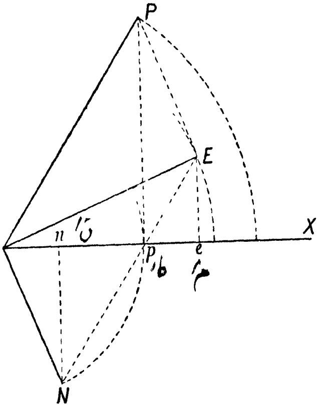


Fig. 466.

the figure $OPEN$ be supposed to revolve round O in the direction of the arrow, the projections of OP, ON and OE on the line OX (Fig. 466) give at any instant the instantaneous values of E, $-L\frac{dI}{dt}$ and IR, and as geometrically the projection Oe always equals $Op + On$ the construction expresses the relation

$$E - L\frac{dI}{dt} = IR,$$

$$\textit{i.e.} \quad E = IR + L\frac{dI}{dt}.$$

The angle POE evidently represents the angle ϕ of the above formula and indicates the lag of the current alternations behind those of the electromotive force. Note that in the Figures the letter C is used for current.

286. The "Square Root of the Mean Square" Current.

—As the strength of an alternating current varies from instant to instant it is necessary to specify how the strength of such a current is measured; the mean or average value of the current for a half-period is not used for this purpose, but a value known as the *square root of the mean square* current is used.

Let the curve *AOB* (Fig. 467) represent the current for a half-period. Since the abscissae of the curve represent time and the ordinates current, the area between the curve *AOB* and the line *AB* evidently represents the *quantity* of electricity carried during the half-period. For in any very short time represented by *ab* the quantity that passes is given by *ac* × *ab*, the area of the shaded strip standing on *ab*, and this is true for every short interval into which the half-period represented by *AB* can be divided. Hence the area *AOBA* represents the total quantity of electricity carried by the current during the half-period, and the average current can be obtained by dividing this quantity by the time of the half-period.

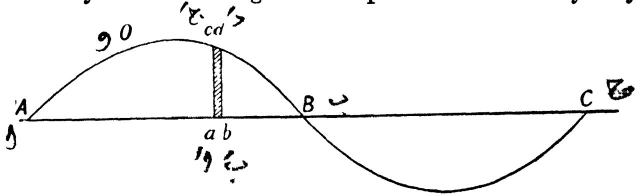


Fig. 467.

Case 1. *To show that the average current is* $2/\pi$, i.e. ·637 *of the maximum current.*—Let t be the half-period, I_a the average current, and I the instantaneous current; then

$$I_a = \frac{\Sigma I \, . \, dt}{t} = \frac{I_0}{t}\int_0^t \sin \omega t \, . \, dt$$

$$= \frac{I_0}{\omega t}\int_0^\pi \sin \omega t \, . \, d\,(\omega t) = \frac{I_0}{\omega t}\Big[-\cos \omega t\Big]_0^\pi$$

$$= \frac{I_0}{\omega t}\Big[-\cos \pi + \cos 0\Big] = \frac{2I_0}{\omega t}$$

$$= \frac{2}{\pi} I_0, \text{ since } \omega = \frac{\pi}{t} \text{ (see also Art. 241).}$$

The *square root of the mean square value,* the value which is always used to measure the current, may be found as follows. Let a curve *AOB* (Fig. 468) be drawn for the half-period, with abscissae representing time but with ordinates representing the *square* of the current at any instant. Then the area of the strip *abcd* represents the value of I^2dt, where dt is the very short time represented by *ab* and I^2 is the square of the current strength represented by *ac* or *bd*. The area of the curve *AOBA* therefore represents $\Sigma I^2 . dt$ for the half-period, and the value of $\Sigma I^2 . dt$ divided by the time of the half-period gives the *mean square* value for the current, and the square root of this gives the square root of the mean square value.

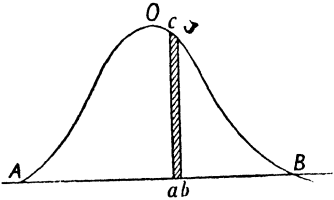


Fig. 468.

The reason for taking this value will readily be understood. The heat developed by a current I in a time dt in a resistance R is $I^2R . dt$ or $R . I^2dt$, hence the heat developed in a resistance R by an alternating current during a half-period is $R\Sigma I^2dt$, and if I be the square root of the mean square current for the half-period t then $I^2t = \Sigma I^2 . dt$. The heat developed in the resistance R by the alternating current whose square root of mean square value is I is therefore the same as the heat developed by a steady continuous current of strength I in the same time. That is, *the alternating current is measured by the strength of the steady current which would produce the same heating effect.* The square root of the mean square value is called the **virtual** value and is the value given by alternating current instruments.

Case 2. *To show that the virtual current is* $1/\sqrt{2}$, i.e. ·707 *of the maximum current.*—If I denote the virtual current,

$$I = \left(\frac{\Sigma I^2 dt}{t}\right)^{\frac{1}{2}} = \left(\int_0^t \frac{I_0^2 \sin^2 \omega t . dt}{t}\right)^{\frac{1}{2}}$$

$$= \left(\frac{I_0^2}{t}\int_0^t \sin^2 \omega t \,.\, dt\right)^{\frac{1}{2}} = \left(\frac{I_0^2}{\omega t}\int_0^{\pi} \sin^2 \omega t \,.\, d(\omega t)\right)^{\frac{1}{2}}$$

$$= \left(\frac{I_0^2}{\omega t} \cdot \frac{\pi}{2}\right)^{\frac{1}{2}}, \qquad (\pi = wt),$$

i.e. $$I = \frac{I_0}{\sqrt{2}} \text{ and } I_0 = \sqrt{2}I.$$

From the above it follows that a **virtual ampere** is one which will produce the same heat in a resistance as a steady current of one ampere will produce in the same time, and **a virtual volt** is one which when applied to the ends of a resistance results in the same heating effect as a steady pressure of one volt applied for the same time.

In practical work the only values ever dealt with are virtual values, *i.e.* instrument readings, except in very special cases. Clearly, however, the formulae of Art. 284 can be used with virtual values, as is illustrated by the following example:—

Example. *An alternating pressure of* 100 *volts* (*virtual understood*) *is applied to a circuit of resistance* 0·5 *ohm and self-induction* 0·01 *henry, the frequency being* 50 *cycles per second* (50 ~). *What will be the reading of an ammeter in the circuit and what will be the lag in time between pressure and current ?*

We have $$I = \frac{100}{\sqrt{(0{\cdot}5)^2 + 4\pi^2 (50)^2 (0{\cdot}01)^2}};$$

$$\therefore I = 31{\cdot}2 \textbf{ amperes} \text{ (nearly)}.$$

$$\therefore \textbf{Ammeter reading} = \mathbf{31{\cdot}2 \text{ amperes.}}$$

Again, $$\tan \phi = \frac{2\pi \times 50 \times 0{\cdot}01}{0{\cdot}5}$$

$$= 6{\cdot}28.$$

$$\therefore \boldsymbol{\phi = 81^\circ} \textbf{ (nearly)}$$

and $$\textbf{lag in time} = \tfrac{81}{360} \times \tfrac{1}{50} \textbf{ second} = \tfrac{1}{222} \textbf{ second.}$$

In this example 31·2 amperes will be the virtual or ammeter reading, because the pressure is given in virtual volts. The current will fluctuate between 31·2 × $\sqrt{2}$ amperes, first one way and then the other, and it will reach this maximum value $\frac{1}{222}$ second after the pressure each time reaches its maximum value of 100 × $\sqrt{2}$ volts.

287. Power.—The rate of doing work or the *activity* of a current is, in the case of a steady current, given by EI, where E is the E.M.F. and I the current. In an alternating current both E and I vary harmonically and they differ in phase. From the relations given above we have

$$E = E_0 \sin \omega t,$$

$$I = I_0 \sin (\omega t - \phi).$$

Hence the activity at any instant is given by

$$EI = E_0 I_0 \sin \omega t \sin (\omega t - \phi) = \tfrac{1}{2} E_0 I_0 [\cos \phi - \cos (2\omega t - \phi)].$$

During a half-period the angle $2\omega t$ and therefore $(2\omega t - \phi)$ changes by 2π, and the mean value of its cosine for the half-period is, therefore, zero; hence the power or activity of the alternating current in which the alternations follow the simple harmonic law may be given as

$$\tfrac{1}{2} E_0 I_0 \cos \phi$$

for a time equal to any integral number of half-periods. Since

$$\tfrac{1}{2} E_0 I_0 = \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} = EI,$$

we may write

$$\text{Power} = EI \cos \phi.$$

Also, since $\cos \phi = \dfrac{R}{\sqrt{R^2 + L^2\omega^2}}$,

$$\text{Power} = EI \frac{R}{\sqrt{R^2 + L^2\omega^2}} = \frac{E^2 R}{R^2 + L^2\omega^2}.$$

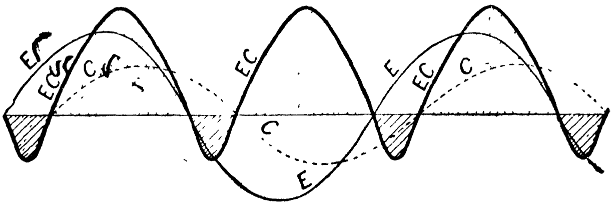


Fig. 469.

If we plot a power curve for a given alternating current by plotting the values of EI as ordinates to times as abscissae, as in Fig. 469, we notice that the work done by the current as represented by the *area* of the curve is partly positive and partly negative, that is, during certain short periods occurring periodically the current generator gives out power, and at other periods it absorbs power. The shaded areas in the figure represent the power given out, the dotted areas the power taken in, and the difference of the two sets of areas during a complete period represents the total work done during the period. Note that in the Figure the letter C is used for current.

288. Choking Coils.—It is evident that for a coil with an iron core the quantity $L\mu$ must generally be large, for μ may be very great. Hence, if an alternating current be sent through a low resistance coil with a soft iron core, the retardation of the current relative to the electromotive force in the coil will be practically a quarter period, for $\cos\phi = \dfrac{R}{\sqrt{R^2 + L^2\omega^2}}$ or, if R be small and L large, $\cos\phi = R/L\omega$, and $R/L\omega$ being very small ϕ is nearly $\pi/2$ and $\cos\phi$ therefore nearly zero. This shows that $EI\cos\phi$, the power absorbed by the coil, is very small, although on account of its large impedance, $\sqrt{R^2 + L^2\omega^2}$, it admits of the passage of ~~only~~ a small current through it. Such a coil is usually called a *choking coil* on account of its effect on the current, and choking coils are largely used in alternating current circuits for the purpose of adjusting the current to any required value without waste of energy such as takes place when a regulating resistance is placed in the circuit of a continuous or alternating current.

289. Circuit with Resistance, Inductance, and Capacity.—When a harmonically varying electromotive force $E_0 \sin \omega t$ is applied to a circuit containing a resistance R with inductance L, and a capacity C in series, we have

$$IR + L\frac{dI}{dt} + V = E_0 \sin \omega t \quad \ldots\ldots\ldots\ldots \text{(1)}$$

where I is the current and V the potential difference on the condenser. If Q be the charge on the condenser at any instant $V = Q/C$, and we get

$$IR + L\frac{dI}{dt} + \frac{Q}{C} = E_0 \sin \omega t \quad \ldots\ldots\ldots\ldots\ldots \text{(2)}$$

Now, as in Art. 284, let $I = Z \sin(\omega t - \phi)$, *i.e.* $$\frac{dQ}{dt} = Z \sin(\omega t - \phi),$$

$$\therefore\ Q = -\frac{Z}{\omega}\cos(\omega t - \phi).$$

Substituting in the equation above,

$$ZR\sin(\omega t - \phi) + LZ\omega\cos(\omega t - \phi) - \frac{Z}{C\omega}\cos(\omega t - \phi) = E_0 \sin \omega t,$$

$$\therefore\ ZR\sin(\omega t - \phi) + \left(L\omega - \frac{1}{C\omega}\right)Z\cos(\omega t - \phi) = E_0\sin\omega t.$$

This is identical with equation (6) of Art. 284, except that in place of $L\omega$ we have $\left(L\omega - \frac{1}{C\omega}\right)$; as in that section the solution is

$$Z = \frac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

$$I = \frac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}\sin(\omega t - \phi) \ldots (3)$$

$$\tan\phi = \frac{L\omega - \frac{1}{C\omega}}{R} = \frac{2\pi nL - \frac{1}{2\pi nC}}{R} \ldots\ldots\ldots\ldots (4)$$

$$I_0 = \frac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} = \frac{E_0}{\sqrt{R^2 + \left(2\pi nL - \frac{1}{2\pi nC}\right)^2}} \ldots (5)$$

and clearly, if $L\omega = 1/C\omega$, *i.e.* if $2\pi nL = 1/2\pi nC$, capacity and inductance neutralise, the angle ϕ is 0°, and

$$I = \frac{E_0 \sin\omega t}{R}, \qquad I_0 = \frac{E_0}{R}.$$

Thus the current and the applied electromotive force are in the same phase, and the current has the same value as in a circuit of resistance R free from inductance and capacity. If $L\omega > 1/C\omega$ the current *lags*, if $L\omega < 1/C\omega$ the current *leads*.

The condition for the neutralisation of the inductance effect by the capacity effect may be obtained without working out the general solution above. From (1) capacity and inductance neutralise if $L\frac{dI}{dt} = -V$. Now V will be a harmonically varying quantity with the same period as the electromotive force, and may be written $V = V_0 \sin(\omega t - \phi)$. Further, $I = dQ/dt = d(CV)/dt = C.dV/dt$; hence $I = C\omega V_0 \cos(\omega t - \phi)$ and $dI/dt = -C\omega^2 V_0 \sin(\omega t - \phi)$. Thus the required condition $L.dI/dt = -V$ becomes

$$LC\omega^2 V_0 \sin(\omega t - \phi) = V_0 \sin(\omega t - \phi),$$

i.e.

$$LC = \frac{1}{\omega^2},$$

$$\therefore L\omega = \frac{1}{C\omega}, \quad \text{or} \quad 2\pi nL = \frac{1}{2\pi nC},$$

as shown above.

Further, since $\omega = 2\pi/T$, where T is the period of the applied electromotive force, we have

$$LC = \frac{T^2}{4\pi^2}, \quad i.e.\ T = 2\pi\sqrt{LC}.$$

This, as shown in Chapter XXII., is the period for electrical oscillations in the circuit; hence *when the period of the applied electromotive force is the same as that of the circuit for electric oscillations the effect of capacity neutralises the effect of inductance.*

If the circuit possesses capacity and resistance *but no inductance,* it may be shown that the law becomes

$$I_0 = \frac{E_0}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}} = \frac{E_0}{\sqrt{R^2 + \frac{1}{4\pi^2 n^2 C^2}}} \quad \text{......} \quad (6)$$

The effect of capacity alone is to make the current *lead* in front of the pressure, and

$$\text{tan angle of lead} = \frac{1}{2\pi nCR} \quad \text{............} \quad (7)$$

The student should prove these statements.

Clearly, if in a circuit containing capacity, resistance and inductance $2\pi nL = 1/2\pi nC$, then $2\pi nL/R = 1/2\pi nCR$, *i.e.* the lag due to inductance is equal to the lead due to capacity and the two neutralise as already proved.

289a. Typical A.C. Problems.—The following worked examples will assist the student to grasp the details of the preceding sections :—

(1) *A coil of wire of negligible resistance and inductance* ·02 *henry, and a wire of zero inductance and resistance* 12 *ohms are in series. If the impressed E.M.F. be* 130 *volts and the frequency* 40 *cycles per second, determine* (*a*) *the current,* (*b*) *the lag,* (*c*) *the P.D. across the inductive resistance,* (*d*) *the P.D. across the non-inductive resistance.*

(*a*) For the current we have :—

$$I = \frac{E}{\sqrt{R^2 + (2\pi nL)^2}} = \frac{130}{\sqrt{12^2 + (2\pi \times 40 \times \cdot 02)^2}} = \mathbf{10\ amperes.}$$

(*b*) The current lags behind the impressed E.M.F. by an angle ϕ such that :—

$$\tan\phi = \frac{2\pi nL}{R} = \frac{2\pi \times 40 \times \cdot 02}{12} = \cdot 417,$$

$$\therefore \boldsymbol{\phi = 22^\circ} \text{ and } \mathbf{Lag\ in\ time} = \frac{22}{360} \times \frac{1}{40} = \frac{\mathbf{1}}{\mathbf{654}} \mathbf{\ second.}$$

(*c*) P.D. (E_1) on the non-inductive resistance is obtained in the usual way, viz. :—

$$I = \frac{E_1}{R}, \text{ i.e. } 10 = \frac{E_1}{12}, \quad \therefore E_1 = \mathbf{120\ volts.}$$

(*d*) P.D. (E_2) for the inductive resistance is obtained from :—

$$I = \frac{E_2}{\sqrt{O + (2\pi nL)^2}}, \quad \text{i.e. } 10 = \frac{E_2}{2\pi nL}, \quad \therefore E_2 = \mathbf{50\ volts.}$$

Note that the relation between the three pressures is represented by the triangle OPE of Fig. 465, OP representing 130, OE representing 120, and PE representing 50 ($130^2 = 120^2 + 50^2$). In the case of a *direct* current we would have, of course, $E_1 + E_2 = E$.

(2) *An A.C. arc lamp needing* 50 *volts is put across mains at a P.D. of* 80 *volts, and a choking coil is in series with the lamp. The lamp takes* 10 *amperes and the frequency is* 50 *per second. Find* (1) *the pressure produced by the choking coil reaction,* (2) *the self-inductance of the choking coil.*

The solution is given by the triangle of Fig. 465. Here OP is the impressed or applied E.M.F. (80), OE is the effective pressure (50) and ON or PE is the pressure produced by the choking coil. Clearly :—

$$PE^2 = OP^2 - OE^2, \quad \text{i.e. } PE = \sqrt{OP^2 - OE^2}.$$

$\therefore$ **Pressure produced by choking coil**

$$= \sqrt{80^2 - 50^2} = \mathbf{62 \cdot 5\ volts.}$$

Again, from (5) Art. 284, $E_0 = \sqrt{(I_0R)^2 + (L\omega I_0)^2}$, and from Fig. 465, $OP = \sqrt{OE^2 + PE^2}$. Here OP represents E_0, OE represents I_0R, and PE represents $L\omega I_0$, *i.e.* $2\pi nLI_0$. But $PE = 62{\cdot}5$ volts.

$\therefore$ **Self-inductance of choking coil**

$$= L = \frac{62{\cdot}5}{2\pi nI_0} = \frac{62{\cdot}5}{2\pi \times 50 \times 10} = {\cdot}\mathbf{02}\ \textbf{henry.}$$

(3) *A certain choking coil of negligible resistance takes a current of* 8 *amperes if supplied at* 100 *volts, at* 50 *periods, per second. A certain non-inductive resistance, under the same conditions, carries* 10 *amperes. If the two are transferred to a supply system working at* 150 *volts, at* 40 *periods per second, what total current will they take* (*a*) *if joined in series,* (*b*) *if joined in parallel?*

The value of R for the non-inductive resistance and the value of L for the inductive resistance must first be found. For R we have $R = 100/10 = 10$ ohms, and for L we have

$$I = \frac{E}{\sqrt{R^2 + (2\pi nL)^2}}, \quad i.e.\ 8 = \frac{100}{\sqrt{0 + (2\pi nL)^2}} = \frac{100}{2\pi nL}.$$

$$\therefore L = \frac{100}{2\pi \times 50 \times 8} = \frac{1}{8\pi}\ \text{henry.}$$

Case (*a*) *:—The two in series.*

$$I_1 = \frac{E}{\sqrt{R^2 + (2\pi nL)^2}} = \frac{150}{\sqrt{10^2 + \left(2\pi \times 40 \times \frac{1}{8\pi}\right)^2}}$$

$$= \mathbf{10{\cdot}6}\ \textbf{amperes.}$$

Case (*b*) *:—The two in parallel.*

If i_1 be the current for the non-inductive resistance,

$$i_1 = \frac{E}{R} = \frac{150}{10} = 15\ \text{amperes.}$$

If i_2 be the choking coil current,

$$i_2 = \frac{E}{\sqrt{0 + (2\pi nL)^2}} = \frac{150}{2\pi \times 40 \times \frac{1}{8\pi}} = 15\ \text{amperes.}$$

But the current i_2 has a phase relationship of 90° with i_1, and the resultant current I_2 is given by :—

$$I_2 = \sqrt{i_1^2 + i_2^2} = \sqrt{15^2 + 15^2}$$

$$= \mathbf{21{\cdot}2}\ \textbf{amperes,}$$

The total currents in the two cases are therefore 10·6 and 21·2 amperes respectively.

(4) *Explain the effect of applying to the terminals of a condenser the resistance of which is many megohms an alternating E.M.F. What E.M.F. is required to drive* 10 *virtual amperes through a*

circuit containing a condenser of resistance 1200 *megohms and capacity* 22 *microfarads, the frequency being* 80? (*C. & G.*)

Note that the condenser resistance in problems of this type should *not* be regarded as an ordinary resistance *in series* with the condenser.

If E be the impressed E.M.F., the current of 10 amperes is the resultant of two currents, viz. a current x equal to $\frac{E}{1200 \times 10^6}$ in step with E, and a current y, equal to $E\Big/\frac{1}{2\pi nC}$, or $2\pi nCE$, the two differing in phase by 90°; hence:—

$$10 = \sqrt{x^2 + y^2}$$

$$100 = x^2 + y^2 = \left(\frac{E}{1200 \times 10^6}\right)^2 + \left(2\pi nCE\right)^2$$

$$100 = E^2\left\{\left(\frac{1}{1200 \times 10^6}\right)^2 + \left(2\pi \times 80 \times \frac{22}{10^6}\right)^2\right\}.$$

The first term within the bracket may evidently be neglected in this problem, and we get:—

$$E^2 = \frac{100}{\left(2\pi \times 80 \times \frac{22}{10^6}\right)^2}$$

$$i.e.\ E = \frac{10}{2\pi \times 80 \times \frac{22}{10^6}} = \mathbf{904\ volts.}$$

The answer to the first part of the question will be gathered from preceding pages.

290. Transformers.—Transformers are used in modern electrical practice (1) for converting an A.C. of high electromotive force and low current strength into a current of lower electromotive force and higher current value; (2) for converting an A.C. of low electromotive force and high current strength into a current of higher electromotive force and lower current value. The former is called a "step down" and the latter a "step up" transformer.

A step down transformer consists essentially of two coils of wire—a primary coil and a secondary coil—coiled round an endless core of soft iron (Fig. 470). The primary coil carries the current to be transformed, and consists of a large number of turns of highly insulated wire of the thickness necessary to carry the current transmitted to it. The secondary coil consists of fewer turns of insulated wire of thickness sufficient to carry the low tension current resulting from the transformation, and is connected directly with the system to which this current is to be supplied. The iron core is usually a core of soft iron wire or a system of thin plates of soft iron, arranged to give an endless magnetic circuit and to avoid loss of energy by Foucault eddy currents.

The principle of action of the transformer will be evident from its construction. The alternating current in the primary coil magnetises the iron core and sets up an alternating flow of induction in the magnetic circuit. This variation of the induction through the secondary coil gives an induced alternating current in the secondary coil. The period of this induced current is the same as that of the current in the primary coil, but the two currents are not, in general, in the same phase.

The construction of a step up transformer is similar to the above, save that the primary has few turns of wire and the secondary many turns (*e.g.* induction coil).

Transformer Theory.—(1) The ratio of transformation depends mainly upon the ratio of the number of turns in the two coils, and practically we have

$$\frac{\text{Pressure in secondary}}{\text{Pressure in primary}} = \frac{\text{Number of turns in secondary}}{\text{Number of turns in primary}};$$

but, since energy cannot be gained by the transformation, we have also

$$E_2 I_2 = E_1 I_1,$$

where E_2 and I_2 are the pressure and current in the secondary, and E_1 and I_1 the corresponding values for the primary. This is only true if we neglect losses due to heating and magnetic frictions.

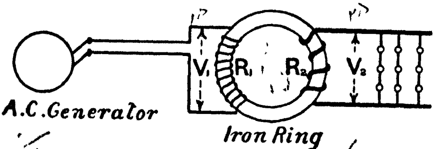


Fig. 470.

Assuming the above, a simple relation may be established between the primary and secondary terminal pressures V_1 and V_2 (Fig. 470). Let E_1 be the E.M.F. operating in the primary, E_2 that in the secondary, and n_1 and n_2 the number of turns on each. Let $n_1/n_2 = K$, then $E_1/E_2 = n_1/n_2 = K$; and, since $E_1I_1 = E_2I_2$, we have

$$\frac{I_1}{I_2} = \frac{E_2}{E_1} = \frac{1}{K}.$$

From the figure we see that

$$E_1 = V_1 - I_1R_1$$

and

$$E_2 = V_2 + I_2R_2;$$

$$\therefore \frac{E_1}{E_2} = K = \frac{V_1 - I_1R_1}{V_2 + I_2R_2}.$$

$$\therefore KV_2 + KI_2R_2 = V_1 - \frac{R_1I_2}{K},$$

and

$$\therefore V_2 = \frac{V_1}{K} - I_2\left(\frac{R_1}{K^2} + R_2\right).$$

This expression gives the secondary terminal pressure in terms of the primary pressure and other constants.

Obviously, since R_1, R_2, and K are constants, the value of V_2 is not a definite fraction of V_1, *but is lessened the more I_2 is increased*—that is, the more current is taken from the secondary. Hence, if we desire the secondary pressure to remain constant, we must arrange matters so that the primary pressure rises somewhat as the secondary load is increased.

(2) When two coils are wound together in such a way that the flux of induction through one all passes through the other there is a simple relation between the coefficients of self-induction and the coefficient of mutual induction of the coils. For, if L_1, L_2 and M denote these coefficients, and n_1 and n_2 the number of turns in the coils, we have, for a current I in the coil to which L_1 and n_1 refer, the flow of induction through one turn measured by L_1I/n_1, and the flow of induction through the other coil is $\frac{L_1In_2}{n_1}$.

That is, $\frac{n_2}{n_1}L_1 = M$. Similarly, beginning with the other coil we get $\frac{n_1}{n_2} L_2 = M$, and we therefore have

$$L_1 L_2 = M^2 \text{ and } \frac{L_1}{L_2} = \left(\frac{n_1}{n_2}\right)^2.$$

In the coils of a transformer these relations hold only approximately, for there is always some magnetic leakage between the two coils, and the variation of the permeability of the iron core with the intensity of magnetisation complicates the result. If, however, we neglect the variation of permeability it may be assumed that $(M^2 - L_1 L_2)$ is a small quantity, and that $\frac{L_1}{L_2} = \left(\frac{n_1}{n_2}\right)^2$ is roughly correct.

(3) If the currents in the primary and secondary coils at any instant be denoted by x and y respectively, and if we represent the harmonic electromotive force applied to the primary circuit by $E \sin \omega t$, the resistances of the primary and secondary coils by R_1 and R_2, and the coefficients of induction by L_1, L_2 and M, as above, we have the following relations for the two coils:—

$$L_1 \frac{dx}{dt} + M \frac{dy}{dt} + R_1 x = E \sin \omega t,$$

$$M \frac{dx}{dt} + L_2 \frac{dy}{dt} + R_2 y = 0.$$

Taking the first of these relations for the primary coil, $L_1 \frac{dx}{dt}$ gives the back electromotive force due to self-induction, $M \frac{dy}{dt}$ the back electromotive force due to mutual induction, and $R_1 x$ the potential difference which determines the current x in the coil. The sum of these quantities must evidently be equal to the impressed electromotive force $E \sin \omega t$. Similarly, for the second relation the sum of the corresponding terms is equal to zero, since there is no impressed electromotive force acting on the secondary coil.

From these equations the values of x and y are found to be

$$x = \frac{E}{\sqrt{R^2 + (L\omega)^2}} \sin (\omega t + \alpha),$$

$$y = \frac{M\omega}{\sqrt{(L_2\omega)^2 + R_2^2}} \cdot \frac{E}{\sqrt{R^2 + (L\omega)^2}} \sin (\omega t + \beta),$$

where

$$L = L_1 - \frac{(M\omega)^2 L_2}{(L_2\omega)^2 + R_2^2},$$

$$R = R_1 + \frac{(M\omega)^2 R_2}{(L_2\omega)^2 + R_2^2},$$

$$\tan \alpha = \frac{L\omega}{R},$$

$$\tan (\alpha - \beta) = \frac{R_2}{L_2\omega}.$$

These results are readily verified by simple differentiations and substitutions. They show that the currents in the two coils are periodic currents of the same period as the impressed electromotive force in the primary coil, but differing in phase. Also, from the form of the value for x, it is evident that the current in the primary coil may be taken as the current in a coil of self-induction L and resistance R. That is, the apparent effect of the secondary coil is to *decrease* the self-inductance of the primary coil from L_1 to $(L_1 - aL_2)$ and to *increase* the resistance from R_1 to $(R_1 + aR_2)$, where

$$a = \frac{(M\omega)^2}{(L_2\omega)^2 + R_2^2}.$$

(4) When the secondary coil of a transformer is open R_2 is infinite, and a therefore is zero, and L and R reduce to their real values L_1 and R_1.

When the secondary coil is closed R_2 is usually small, and may be negligibly small compared with $L_2\omega$ if ω is large. If this is so, the value of a reduces to $\left(\frac{M}{L_2}\right)^2$ and therefore

$$L = L_1 - \frac{M^2}{L_2}$$

and

$$R = R_1 + \left(\frac{M}{L_2}\right)^2 R_2.$$

Also, since $\tan (\alpha - \beta) = \frac{R_2}{L_2\omega}$, we have, when R_2 is negligibly small compared with $L_2\omega$, $\alpha - \beta = 0$ or π. If we also assume that the transformer is so well designed that $M^2 - L_1L_2$ is negligibly small, then we may take $M^2 = L_1L_2$ as approximately true, and this gives us $L_1 - \frac{M^2}{L_2} = 0$ or $L = 0$, $R = R_1 + \frac{L_1}{L_2}R_2$, and $\alpha = 0$. Hence, when the secondary coil is closed the apparent self-inductance of the primary coil is practically zero, the apparent resistance R is equal to $R_1 + \frac{L_1}{L_2}R_2$, where R_1, R_2, L_1 and L_2 are as defined above, and the current in the primary coil is in the same phase as the impressed electromotive force.

Under the above conditions, the current in the primary coil is $\frac{E \sin \omega t}{R}$, and the power spent in the coil is $\frac{1}{2}E^2/R$. This indicates that the power absorbed increases as R decreases, that is, as R_2 decreases, for $R = R_1 + \frac{L_1}{L_2}R_2$ and R_1, L_1 and L_2 are fixed. This is true, however, only if $M^2 = L_1L_2$; when $M^2 - L_1L_2$ is a small but appreciable quantity the power absorbed can be shown to increase as R_2 decreases to a critical value which depends upon the frequency of the primary current. Below this critical value of R_2 the power absorbed decreases.

(5) If we apply the above approximations to the values of x and y previously given we get

$$x = \frac{E \sin \omega t}{R}, \qquad \text{where} \qquad R = R_1 + \frac{L_1}{L_2}R_2$$

and

$$y = \frac{M}{L_2} \cdot \frac{E \sin \omega t}{R} = \frac{M}{L_2}x.$$

We have seen that $M = \frac{n_1}{n_2}L_2$, and therefore $\frac{M}{L_2} = \frac{n_1}{n_2}$. Hence we have $\frac{y}{x} = \frac{n_1}{n_2}$, that is—

$$\frac{\text{Secondary current}}{\text{Primary current}} = \frac{\text{Number of turns in primary}}{\text{Number of turns in secondary}}.$$

The electromotive force in the secondary coil is

$$R_2 y \text{ or } R_2 \cdot \frac{M}{L_2} \cdot \frac{E \sin \omega t}{R}.$$

Substituting $R_1 + \frac{L_1}{L_2} R_2$ for R this gives

$$\frac{R_2 M}{L_2 (R_1 + \frac{L_1}{L_2} R_2)} E \sin \omega t,$$

and assuming R_1 to be small compared with $\frac{L_1}{L_2} R_2$ we get

$$\frac{L_2}{M} E \sin \omega t$$

as the electromotive force in the secondary circuit. Since $L_2/M = n_2/n_1$ we see that

$$\frac{\text{Secondary E.M.F.}}{\text{Primary E.M.F.}} = \frac{n_2}{n_1} = \frac{\text{Number of turns in secondary}}{\text{Number of turns in primary}}.$$

(6) The power absorbed in the primary is, as given above, $\frac{1}{2} \frac{E^2}{R}$. The power given out in the secondary is evidently given by $\frac{1}{2} \frac{M^2 E^2 R_2}{L_2^2 R^2}$. Hence the efficiency of the transformer is therefore given by

$$\frac{M^2 R_2}{L_2^2 R} \quad \text{or} \quad \frac{M^2}{L_2^2} \cdot \frac{R_2}{R_1 + \frac{L_1}{L_2} R_2}$$

If R_1 be taken as small compared with $\frac{L_1}{L_2} R_2$, this result reduces to $\frac{M^2}{L_1 L_2}$. This is unity when $M^2 = L_1 L_2$, and is nearly unity in a well designed transformer.

آخر ہم سمجھے

290a. Miscellaneous.—(1) The principle of the alternating current dynamo, or **alternator** has been dealt with in Chapter XVII. In practice the field magnets are sometimes stationary and the armature rotates, while in other types the armature is stationary and the field magnets rotate; hence to avoid confusion the rotating part is called the **rotor** and the stationary part the **stator**. The field poles of an alternator must of course be separately magnetised by say a direct current machine.

In **single-phase alternators** all the conductors are connected in series, and the ends joined to the two slip rings. In **two-phase alternators** there are two different sets of conductors, the constituents of one set being spaced half way between the constituents of the other set, so that the E.M.F. in one set is a maximum when it is zero in the other; thus there is a phase difference of 90°, and there are four terminals to the machine. In **three-phase alternators** there are three sets of conductors arranged so that, while set (1) is producing the maximum E.M.F., set (2) is producing E.M.F. lagging 120° behind set (1), and set (3) is producing E.M.F. lagging 120° behind set (2), and therefore 240° behind set (1). Instead of six terminals, the starting point of all three windings is usually a common junction; this is earthed to the shaft of the generator, and the other three ends after forming the windings are connected to *three terminals* on the machine.

(2) Since the power supplied (direct current) to a circuit is EI watts, and the power wasted in a main of resistance R ohms is I^2R watts, it is advisable to keep the current as small as possible (thus reducing the I^2R loss) while transmitting the same power, and the best method of doing this is to *raise the pressure.* For example, we can transmit 100 amperes at 100 volts or 1 ampere at 10,000 volts, and in each case the *power* is the same; but in the second case, since the current is $\frac{1}{100}$ of its first value, the loss in the conductor, if the size of the latter remains unaltered, is $\frac{1}{10000}$ of its former value. Hence, if long distances have to be covered the rule is to transmit at a high pressure, and, since smaller currents flow, to have a thinner conductor, which results in economy of copper.

In such cases it is usual to generate alternating current at an ordinary pressure, raise to a high pressure, say 6,000 volts, by step-up transformers, transmit at that pressure to the far end, and then transform down again by step-down transformers. (If D.C. is required, we pass the A.C. into a special machine called a rotary converter and produce D.C. by its means). In modern stations for traction purposes it is frequently arranged that the alternators generate direct at 6,000 volts, thus saving the first transformer with its attendant losses.

For such very high pressure transmission alternating current is always selected, for owing to insulation difficulties at the commutator it is impossible to make a satisfactory D.C. dynamo to generate at a very big pressure.

(3) Alternating and rapidly altering currents tend to confine themselves to the surface of the conductor and this is more marked the higher the frequency. Hence, conductors for such should have a large surface compared with the cross section, a thin flat ribbon being a good pattern to adopt. This is known as the "skin effect," and on account of it **the effective resistance of a conductor is increased.** In the case of a straight circular wire carrying alternating current of very high frequency it can be shown that:—

$$R = R_s\sqrt{\frac{\pi^2 n\mu r^2}{\rho}}$$

where R = effective resistance, R_s = resistance for steady currents, ρ = specific resistance in absolute units, r = radius in cms. and n = frequency.

(4) Alternating current and pressure curves may be seen or photographed by means of an **oscillograph** which is really a dead beat moving coil galvanometer. One type consists of a phosphor bronze strip which passes over a pulley, the two halves of the strip hanging down between the poles of a powerful magnet. The two bottom ends of the strip are fastened to terminals and the pulley to a spring so that a considerable tension is maintained in the strip. A mirror is attached to the two limbs of the strip. On passing a current one limb moves inwards, the other outwards, thus rotating the mirror, the deflection of which at any instant is proportional to the current at that instant. The dead beat character is increased by the presence of an oil bath. In order to obtain a record of the movement of the mirror we may receive the spot of light reflected from it on a photographic plate which is moving in a direction at right angles to the direction of vibration so that the curve connecting "current" and "time" is obtained on the plate. Frequently there are two strips, the "current strip" which is shunted, and the "pressure strip" which is in series with a high resistance, so that both current and pressure curves can be obtained simultaneously.

Exercises XIX.

Section B.

(1) Distinguish between the mean value and the root mean square value of an alternating current, and find the relation between them.

Prove that the power absorbed by a coil traversed by an alternating current is $EC \cos \theta$, where E and C are the root mean square values of the E.M.F. and current respectively, and θ is the difference in phase between these two quantities. (B.E. Hons.)

(2) Explain why the primary current in a transformer, such as an ordinary house transformer, is so much greater when the secondary circuit is closed. (B.E. Hons.)

Section C.

(1) Describe the construction of an electrostatic voltmeter. An electrostatic voltmeter gives deflections of 15, 18, and 21 scale divisions for constant potentials of 50, 60, and 70 volts respectively. What deflections will be produced by an alternating electromotive force $E \sin pt$ (*a*) when the amplitude E is 70 volts, and (*b*) when E is 90 volts? (B.Sc.)

(2) An alternating E.M.F. of 200 volts and 50 periods per second is applied to a condenser in series with a 20 volt 5 watt metal filament lamp. Find the capacity of the condenser required to run the lamp. (B.Sc. Hons.)

CHAPTER XXI.

THEORY OF UNITS.

291. Dimensions of Units.—The dimensions of any quantity express by a formula the extent to which the fundamental units of mass (M), length (L), and time (T) are involved in the unit selected to measure the given quantity. Thus *velocity*, from the usual relation in Mechanics, is distance/time, and the unit of velocity is such that the body moves through a distance of one centimetre in one second; this fact, written as a dimensional equation, becomes

$$[\text{Velocity}] = \frac{L}{T} = LT^{-1} \quad or \quad [v] = LT^{-1}.$$

Again, *acceleration* is defined as rate of change of velocity, and a body has unit acceleration if its velocity changes by unity in unit time; thus for the dimensional equation we have

$$[\text{Acceleration}] = \left[\frac{v}{t}\right] = LT^{-2} \quad or \quad [a] = LT^{-2}.$$

Further, in connection with *force* it is shown in Mechanics that $f = ma$, where f denotes the force, m the mass, and a the acceleration; and unit force is defined as that force which develops unit acceleration in unit mass; thus for the dimensional equation we get

$$[\text{Force}] = [m][a] = MLT^{-2} \quad or \quad [f] = MLT^{-2}.$$

In a similar manner, remembering that *work* = force × distance, we have

$$[\text{Work}] = ML^2T^{-2} \quad or \quad [W] = ML^2T^{-2}.$$

Examples. (1) *Show how to convert poundals into dynes.*

The *poundal* "fundamental" units are 1 pound, 1 foot, and 1 second; and the *dyne* units are 1 gramme, 1 centimetre, and

1 second. Further, 1 pound = 453·59 grammes, and 1 foot = 30·48 cm. Dimensions of force = $MLT^{-2} = ML/T^2$.

Hence, to change poundals into dynes we must multiply by $\frac{453{\cdot}59 \times 30{\cdot}48}{1 \times 1} = 13825$, *i.e.* **1 poundal = 13825 dynes.**

(2) *Show how to convert foot-poundals into ergs, and find the number of watts in a horse-power.*

In this case the dimensions are $ML^2T^{-2} = ML^2/T^2$ and the multiplier becomes $\frac{453{\cdot}59 \times (30{\cdot}48)^2}{1 \times 1} = 421390$, *i.e.* **1 foot-poundal = 421390 ergs.**

Again, 1 H.P. = 550 foot-pounds per sec. = 550 × 32·2 foot-poundals per sec. = 550 × 32·2 × 421390 ergs per sec. But 1 watt = 10^7 ergs per sec.

$$\therefore \mathbf{1\ H.P.} = \frac{550 \times 32{\cdot}2 \times 421390}{10^7} = \mathbf{746\ watts.}$$

292. Dimensions of Electrostatic Units.—To obtain the dimensions of the e.s. unit *quantity* we begin with the relation $f = qq'/d^2$. Taking $q' = q$ we get $f = q^2/d^2$, or

$$q^2 = d^2 f,$$

that is $$q = d\sqrt{f}.$$

Hence $$[q] = [d] \, . \, [f]^{\frac{1}{2}}$$

$$[q] = L \, . \, (MLT^{-2})^{\frac{1}{2}}$$

or $$[q] = L \, . \, M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}.$$

That is $$[q] = M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}.$$

Again, the *potential* at any point in the electric field is the potential energy *per unit quantity of electricity.*

That is

$$[V] = \frac{[\text{Energy}]}{[\text{Quantity of electricity}]},$$

that is $$[V] = \frac{ML^2T^{-2}}{M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}},$$

or $$[V] = M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}.$$

Similarly, for *capacity* we have

$$q = VC \text{ or } C = \frac{q}{V},$$

and therefore $$[C] = \frac{[q]}{[V]},$$

that is $$[C] = \frac{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}}{M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}}$$

or $$[C] = L.$$

It will be noticed that the dimensions of capacity are the same as those of length. This does not mean that capacity and length are similar quantities. It is a result of the fact that the system of electrostatic units is a conventional one based upon the definition of unit quantity of electricity given in Art. 80. The conditions of this definition involve the suppression of the dimensions of the quantity K for air. This question will be dealt with later on.

By an extension of the method indicated above it is easy to obtain the dimensions of all the electrostatic quantities. The more important of these are given below.

Quantity	$[Q]$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$
Electric force	$[F]$	$M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$
Potential	$[V]$	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$
Capacity	$[C]$	L
Electric polarisation	$[P]$	$M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$.

293. Dimensions of Magnetic Units.—The magnetic units are based upon the definition of the unit pole given in Art. 17. Taking the relation $f = mm'/d^2$ and putting $m' = m$ we get $f = m^2/d^2$; hence

$$m^2 = fd^2$$

or $$m = d\sqrt{f}.$$

Proceeding as in the case of electrostatic units we get

$$[m] = L \, . \, M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}$$

or

$$[m] = M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}.$$

From this result as a starting point we readily obtain the dimensions of the magnetic quantities tabulated below.

Strength of pole	$[m]$	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$
Magnetic force	$[H]$	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1}$
Magnetic potential	$[P]$	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}$
Flow of force	$[F]$	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{1}$
Reluctance	$[Z$ or $S]$	L^{-1}
Magnetic moment	$[M]$	$M^{\frac{1}{2}} L^{\frac{5}{2}} T^{-1}$
Intensity of magnetisation	$[I]$	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1}$
Magnetic induction	$[B]$	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1}$

It should here be noticed that as the dimensions of K are suppressed in the electrostatic system, so the dimensions of μ are suppressed in the system of magnetic and electromagnetic units.

294. Dimensions of Electromagnetic Units.—To obtain these we start with the definition of the unit of current given in Art. 151. From this we get the relation

$$H = \frac{2\pi n I}{r},$$

where H denotes the strength of the magnetic field at the centre of the coil. This gives the dimensional relation

$$[H] = \frac{[I]}{[r]}$$

or

$$[I] = [H] \, [r].$$

That is
$$[I] = M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1} . L$$

or
$$[I] = M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}.$$

From this it is easy to build up the dimensions of the electromagnetic units given below.

Current	$[I]$	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$
Quantity	$[It]$	$M^{\frac{1}{2}}L^{\frac{1}{2}}$
Electromotive force	$[E]$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$
Resistance	$[R]$	LT^{-1}
Capacity	$[C]$	$L^{-1}T^{2}$
Inductance	$[L]$; $[M]$	L

295. Irrationality of Electromagnetic and Electrostatic Units.—It will be seen by reference to the tables given in the foregoing article that the dimensions of the *same* quantity are not the same in the two sets of units. For example, the dimensions of Quantity of Electricity are $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$ in the electrostatic system, and $M^{\frac{1}{2}}L^{\frac{1}{2}}$ in electromagnetic units; this results, as already explained, from the suppression of the dimensions of K and μ. If we include these quantities in determining the dimensions of any quantity in the two sets of units it will be possible to determine a condition that the dimensions of K and μ must satisfy in order that the dimensions of the given quantity shall be the same for the two sets of units. Taking quantity of electricity for example, we have for the electrostatic units

$$f = \frac{1}{K}\frac{q^2}{d^2}$$

or
$$q^2 = Kfd^2$$

or
$$q = \sqrt{K} . \sqrt{f} . d.$$

That is $$[q] = [\sqrt{K}]\,[\sqrt{f}]\,[d]$$

or $$[q] = [\sqrt{K}]\,M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}.$$

Similarly, for the electromagnetic units

$$[Q] = [It] = [I]\,T.$$

Now to introduce μ into $[I]$ we have

$$f = \frac{1}{\mu}\cdot\frac{m^2}{d^2}$$

or $$m = \sqrt{\mu}\,.\,\sqrt{f}\,.\,d.$$

That is, $$[m] = [\sqrt{\mu}]\,M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1},$$

then the dimensions of magnetic potential, V, are those of work per unit pole or

$$[V] = \frac{1}{[\sqrt{\mu}]}\cdot\frac{ML^2T^{-2}}{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}} = \frac{1}{[\sqrt{\mu}]}\cdot M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}.$$

Also magnetic force or strength of field H is measured by rate of change of potential with distance, and therefore

$$[H] = \frac{[V]}{L} = \frac{1}{[\sqrt{\mu}]}\,M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}.$$

And, as in Art. 294,

$$[I] = [H]\,L = \frac{1}{[\sqrt{\mu}]}\cdot M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$$

and therefore $$[Q] = \frac{1}{[\sqrt{\mu}]}\,M^{\frac{1}{2}}L^{\frac{1}{2}}.$$

If the two sets of units are consistent the two dimensions for quantity should be the same, that is

$$[\sqrt{K}]\,M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$$

and $$\frac{1}{[\sqrt{\mu}]}\cdot M^{\frac{1}{2}}L^{\frac{1}{2}}$$

should be identical. This evidently involves

$$[\sqrt{K}]\,[\sqrt{\mu}] = \frac{M^{\frac{1}{2}}L^{\frac{1}{2}}}{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}}$$

or $$[\sqrt{K\mu}] = L^{-1}T.$$

That is $$\left[\frac{1}{\sqrt{K\mu}}\right] = LT^{-1},$$

or the dimensions of $\frac{1}{\sqrt{K\mu}}$ are those of a velocity. If we work out the result for any other quantity whose dimensions can be expressed in both sets of units we always get the same result. We are therefore not able to give the dimensions of K or μ separately, but only to state that

$$\left[\frac{1}{\sqrt{K\mu}}\right] = LT^{-1} = [v].$$

If we extend the method indicated in this article it is possible to express the dimensions of the electrical quantities in terms of M, L, T, and K for the electrostatic system, and in terms of M, L, T, and μ for the electromagnetic system. If in these dimensions we suppress K or μ we get the usual electrostatic or electromagnetic dimensions. The table on p. 341 gives these dimensions for the more important quantities.

296. Practical Units.—The practical units are again given below and their magnitude in terms of electromagnetic and electrostatic C.G.S. units specified. The quantity $1/\sqrt{K\mu}$ is indicated by v, and its measure in air may be taken as 3×10^{10}.

Quantity.	Practical Unit.	Electromagnetic C.G.S. units.	Electrostatic C.G.S. units.
Electromotive Force	Volt	10^8	$10^8/v$
Resistance	Ohm	10^9	$10^9/v^2$
Current	Ampere	10^{-1}	$10^{-1}v$
Quantity of Electricity	Coulomb	10^{-1}	$10^{-1}v$
Capacity	Farad	10^{-9}	$10^{-9}v^2$
	Micro-farad	10^{-15}	$10^{-15}v^2$
Inductance	Henry	10^9	$10^9/v^2$

Quantity.	Electrostatic (M, L, T, K) Dimensions.	Electromagnetic (M, L, T, μ) Dimensions.	Quantity.	Electromagnetic (M, L, T, μ) Dimensions.	Electrostatic (M, L, T, K) Dimensions.
Specific Inductive Capacity	K	$L^{-2} T^{2} \mu^{-1}$	Permeability ...	μ	$L^{-2} T^{2} K^{-1}$
Quantity of Electricity	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} K^{\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{1}{2}} \mu^{-\frac{1}{2}}$	Quantity of Magnetism	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} \mu^{\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{1}{2}} K^{-\frac{1}{2}}$
Electromotive force Potential Difference	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} K^{-\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2} \mu^{\frac{1}{2}}$	Magnetic Potential	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} \mu^{-\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2} K^{\frac{1}{2}}$
Electric force ...	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} K^{-\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-2} \mu^{\frac{1}{2}}$	Magnetic force ...	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} \mu^{-\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-2} K^{\frac{1}{2}}$
Surface Density Electric Polarisation or Displacement	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} K^{\frac{1}{2}}$	$M^{\frac{1}{2}} L^{-\frac{3}{2}} \mu^{-\frac{1}{2}}$	Intensity of Magnetisation Magnetic Induction	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} \mu^{\frac{1}{2}}$	$M^{\frac{1}{2}} L^{-\frac{3}{2}} K^{-\frac{1}{2}}$
Capacity	$L K$	$L^{-1} T^{2} \mu^{-1}$	Inductance ...	$L \mu$	$L^{-1} T^{2} K^{-1}$
*Conductance ...	$L T^{-1} K$	$L^{-1} T \mu^{-1}$	*Resistance ...	$L T^{-1} \mu$	$L^{-1} T K^{-1}$
Current	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2} K^{\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} \mu^{-\frac{1}{2}}$	Current	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} \mu^{-\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2} K^{\frac{1}{2}}$

---

* Conductance and Resistance are reciprocals, not analogues. The apparent analogy here results from the fact that, as $[K \mu]$ are identical with $[1/v^2]$, the dimensions of the reciprocal of a quantity of dimensions $LT^{-1} K$, that is vk, must be $LT^{-1} \mu$ or $v \mu$.

297. Ratio of Electrostatic and Electromagnetic Units.—If we take the ratio of the dimensions of any electrical quantity in the ordinary electrostatic and electromagnetic units we find that the result is of the dimensions of a velocity or a power of a velocity. Thus, for quantity of electricity, the ratio of the electrostatic to the electromagnetic *dimensions* is

$$\frac{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}}{M^{\frac{1}{2}}L^{\frac{1}{2}}} \text{ or } LT^{-1} \text{ or } [v].$$

Similarly, for electromotive force or difference of potential we get

$$\frac{M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}}{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}} \text{ or } \frac{1}{LT^{-1}} \text{ or } \left[\frac{1}{v}\right]$$

and for capacity we get

$$\frac{L}{L^{-1}T^{2}} \text{ or } L^{2}T^{-2}. \text{ That is } \left(LT^{-1}\right)^{2} \text{ or } [v^{2}],$$

and so on for the other quantities.

Now let *s* and *m* be the **numbers** representing the *same quantity* on the electrostatic and electromagnetic systems. Then clearly

$$s\,[M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}K^{\frac{1}{2}}] = m\,[M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}],$$

$$\therefore \frac{s}{m}\left[LT^{-1}\right] = \left[\frac{1}{\sqrt{K\mu}}\right].$$

In this equation s/m is a pure number, $[LT^{-1}]$ means the unit of velocity; hence

$$\frac{s}{m} = \frac{1}{\sqrt{K\mu}} = v \text{ centimetres per second,}$$

$$\therefore \frac{\text{A quantity measured in e.s. units}}{\text{The same quantity measured in e.m. units}} = v.$$

It should be noted that *s* and *m* being the magnitudes of the same quantity in the two units, their ratio is *the inverse ratio* of the size of the units, *i.e.*

$$\frac{\text{The e.m. unit of quantity}}{\text{The e.s. unit of quantity}} = v,$$

$$\therefore \text{ The e.m. unit quantity} = v \text{ (the e.s. unit quantity).}$$

It was stated in Art. 151 that the e.m. unit quantity was 3×10^{10} e.s. units (this means that $v = 1/\sqrt{K\mu} = 3 \times 10^{10} =$ velocity of light).

Again, in the case of capacity, if s and m be the **numbers** representing the *same capacity* on the two systems,

$$s\,[LK] = m\,[L^{-1}T^2\mu^{-1}]$$

$$\frac{s}{m}\left[LT^{-1}\right]^2 = \left[\frac{1}{K\mu}\right],$$

$$\therefore \frac{s}{m} = \frac{1}{K\mu} = v^2,$$

$$\textit{i.e.} \quad \frac{\text{A capacity measured in e.s. units}}{\text{The same capacity measured in e.m. units}} = v^2.$$

Similarly, as above,

$$\frac{\text{The e.m. unit of capacity}}{\text{The e.s. unit of capacity}} = v^2,$$

$$\therefore \text{ The e.m. unit of capacity} = v^2 \text{ (the e.s. unit of capacity).}$$

It was stated in Art. 84 that the e.m. unit capacity was 9×10^{20} e.s. units (this again means that $1/\sqrt{K\mu} = 3 \times 10^{10} =$ velocity of light).

From the preceding it follows that in order to determine practically the value of v it is only necessary to measure the same electrical quantity in both systems of units ; the most convenient quantity in practice is capacity.

298. Determination of "v."—In order to determine v from measurements of the capacity of a condenser the following methods may be adopted :—

First Method.—A very accurately made condenser of definite geometrical form—plane, spherical, or cylindrical—is carefully measured, and its capacity calculated from its dimensions by the appropriate formula (Chapter VII.). This gives s, the *electrostatic measure* of the capacity in C.G.S. electrostatic units.

To determine the *electromagnetic measure* of its capacity the condenser is charged to a known difference of potential V and then discharged through a ballistic galvanometer. If δ denotes the angular throw of the galvanometer needle, corrected for damping, we have

$$Q = \frac{HT}{\pi G} \cdot \frac{\delta}{2}.$$

But $Q = mV$, where m is the electromagnetic measure of the capacity and V is expressed in C.G.S. electromagnetic units.

Hence we get $$mV = \frac{HT}{\pi G} \cdot \frac{\delta}{2}$$

or $$m = \frac{HT}{\pi G V} \cdot \frac{\delta}{2}.$$

The measures s and m being thus found, the value of v is given, as shown above, by the relation

$$v = \sqrt{\frac{s}{m}}.$$

Second Method.—The principle of this method (which was suggested by Maxwell and has been used by Thomson and Searle) will be gathered from Fig. 471, where C is the condenser of capacity m electromagnetic units. Imagine Q to be a vibrating piece making n contacts with X and Y per second. At each contact with X the condenser is charged and at each contact with Y it is discharged. The charge at each contact will be Em units, where E is the E.M.F. of the cell, and the *charge per second* passing through the galvanometer will be nEm units. If the time of swing of the galvanometer be long compared with $1/n$ of a second these intermittent currents

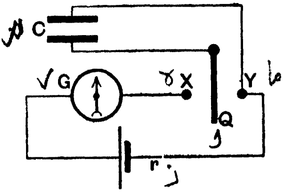


Fig. 471.

will produce the same deflection θ as a steady current of strength nEm units.

Now let the condenser and Q be replaced by a resistance R e.m. units and let this be adjusted until the deflection is again θ so that the current is again numerically equal to nEm, hence

$$nEm = \frac{E}{G + r + R},$$

$$m = \frac{1}{n(G + r + R)},$$

from which, knowing n and the various resistances, m is determined. If m is small $(G+r+R)$ will be large for medium values of n, and if $(G + r)$ is small R will be large, hence we may write

$$m = \frac{1}{nR} \quad \text{or} \quad R = \frac{1}{n \, . \, m},$$

so that the capacity m behaves like a resistance $R = 1/nm$.

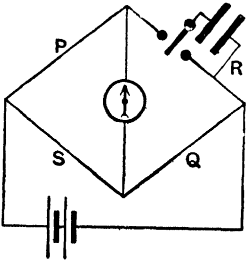


Fig. 472.

Hence Maxwell pointed out that instead of substituting a resistance for C and Q, these might be placed in one arm of the Wheatstone Bridge and a balance obtained in the usual way (Fig. 472). In this way R, i.e. $\frac{1}{n \, . \, m}$, and therefore m, is determined.

The capacity s in e.s. units is obtained from the dimensions of the condenser, and therefore $v = \sqrt{\frac{s}{m}}$ is found.

The preceding method is specially suited to the measurement of small capacities.

The values obtained for v by various methods all approximate very closely to 3×10^{10} cm. per sec. This is the same as the velocity of light, and it is shown in Chapter XXII. that the velocity v really is the velocity of transverse waves in the aether, and is therefore identical with the velocity of light.

299. Determination of the Ohm. The B.A. Method.—The problem is to find the exact specification of a resistance which is equal to 10^9 C.G.S. electromagnetic units, *e.g.* to find the length of a column of mercury of 1 sq. mm. cross-section and at the temperature of melting ice which has a resistance of 10^9 C.G.S. electromagnetic units. The general principle of the *British Association Rotating Coil Method* is as follows:—

A thin circular coil is rotated rapidly and uniformly about a vertical axis in the earth's field. As a result currents are induced in the coil, and if a magnetic needle be suspended exactly at the centre of the coil it will be deflected by the action of the induced currents in the direction of the rotation of the coil. From the constant of the coil, the speed of rotation, and the deflection of the needle it is possible to obtain an absolute measure of the resistance of the coil.

Let H denote the horizontal component of the earth's field, A the area of the coil, n the number of turns, and α the angle between the plane of the coil at any instant and the plane at right angles to the magnetic meridian. Then F, the flow of force through the coil, is given by $F = nHA \cos \alpha$ or, since $\alpha = \omega t$, by $F = nHA \cos \omega t$, and the induced electromotive force is $nHA\omega \sin \omega t$.

This gives

$$I = \frac{nHA\omega}{\sqrt{R^2 + L^2\omega^2}} \sin(\omega t - \phi).$$

But $$\sin(\omega t - \phi) = \sin \omega t \cos \phi - \cos \omega t \sin \phi.$$

Also $$\cos \phi = \frac{R}{\sqrt{R^2 + L^2\omega^2}} \quad \text{and} \quad \sin \phi = \frac{L\omega}{\sqrt{R^2 + L^2\omega^2}}.$$

Therefore we may write

$$I = \frac{nHA\omega}{R^2 + L^2\omega^2} [R \sin \omega t - L\omega \cos \omega t].$$

Now, if G be the constant of the coil, the field at the centre of the coil at right angles to its plane will be GI, and the components of this field in and at right angles to the meridian are $GI \cos \omega t$ and $GI \sin \omega t$. Substituting the value of I we get

$$\frac{GnHA\omega}{R^2 + L^2\omega^2} [R \sin \omega t \cos \omega t - L\omega \cos^2 \omega t],$$

or $$\frac{GnHA\omega}{R^2 + L^2\omega^2} \cdot \frac{1}{2} \left[R \sin 2\,\omega t - \frac{L\omega(1 + \cos 2\,\omega t)}{4} \right],$$

or $$-\frac{GnHAL\omega^2}{2(R^2+L^2\omega^2)}+\frac{GnHA\omega}{2(R^2+L^2\omega^2)}[R\sin 2\,\omega t - L\omega\cos 2\,\omega t],$$

or $$-\frac{GnHAL\omega^2}{2(R^2+L^2\omega^2)}+\frac{GnHA\omega}{2\sqrt{R^2+L^2\omega^2}}\sin(2\omega t-\phi$$

for the component parallel to the meridian, and

$$\frac{GnHA\omega}{R^2+L^2\omega^2}[R\sin^2\omega t - L\omega\cos\omega t\sin\omega t],$$

or $$\frac{GnHA\omega}{R^2+L^2\omega^2}\,\tfrac{1}{2}[R(1-\cos 2\,\omega t)-L\omega\sin 2\,\omega t],$$

or $$\frac{GnHAR\omega}{2(R^2+L^2\omega^2)}-\frac{GnHA\omega}{2(R^2+L^2\omega^2)}[R\cos 2\,\omega t + L\omega\sin 2\,\omega t],$$

or $$\frac{GnHAR\omega}{2(R^2+L^2\omega^2)}-\frac{GnHA\omega}{\sqrt{R^2+L^2\omega^2}}\cos(2\omega t-\phi)$$

for the component at right angles to the meridian.

These expressions for the components of the field due to the induced currents consist each of two parts, a constant part and a periodic part of period double that of the rotation of the coil, and if the needle suspended at the centre be assumed to have a sufficiently large moment of inertia the effect of the periodic parts of the components will have a negligible effect in determining its position, and we may write the components effectively determining the position of the needle as

$$H-\frac{GnHAL\omega^2}{2(R^2+L^2\omega^2)}$$

parallel to the meridian and

$$\frac{GnHAR\omega}{2(R^2+L^2\omega^2)}$$

at right angles to the meridian.

If therefore δ be the observed deflection of the needle from the meridian, we have

$$\tan\delta=\frac{\dfrac{GnHAR\omega}{2(R^2+L^2\omega^2)}}{H-\dfrac{GnHAL\omega^2}{2(R^2+L^2\omega^2)}}$$

or $$\tan\delta=\frac{GnAR\omega}{2(R^2+L^2\omega^2)-GnAL\omega^2}\cdot$$

If $L\omega$ is very small, then this reduces to

$$\tan\delta=\frac{GnA\omega}{2R},\qquad\therefore\ R=\frac{GnA\omega}{2\tan\delta}.$$

From these relations R can be determined absolutely, for all the other quantities can be calculated from the dimensions of the coil and the speed of rotation. The resistance of the coil circuit thus determined in electromagnetic C.G.S. units can then be compared with the specific resistance of pure mercury and the resistance equivalent to the ohm, or 10^9 C.G.S. electromagnetic units deduced from the results. In this way the ohm can be specified in terms of the specific resistance of pure mercury.

The ohm determined from the results of these experiments was called the B.A. unit, and its value is about ·9863 of the ohm as now specified.

300. Determination of the Ohm. The Lorenz Method.—When a disc of conducting material is rotated in its own plane in a magnetic field about its geometrical axis a difference of potential is set up, as already explained, between the axis and the circumference of the disc. As in Art. 244, if H denote the component of the field at right angles to the plane of the disc, and if the field be assumed uniform, we have

$$e = \frac{\pi r^2 H}{T},$$

where r denotes the radius of the disc, T the time of rotation, and e the difference of potential set up between the axis and the circumference of the disc. If, then, a disc be set up in the interior of a long carefully-wound solenoid with its axis parallel to the axis of the coil (which should be at right angles to the meridian in order to eliminate the earth's field) and made to rotate uniformly we shall have

$$e = \frac{\pi r^2 \,.\, 4\pi n I}{T} = \frac{4\pi^2 r^2 n I}{T},$$

where I is the current in the coil, for the plane of the disc is at right angles to the field $4\pi n I$ inside the coil due to the current I passing through the coil. It is easily possible by the usual arrangement to balance the difference of potential e against the difference of potential between two points on a conductor forming part of the circuit of the coil and carrying therefore the *same* current. If R be resistance between these two points when the balance is exact we evidently have

$$\frac{4\pi^2r^2nI}{T} = IR,$$

or

$$\frac{4\pi^2r^2n}{T} = R.$$

In this way I is eliminated and the resistance R is determined in absolute units, for r is a length, n the number of turns *per unit length,* and T a time.

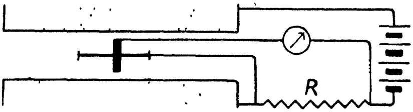


Fig. 473.

The arrangement of connections for this method is shown diagrammatically (Fig. 473). The elementary theory of the method is simple, but there are a number of details and corrections. The method is due to Lorenz, and careful determinations made by Lord Rayleigh and Mrs. Sidgwick and by the late Principal Viriamu Jones show that it is capable of giving very good results.

301. Absolute Measurement of Current, Resistance, and Electromotive Force.

301. Absolute Measurement of Current, Resistance, and Electromotive Force.—Most of the measurements dealt with in previous chapters have been *comparative* and not absolute; to measure any electrical quantity "absolutely" no units are to be assumed known but the necessary fundamental units of length, mass, and time.

(1) *Current.*—The current to be measured is, for example, passed through a standard tangent galvanometer set in the earth's field, and from the theory of the galvanometer we have

$$I = \frac{rH}{2\pi n}\tan\delta.$$

But this relation involves H, the horizontal component of the earth's field, the value of which must not be assumed but determined absolutely. This can, however, be done by the deflection and oscillation method of Art. 43. This gives

$$H = \frac{2\pi}{t}\sqrt{\frac{2K}{d^3 \tan a}}.$$

Substituting this value of H in the expression for I we get

$$I = \sqrt{\frac{2K}{d^3 \tan a}} \cdot \frac{r}{tn} \tan \delta.$$

In this expression for I there is no quantity which cannot be measured directly in one or more of the fundamental units of mass, length, and time; and so I can be determined absolutely by measurement made with the balance, the scale of length, and a seconds clock. To make an accurate absolute measurement of current it would be necessary to adopt less approximate relations than those given above and to secure the utmost accuracy possible in making the fundamental measurements, but the outline just given sufficiently indicates the method.

The absolute measurement of current can also be made by measuring the attraction between two coils carrying the same current and placed with their planes parallel, at right angles to the line joining their centres. If I be the current in the coils and M the coefficient of mutual induction, the force of attraction is given by $I^2 \, (dM/dx)$, where dM/dx denotes the *rate* of change of M with x, the distance between the centres of the coils. This rate of change of M can be calculated from the dimensions of the coil system, and the force of attraction between the coils can be determined in dynes directly. One coil is attached to one pan of an accurate balance and the other is fixed in the specified position under it. When the current passes the force of attraction between the coils can be exactly balanced by placing weights in the other pan of the balance, and from these weights the measure of the force is at once obtained in dynes.

Lord Rayleigh and Mrs. Sidgwick in their determination of the electrochemical equivalent of silver measured the current in this way; as a mean result it was found that the C.G.S. unit current deposited ·0111794 gramme of silver per second.

(2) *Resistance.*—Two methods for the absolute measurement of resistance have already been dealt with in Arts. 299 and 300, and a third, known as the *mutual inductance method*, may be briefly indicated. Two coils A and B are connected, one with a battery giving a steady current I, the other with a ballistic galvanometer. On reversing the current I in A an induced quantity Q flows in B, such that $Q = 2MI/R$, where M is the coefficient of mutual induction and R the resistance of B and the ballistic galvanometer circuit. If δ be the throw corrected for damping, $Q = \frac{HT}{\pi G} \cdot \frac{\delta}{2}$;

hence $$\frac{2MI}{R} = \frac{HT}{\pi G} \cdot \frac{\delta}{2}, \quad \text{i.e. } R = \frac{4\pi MGI}{HT\delta}.$$

If a tangent galvanometer be in series with A and δ' be the deflection, $I = \frac{H}{G'} \cdot \tan \delta'$; substituting—

$$R = \frac{4\pi M}{T} \cdot \frac{G}{G'} \cdot \frac{\tan \delta'}{\delta}.$$

The ratio G/G' may be obtained experimentally by passing the same current through the two galvanometers in series, and the other quantities involved can be directly determined in fundamental units.

(3) *Electromotive Force.*—In practice, having obtained the two preceding quantities independently in absolute measure, an E.M.F. can be determined from them; but E.M.F. might be measured absolutely as follows. If a thin circular conductor be made to rotate in a uniform field about an axis at right angles to the direction of the field with a uniform velocity ω, then, neglecting self-induction, the electromotive force induced in the conductor at any instant is, as in Arts. 241, 283, given by

$$e = HA\omega \sin \omega t.$$

The maximum value of e obtains when the plane of the conductor is parallel to the direction of the field, so that if the circular conductor be cut, and the ends made to connect automatically with the terminals of a voltaic cell every time the conductor passes through this position, it would be possible to arrange that the E.M.F. of the cell should be opposed to the induced E.M.F. in the conductor, and to adjust the speed of rotation until the two opposed electromotive forces should exactly balance. We should then have

$$E = HA\omega \sin \omega t,$$

where H can be determined absolutely as before and the other quantities involved are fundamental. The electromotive force of the cell can thus be determined in absolute units.

Examples.

1. *The value of H at Kew is* ·18 *C.G.S. unit; express this in mm., mgm., min. units.*

$$\text{Dimensions of field} = M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1} = \frac{M^{\frac{1}{2}}}{L^{\frac{1}{2}}T};$$

1 mm. = $\frac{1}{10}$ cm.; 1 mgm. = $\frac{1}{1000}$ gramme; 1 min. = 60 sec.

To change from the new units into C.G.S. units we must therefore multiply by

$$\frac{\frac{1}{1000})^{\frac{1}{2}}}{(\frac{1}{10})^{\frac{1}{2}}.60} = \frac{1}{600},$$

$$\therefore \; 1 \text{ new unit} = \tfrac{1}{600} \text{ C.G.S. unit},$$

i.e. $$1 \text{ C.G.S. unit} = 600 \text{ new units};$$

$$\therefore \; \cdot 18 \text{ C.G.S. unit} = 108 \text{ new units},$$

i.e. $$H = 108.$$

2. *Find the connection between the volt and* (1) *the e.m. unit,* (2) *the e.s. unit of potential.*

The definitions of the practical units in terms of the C.G.S. units (Chapter XI.) really amount to taking 10^9 cm. (an arc of the earth's surface), 10^{-11} grammes, and the second as the units of length, mass, and time; hence the practical system is sometimes called the **quadrant-eleventh-et-gramme-second system.**

$$\text{Dimensions} = M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2},$$

$$\therefore \; \text{Multiplier} = (10^{-11})^{\frac{1}{2}} (10^9)^{\frac{3}{2}} = 10^8,$$

i.e. $$1 \text{ volt} = 10^8 \text{ e.m. units}.$$

Again $$1 \text{ e.m. unit} = \frac{1}{v} \text{ e.s. units},$$

$$\therefore \; 10^8 \text{ e.m. units} = \frac{10^8}{3 \times 10^{10}} = \frac{1}{300} \text{ e.s. unit},$$

i.e. $$1 \text{ volt} = \tfrac{1}{300} \text{ e.s. unit}.$$

Note.—The lack of uniformity in the systems of electrical units and the complication of electrical equations by the "4π monstrosity" have led to several suggestions for a more rational system. **Mr. Oliver Heaviside** suggests that unit pole should be taken as that which exerts a force of 4π dynes on an equal pole one centimetre distant, and evolves a system in which

One Heaviside Unit of Pole Strength $= \dfrac{1}{\sqrt{4\pi}} \times$ C.G.S. Unit Pole,

,, ,, ,, ,, Current $= \dfrac{10}{\sqrt{4\pi}} \times$ The Ampere,

,, ,, ,, ,, E.M.F. $= \sqrt{4\pi} \times$ The Volt,

,, ,, ,, ,, Resistance $= \dfrac{4\pi}{10} \times$ The Ohm,

and so on. **Professor Fessenden** suggests rationalising our formulae by taking the permeability of air as 4π. Other suggestions have been made, but so far none has met with decided approval; so that we are still worried with two distinct systems of units which do not in the least coincide, two distinct sets of dimensions neither of which can possibly be correct, and an awkward 4π in our formulae.

Exercises XX.

Section C.

(1) Give Lorenz's method of determining the ohm in absolute measure, and describe in detail the apparatus required for the purpose. (B.E. Hons.)

(2) What is meant by the expression "the dimensions of a physical quantity"?

Taking as your fundamental quantities time, length, and mass, deduce the dimensions of energy and electrostatic potential. (B.Sc.)

(3) Describe some methods by which the units of current and electromotive force may be found in the electromagnetic system. (B.Sc.)

(4) According to the usual definitions, the dimensions of capacity on the electromagnetic system are those of the reciprocal of an acceleration, while on the electrostatic system they are simply a length. Show how these results are obtained, and explain the apparent discrepancy. (B.Sc.)

(5) Deduce the dimensions in terms of those of mass, length, and time, of quantity, potential difference, and capacity, both in the electrostatic and electromagnetic systems. Show that the ratio is expressed by some power of a velocity, and that to reduce this ratio to a pure number it is necessary to include the dimensions of dielectric constant and permeability. (B.Sc. Hons.)

(6) Define the terms magnetomotive force, magnetic flux, and reluctance of a magnetic circuit. Find the dimensions of these quantities. (B.Sc. Hons.)

(7) Describe a method for determining the relation between the electromagnetic and electrostatic system of units, and state the dimensions of the several quantities involved in their measurements. (D.Sc.)

(8) What are the relations between the several absolute units of the electromagnetic system and the units adopted in practice? A foot-pound is 13,560,000 ergs. What horse-power is required to maintain a current of one ampere through 100 ohms, and what is the efficiency of a system in which a current of one ampere is sent through 1,000 lamps, each of 100 ohms resistance, by the expenditure of 170 horse-power? (D.Sc.)

CHAPTER XXII.

ELECTRIC OSCILLATIONS, ELECTROMAGNETIC WAVES, AND WIRELESS TELEGRAPHY.

302. Oscillatory Discharge.—When the plates of a charged condenser are connected by a large resistance the discharge consists of a steady flow from one plate to the other, the charges thus neutralising and the potentials of the plates both becoming zero. If, however, the resistance be below a certain value, the discharge consists, not of a steady flow, but of a number of rapid oscillations or surgings to and fro: the first "rush" more than neutralises the opposite charge and charges the condenser in the opposite direction; this is followed by a reverse rush which again "overshoots the mark" and charges the condenser in the same way as it was originally, and so on. Each successive oscillation is weaker than the preceding; thus, after a number of such surgings the discharge is complete and the potentials of the plates equalised. At each oscillation the electrostatic energy of the condenser field is converted into electromagnetic energy accompanied by the dissipation of a small quantity of energy as heat in the conductor (and also by electric radiation as explained below). These oscillations take place very rapidly; the time of each is the same, and this time is known as the period of the oscillations.

As the result of these transformations of energy it is evident that the portion of the medium involved primarily in the transformations *must undergo a definite periodic variation of state.* This variation of state is propagated with finite velocity throughout the surrounding medium, that is, **it is the origin of a set of electric or electromagnetic waves** *which travel outward in all directions into the medium with a definite finite velocity.*

Fedderson in 1857 verified the oscillatory character of the spark from a Leyden jar by viewing it in a rapidly rotating mirror; the image consisted of a band with bright and dark spaces. With a high resistance in the circuit the image became simply a band, the bright and dark alternations being absent.

303. Period of Oscillation. First Approximation.—A first approximation to the time of oscillation in any particular case is readily obtained if we consider a case in which the dissipation of energy at each oscillation is negligibly small. Let C denote the capacity of the condenser, L the self-inductance of the circuit, Q_0 the maximum charge, and I_0 the maximum current during discharge, then the energy in the medium is given by

$$E = \tfrac{1}{2}\frac{Q_0{}^2}{C} = \tfrac{1}{2}LI_0{}^2.$$

Hence, if the dissipation of energy be neglected and Q and I denote the charge and current *at any instant*, we have

$$\tfrac{1}{2}\frac{Q^2}{C} + \tfrac{1}{2}LI^2 = E,$$

where E is constant.

If this is differentiated with respect to time we get

$$\frac{Q}{C}\cdot\frac{dQ}{dt} + LI\frac{dI}{dt} = 0.$$

Now $\qquad I = \dfrac{dQ}{dt}$, and therefore $\dfrac{dI}{dt} = \dfrac{d\left(\dfrac{dQ}{dt}\right)}{dt}$.

Hence $\qquad \dfrac{Q}{C}\cdot\dfrac{dQ}{dt} + L\dfrac{dQ}{dt}\cdot\dfrac{d\left(\dfrac{dQ}{dt}\right)}{dt} = 0,$

$$\therefore \frac{Q}{C} + L\frac{d\left(\dfrac{dQ}{dt}\right)}{dt} = 0,$$

i.e. $\qquad \dfrac{d\left(\dfrac{dQ}{dt}\right)}{dt} = \dfrac{1}{-CL}Q$

This evidently means that Q is a quantity such that if we differentiate it with respect to time to get dQ/dt and then differentiate

dQ/dt with respect to time we get the initial result multiplied by a constant.

This at once suggests that

$$Q = Q_0 \sin \omega t,$$

for

$$\frac{dQ}{dt} = Q_0\omega \cos \omega t, \qquad \therefore \frac{d\left(\frac{dQ}{dt}\right)}{dt} = -Q_0\omega^2 \sin \omega t,$$

i.e.

$$\frac{d\left(\frac{dQ}{dt}\right)}{dt} = -\omega^2 Q.$$

This result agrees with the one given above, the constant $1/CL$ being represented by ω^2.

Now the expression $Q = Q_0 \sin \omega t$ indicates that Q is a periodic quantity varying harmonically between the limits Q_0 and $-Q_0$, and the period of its variation is $2\pi/\omega$, for ωt changes from zero value to 2π in that time and goes through its full cycle of values within these limits. Now this period $2\pi/\omega$ is evidently the period of oscillation of the discharge, and since $\omega^2 = 1/CL$ the period of oscillation is given by

$$t = 2\pi\sqrt{CL} \quad \text{.................................(1)}$$

and the frequency (n), viz. $1/t$, is

$$n = \frac{1}{2\pi\sqrt{CL}} \quad \text{................................ (2)}$$

If L be in C.G.S. electromagnetic units and C in C.G.S. electrostatic units, then, since the latter is equal to $C/(9 \times 10^{20})$ electromagnetic units,

$$n = \frac{3 \times 10^{10}}{2\pi\sqrt{CL}}.$$

304. Period of Oscillation. Second Approximation.—In the preceding we have neglected the energy dissipated by the Joule heating effect. If we include this we must change the equation given above by adding I^2R, the rate of dissipation of energy, to the two other rates of change of energy before equating their algebraic sum to zero. This gives, after substituting $\left(\frac{dQ}{dt}\right)^2 R$ for I^2R and simplifying,

$$\frac{Q}{C} + L\frac{d\left(\frac{dQ}{dt}\right)}{dt} + R\frac{dQ}{dt} = 0$$

or

$$L\frac{d\left(\frac{dQ}{at}\right)}{dt} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

The expression for Q for which this relation is true depends upon whether the roots of the quadratic equation

$$Lx^2 + Rx + \frac{1}{C} = 0$$

are real or imaginary.

Case 1. If the roots are imaginary, that is if $\frac{4L}{C} > R^2$, the value of Q is given by

$$Q = Ae^{-\frac{Rt}{2L}} \cos(\omega t + a),$$

where

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{4L^2}.$$

By this result Q is a periodic function of the time, its successive maximum values decreasing in geometrical progression as t increases in arithmetical progression, and, as above, the period of oscillation is

$$t = 2\pi \Big/ \sqrt{\frac{1}{CL} - \frac{R^2}{4L^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

and the frequency is

$$n = \frac{1}{2\pi} \sqrt{\frac{1}{CL} - \frac{R^2}{4L^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

When $R = 0$ these evidently reduce to the simpler results, $t = 2\pi\sqrt{LC}$, etc., given above.

Case 2. If the roots of the quadratic equation are real, that is if $R^2 > \frac{4L}{C}$, then the value for Q which satisfies the above relation is

$$Q = Ae^{at} + Be^{\beta t},$$

where a and β are the roots of the equation. From this equation it will be seen that as t increases Q never changes sign, but steadily decreases, reaching zero value only after an infinite time, but becoming negligibly small in a very short time.

It follows from this that the relation $R^2 = \frac{4L}{C}$ or $R = \sqrt{\frac{4L}{C}}$ gives the limiting value of R for oscillatory discharge. If R exceeds this value the discharge is continuous, if less than this it is oscillatory.

The above value of t is only a second approximation. The loss of energy by radiation and the uncertainty as to the value of R for the rapidly alternating discharge current have not been considered.

305. Hertz's Experiments.—Hertz investigated electric oscillations and waves by means of apparatus in which the *oscillator* or *vibrator* was a dumb-bell-shaped brass conductor. The rod connecting *A* and *B* was cut so as to give a small gap at *ab* (Fig. 474) between the two small brass balls *a* and *b*. The portion *Aa* was then connected to one pole of an induction coil and *Bb* to the other pole as shown.

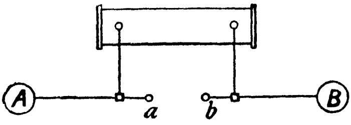


Fig. 474.

When the induction coil is worked *Aa*, say, is charged to a higher potential than *Bb*, then a spark passes at *ab*, and *the path of this spark* on account of its comparatively low resistance practically connects *Aa* and *Bb* as one conductor, and the potential is equalised by very rapid electric oscillations in this conductor. The cycle of changes that go on in the medium near the vibrator during one complete vibration is that associated with the transformation of electrostatic to electromagnetic energy, the re-transformation of electromagnetic to electrostatic energy, and, during these, a gradual dissipation of energy as heat. This cycle of changes is transmitted out into the medium, and the sequence of states so transmitted constitutes the electric waves. When these waves pass a conductor they tend to reproduce the electrical conditions in it, that is, electrical oscillations are induced in the conductor and small sparks may be caused between conductors placed close together.

Hertz took advantage of this to detect the waves. One form of *detector* used by him was a piece of thick copper wire bent into a circle, but with a small air-gap between the ends of the wire; the width of the gap could be adjusted. As the waves passed through this circuit small sparks were observed at the gap, and it was found that the sparks were strongest when the dimensions of the detector were such that its period of oscillation as an oscillator was the same as that of the oscillator originating the waves. This is an example of the general principle of *resonance*, and a circuit showing electric resonance is specially effective in detecting electric waves. By the proper adjustment of inductance and capacity it is possible to adjust the period of the detector within wide limits, and so *tune* it to resonance in any particular case. Further, it was found that the gap in the detector must be parallel to the gap in the oscillator, for the wave is *polarised.*

(*a*) *Determination of Wave Length and Velocity.*—Electric waves are found to pass through stonework, woodwork, and other similar substances, but suffer reflexion at a wall of good conducting material. Hence, if a large metal sheet be set up with its plane at right angles to the line of propagation of the waves, a stationary wave is produced between the vibrator and the reflector. The reflecting surface marks a node in the wave, and successive nodes or internodes are at a distance apart equal to one-half the wave length. By means of a detector the positions of the nodes and internodes are readily detected by its quiescence or activity, and the wave length determined. Hertz adopted this method.

If γ be the wave length, n the frequency calculated from the data of the oscillator, and v the velocity of transmission of the waves, then $v = n\gamma$, so that v is determined. Hertz found the value to be 3×10^{10} cm. per second, that is the velocity of transmission of electric waves is the same as that of light waves. The medium in which the electric waves are propagated is the ether.

Sarasin and De la Rive have pointed out that the interpretation of the above experiment is faulty unless the detector is syntonised with the oscillator, for the distance between the nodes in the experiment with Hertz's apparatus depends more upon the period of the detector than of the oscillator.

(*b*) *Illustrations of Reflection, Refraction, etc.*—Direct experiment has shown that electric waves can be reflected, refracted, etc., in accordance with the same laws that apply to light. Hertz used a metallic mirror of parabolic cross-section with a simple vibrator fixed at its focal line as shown in Fig. 475. The waves originated at the vibrator are reflected from the mirror in a direction parallel to the axis of the cross-section, giving a beam of parallel radiation. If the beam is received on a similar mirror the waves are found to be focussed at the focal lines and well marked sparks are produced in the vibrator fixed at the focus.

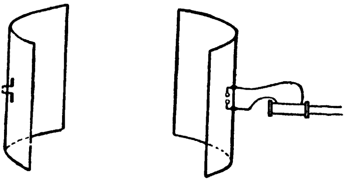


Fig. 475.

To exhibit refraction Hertz passed the beam through a prism of pitch, using one parabolic mirror *M* (Fig. 476), fitted with a vibrator, to originate the waves, and another *M'*, also fitted with a vibrator, as a receiver of the refracted beam. The beam of electric radiation from *M* was found to be refracted in the same way as a beam of light, the direction of the refracted beam being readily detected by the position of maximum spark activity at the vibrator attached to *M'*.

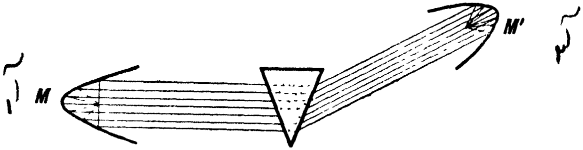


Fig. 476.

That the beams consist of polarised waves may be shown by arranging the reflectors (Fig. 475) first with the focal lines parallel and then with the focal lines at right angles; in the former case the detector responds, but it does not do so in the latter case.

306. Laboratory Methods for the Production, Detection, and Investigation of Electromagnetic Waves.—For experimental work some form of Hertz oscillator is frequently employed. If *A* and *B* (Fig. 474) be circular plates of radius *r* cm. the electrostatic capacity of each is $2r/\pi$, and since the two are really in series the total capacity is one half of this; hence, assuming this to be the whole capacity, we have $C = r/\pi$ e.s. units. The inductance *L* may be calculated from the relation $L = 2l\left(\log_e \frac{4l}{d} - 1\right)$, where *l* is the total length of the two rods and *d* the diameter, both in centimetres; this determines *L* in e.m. units.

A form of oscillator due to Lodge consists of two small brass spheres ·3 cm. diameter and 1·2 cm. apart with a larger one ·8 cm. diameter fixed between them, the smaller spheres being connected to the induction coil and therefore sparking across the diameter of the larger one; the wave length with this oscillator is about 1·85 cm.

Another type, also due to Lodge, consists of two spheres

·5 inch diameter sparking to the interior of a copper cylinder 2 inches long and 2 inches diameter; the wave length with this oscillator is about 3 inches.

For the detection of the waves, a coherer the principle of which was discovered by Lodge and utilised by Branley, Marconi, and others may be used. A simple form is shown in Fig. 477. Two brass, copper, or silver discs *d*, *d*, each soldered to the end of a copper wire *w*, are fitted into the glass tube *gg*, and a thin layer of filings rests lightly between the discs. If this coherer is placed in the circuit of a battery and an electric bell, the contacts made by the filings between the discs may be so adjusted that the current passing will not be strong enough to ring the bell. If, however, waves from a distant vibrator fall upon the coherer after this adjustment is made, the contact at once becomes good and the bell in the circuit rings. If the coherer is slightly tapped the contact again fails and the arrangement is again ready to detect the incidence of the waves. In Marconi's coherer the plugs *d*, *d* are of silver, the filings are a mixture of nickel and silver, and the tube is exhausted and sealed. In laboratory work a suitable galvanometer may take the place of the bell above.

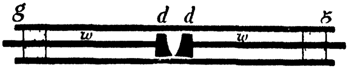


Fig 477.

Even from the facts so far mentioned the reader will be prepared for the statement that light and electromagnetic waves are identical except as regards wave length and frequency, the latter consisting of slower vibrations and much longer waves than those of light. The wave length of the *visible spectrum* ranges from ·43μ (violet) to ·75μ (red), where μ is the symbol for one micron or ·001 millimetre, In the *ultra violet* waves have been detected as short as ·1μ possessing properties similar to light waves. In the *infra red* waves have for some time been known as long as 61·3μ. Some distance beyond the infra red come the electromagnetic waves dealt with above, the shortest being about 3 *mm.* (3000μ), whilst those used in wireless telegraphy range from a few hundred feet to four or five miles.

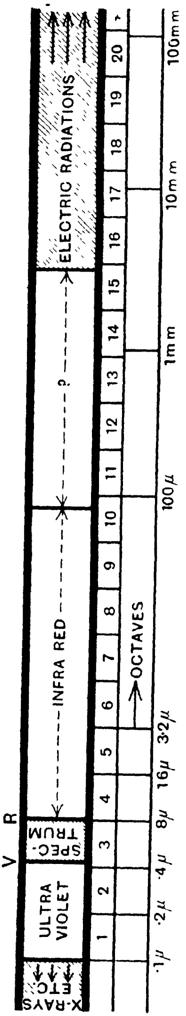


Fig. 478.

Fig. 478 shows diagrammatically, within the limits of the sketch, the range of the ether waves referred to.

307. Commercial Methods for the Production and Detection of Electromagnetic Waves. Wireless Telegraphy.—For the history of the development of wireless telegraphy from the initial experimental stages indicated above to its present position the student must refer to specialised works on the subject.

One form of transmitting circuit is shown in Fig. 479. The condenser C and spark gap S are in series with one winding of the transformer T, the other winding of the transformer being joined to a long vertical conductor A (known as the *antenna* or *aerial*) and to earth. The contact K and therefore the inductance in series with the aerial is adjusted until the aerial circuit and condenser circuit are syntonised, *i.e.* until the period of oscillation of the aerial circuit is equal to that of the condenser circuit. The "charging" is effected from the secondary of the transformer T_1, the primary of which is connected to an alternator; this circuit also includes an inductance L and a switch S_1, the latter being used to produce the required "longs" and "shorts" of the Morse code. At each discharge oscillations occur in the condenser circuit; these induce oscillating electromotive forces and currents in the aerial circuit, and the two being syntonised the aerial becomes a powerful source of radiation of electric waves. At the receiving station the waves encounter a similar aerial and produce electric oscillations in it.

In place of the simple spark gap *S* a **rotary spark gap** is often employed. A wheel carrying a number of projecting studs is rotated between two fixed studs in the condenser circuit and arrangements are such that the fixed and moving studs come opposite each other at the instant when the condenser is fully charged, *i.e.* just when the spark is required.

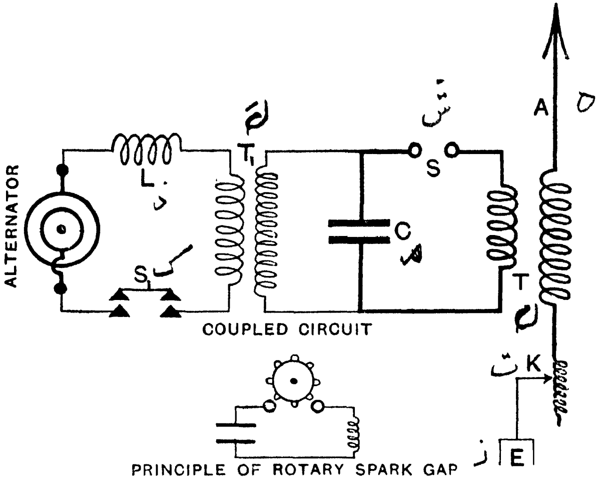


Fig. 479.

The coherer as a detector has been replaced by others more suitable for the purpose and known as magnetic, thermal, and rectifying detectors. The principle of *Marconi's magnetic detector* will be gathered from Fig. 480 It consists of an iron wire band *I* which passes over two pulleys *P*, *P* and is rotated by clockwork. In one part of its journey it passes through a coil *C* which is either directly or inductively coupled to the aerial; a second coil *D* is connected to a telephone. Magnets *M*, *M* are placed as indicated, so that the band becomes magnetised in passing; but owing to hysteresis the magnetised portion of the band is not opposite the poles, but is displaced in the direction of rotation. When an electric oscillation passes through C, due to waves from the transmitting station, hysteresis is wiped out and the magnetised portions of the iron band are then opposite the poles. This change in the position of the magnetised portions results in an induced current in the coil D and the telephone is affected. The aerial circuit is of course "tuned" to the arriving waves.

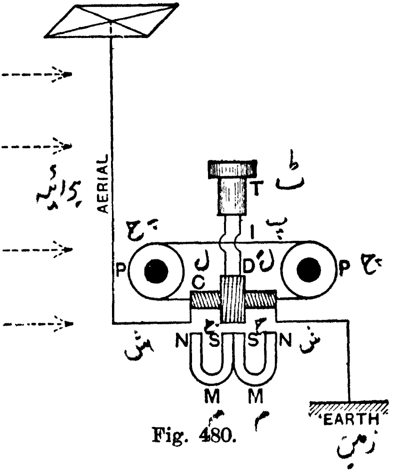


Fig. 480.

Fessenden's thermal detector consists of a platinum wire in a vacuum tube. When an oscillatory current passes through the wire its resistance is increased, and this change in resistance is utilised, indirectly, to operate a telephone. Other types are in use.

Rectifying detectors, one type of which consists of a carborundum crystal between two brass plates, depend for their action on the fact that their conductivity is different in different directions, so that an oscillatory current may be partially changed into a direct current by choking down the flow in one direction. These are now extensively used in practice, and Fig. 481 shows the arrangement of a receiving circuit fitted with such a detector.

The aerial circuit is adjusted to syntony with the arriving waves. The other coil of the transformer T and the condenser C_1 form an oscillatory circuit syntonised to the aerial circuit. The detector D in series with a second condenser C_2 is in parallel with C_1 and the connections to the telephone t are as indicated. The waves arriving at the aerial set up oscillations in it which induce oscillations in the condenser circuit. This tends to cause currents to flow to and fro through *D* into and out of C_2, but owing to the property of *D* mentioned above current in one direction only passes, so that C_2 is charged. This charge in one direction, due to the arrival of a train of oscillations, is given out to the telephone circuit as a single unidirectional current flow. Many such detectors work better if they have a small current flowing through them; hence the insertion of the battery *B*. Other types of crystals are in use.

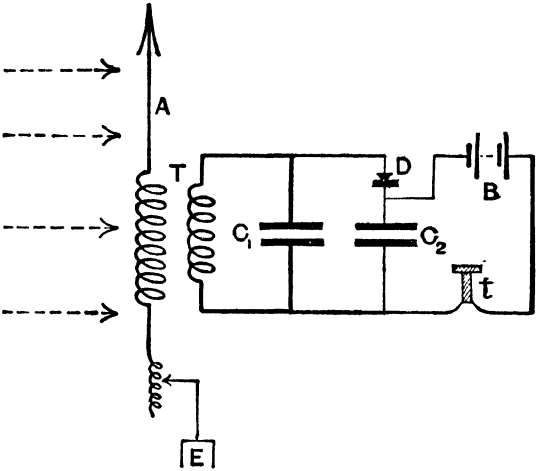


Fig. 481.

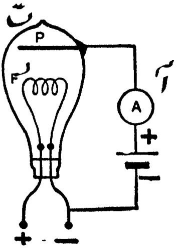


Fig. 481*a*.

Valves are now largely used in wireless. An early type of valve known as the **diode** is shown in Fig. 481*a*. It consists of a filament lamp fitted with an electrode *P* known as the **plate** or **anode.** When the filament is incandescent it emits (negative) electrons, so that if *P* is joined to the positive pole of the cell as shown and therefore maintained at a positive potential, the electrons are attracted towards it. This movement of electrons from filament to plate constitutes an electric current from plate to filament, and the measuring instrument *A* will be deflected. If the cell connections be reversed, however, the current stops. If an oscillatory current be used instead of the cell *C*, current will flow when the upper terminal is positive but not when the upper terminal becomes negative. Thus we may say that the lamp or valve possesses unilateral conductivity similar to the crystal dealt with above, and this indicates its use as a detector. Note particularly that *in order to get a current through A the positive pole of the battery must be joined to the plate:* this current is referred to as the plate current.

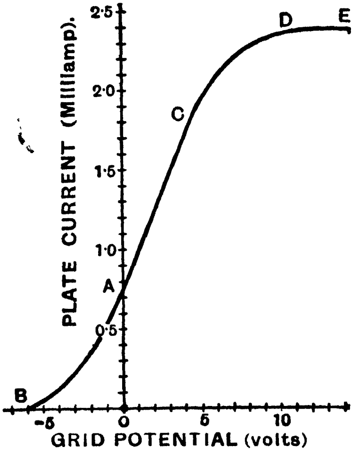


Fig. 481*b*.

The modern valve used in "wireless" and known as the **triode** consists of a tungsten filament and a plate with a wire **grid** between them, the three being quite separate from each other. Imagine that the plate and filament are joined to a battery (from 50 to 100 volts in practice) as in Fig. 481*a* so that electrons are passing from filament to plate and therefore a plate current is flowing through the valve from plate to filament. Imagine now that a grid between *P* and *F* acquires from some outside source a negative potential. It will repel the electrons coming from the filament and stop many of them from reaching the plate; hence the plate current will decrease and so will the deflection of the instrument *A* (Fig. 481*a*). If the potential of the grid becomes positive it will attract the electrons and (as

P is at a higher potential in practice) they continue their motion to the plate; thus the plate current will increase. There are many other important points about this which, however, cannot be treated here; for the present we merely wish the student to see in a general and elementary way how the grid potential affects the plate current.

Fig. 481*b* really shows the effect of the grid potential on the plate current, potentials to the left of *O* being negative. Thus when the grid has the negative potential *OB* it stops all the electrons and the plate current is zero. As the grid potential rises to zero the plate current increases to the value *OA*, and as the grid potential becomes more and more positive the plate current rises according to *ACDE*.

Fig. 481*c* shows the principle of one method of connecting up a triode as a simple detector. The high tension battery (as it is called) in the plate and telephone circuit is shown on the right and the (1·5 to 6 volt) filament battery is shown at the bottom of the figure. The grid circuit is as indicated and the student must note this carefully. The oscillatory P.D. between the grid and filament set up by the arriving waves causes *a corresponding (but greater) variation in the direct current flowing between plate and filament and through the telephones;* thus the messages are received. For further details the student should consult the author's *First Course in Wireless.*

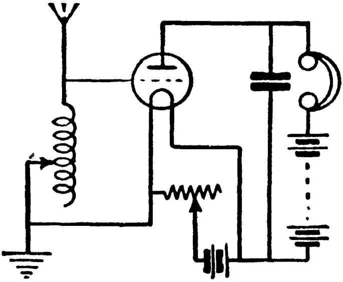


Fig. 481*c*.

We must now leave the "experimental" and "commercial," and pass to the "theoretical" aspect of electromagnetic waves.

308. Magnetic Field due to the Motion of Faraday Tubes.

—The inferences, as has been indicated in previous chapters, from the results of many experiments and considerations go to show that the seat of electrical effects is in the medium. The flow of an electric current has been shown to be the motion of Faraday tubes, the "current-carrying conductor" is the surface along which the ends of the tubes move, and the distinction between a conductor and an insulator is that the former is a substance along the surface of which the ends of the tubes can move, whilst the latter does not permit this. Further, energy is really transferred *from the medium to the circuit*, and Poynting has shown that the paths along which the energy passes into the circuit are the intersections of the electrostatic and magnetic equipotential surfaces.

Let A and B (Fig. 482) be two insulated parallel plates forming a condenser. A is charged positively to a uniform surface density σ, and B negatively to a uniform surface density $-\sigma$. There is at present only electrostatic force between the plates, and Faraday tubes stretch from plate to plate; let D be their number per unit area. Now let A and B be joined by a wire of high resistance. At once the Faraday tubes joining A and B move towards the wire, their ends meeting in the wire, and besides this drifting there is a current down one plate and up the other. Hence a magnetic field is created, *i.e.* a field is created due to the motion of Faraday tubes. This field is perpendicular to the direction of the tubes and to the direction of their motion, *i.e.* perpendicular to the plane of the paper.

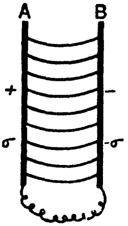


Fig. 482.

Let i equal the current per unit length of A and B measured perpendicular to the paper. Then if H is the magnetic field between A and B we have by Ampère's Theorem

$$H = 4\pi i.$$

If v is the velocity of the ends of the Faraday tubes we have

$$\begin{aligned} i &= \sigma v \\ &= Dv, \\ \therefore\ H &= 4\pi Dv. \end{aligned}$$

It is obvious that if the direction of motion is inclined at an angle θ to the length of the tubes,

$$H = 4\pi Dv \sin\theta.$$

309. Energy and Mass associated with Faraday Tubes.—Since there is a magnetic field between A and B there is also magnetic energy stored between A and B. This energy (Art. 274) is given by the expression

$$\begin{aligned} E &= \frac{\mu H^2}{8\pi} \text{ per unit volume} \\ &= 2\pi\mu v^2 D^2 \text{ per unit volume} \\ &= \tfrac{1}{2}\{4\pi\mu D^2\}v^2 \text{ per unit volume,} \end{aligned}$$

which may be written as $\frac{1}{2}Mv^2$, *i.e.* the tubes may be supposed to drag up through the space between A and B something whose mass per unit volume is given by

$$M = 4\pi\mu D^2 \text{ per unit volume.}$$

As an analogy we may consider the hydrodynamical cases of spheres and cylinders moving through liquids. With a moving sphere is associated a volume of liquid equal to half the volume of the sphere, while with a cylinder moving perpendicular to its length is associated a volume of liquid equal to the volume of the cylinder. Therefore, regarding these tubes as cylinders moving through the aether in a direction perpendicular to their length, we can say that $4\pi\mu D^2$ is the mass of the aether per unit volume carried up by the D tubes passing through unit area. The mass of the bound aether per unit length of a single tube is therefore given by

$$M_1 = 4\pi\mu D \text{ per unit length.}$$

Clearly, if the direction of motion be at an angle θ to the length of the tube,

$$\begin{aligned} E &= 2\pi\mu D^2 v^2 \sin^2\theta \text{ per unit volume,} \\ M_1 &= 4\pi\mu D \sin^2\theta \text{ per unit length of a tube.} \end{aligned}$$

The pull in a Faraday tube is, by Art. 98, equal to $2\pi D/K$, so that the pull bears a constant ratio to the mass per unit length. Faraday tubes may, therefore, be regarded as stretched strings of variable tension and mass per unit length, the ratio between pull and mass per unit length being constant at all points along the tube.

310. Velocity of a Transverse Pulse along a moving Faraday Tube.—Consider a Faraday tube stretching between two points P and Q. It has been shown that the tube bears a considerable resemblance to a stretched string in that it has tension in the direction of its length and possesses mass. Suppose the end P to be moved in a direction perpendicular to the tube; as the latter possesses mass, a finite time will be required for the parts near Q to take up the new conditions consequent upon the transverse motion of P, *i.e.* the transverse disturbance will be propagated along the tube with a definite velocity.

It is well known that the velocity of propagation of a transverse wave along a stretched string

$$= \sqrt{\frac{\text{Pull in the string}}{\text{Mass of unit length of string}}},$$

but before we can apply this formula to find the velocity of a transverse displacement occurring in a Faraday tube we must remember that, as the Faraday tube is bounded by other tubes, the problem is not exactly analogous to that of finding the velocity of a transverse wave in a stretched string.

In Arts. 96 and 98 it is shown that the tension (*i.e.* pull per unit area) in the Faraday tube system $= \frac{KF^2}{8\pi}$, while the transverse pressure (*i.e.* force per unit area) on a tube is also $\frac{KF^2}{8\pi}$.

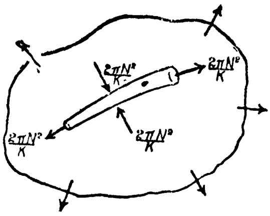


Fig. 483.

Now $F = \frac{4\pi D}{K}$, therefore the tension and lateral pressure are each equal to $\frac{2\pi D^2}{K}$ per unit area at points where D is the number of Faraday tubes per unit area taken perpendicular to their length.

To find what the tension would be if there were no side pressures, let us imagine a portion of the tube placed in an envelope (Fig. 483) and a negative pressure equal to $\frac{2\pi D^2}{K}$ applied to the whole of the contained aether. This neutralises the lateral pressures and makes the tension along the tube equal to $\frac{4\pi D^2}{K}$ at points where there are D tubes per unit area, *i.e.* the pull in a single tube $= \frac{4\pi D}{K}$.

The velocity (v) of the transverse disturbance is therefore given by

$$v = \sqrt{\frac{4\pi D/K}{4\pi\mu D}} = \frac{1}{\sqrt{\mu K}}.$$

The above formula for the velocity of a transverse displacement also holds for the case of a tube moving at any angle θ to its length; for though the bound mass per unit length of such a tube is $4\pi\mu D \sin^2 \theta$, yet in a transverse displacement a portion of the tube is moving at right angles to its length, and therefore the bound mass of aether for such a portion is equal to $4\pi\mu D$.

The next step is to determine the value of $1/\sqrt{\mu K}$ for some medium, say air. This has been done by the methods outlined in Chapter XXI., and v has been found to be 3×10^{10} cm. per second, the velocity of light; thus *the disturbance travels along the Faraday tube with the velocity of light.*

Twenty years after Maxwell had published his views as to the velocity of propagation of electrical disturbances came the experiments of Hertz which confirmed his deductions. The sequence of states due to the transverse vibrations of the ends of the tubes attached to the oscillator is propagated with the velocity v deduced above. Hertz experimentally determined the velocity of propagation and found it to be 3×10^{10} cm. per second.

311. Electric Elasticity and Density of the Aether.—An alternative investigation of the value of v may be given. The velocity of wave transmission in any medium is expressed by the relation $v = \sqrt{e/d}$, where e is the modulus of elasticity for the medium and d the density.

In order to find the values of e and d, which determine the velocity of transmission of electric waves, we may compare the expressions for the energy per unit volume of a strained medium and the energy per unit volume of an electric field. In the first case, if P be the *stress*, s the *strain*, and e the *modulus of elasticity*, the energy of strain per unit volume of the medium is given by

$$\tfrac{1}{2}Ps \quad \text{or} \quad \tfrac{1}{2}\frac{P^2}{e} \quad \text{or} \quad \tfrac{1}{2}es^2.$$

Now the energy per unit volume of an electric field is $F^2K/8\pi$, and F is $4\pi\sigma/K$. Hence, combining these two expressions, we may express the energy per unit volume as

$$\tfrac{1}{2}F\sigma \quad \text{or} \quad \tfrac{1}{2}F^2\frac{K}{4\pi} \quad \text{or} \quad \tfrac{1}{2}\,.\,\frac{4\pi}{K}\,.\,\sigma^2.$$

If now F be taken to represent the stress in the electric field, and σ to represent the strain, then the three expressions just given correspond exactly with the three similar ones given higher up, and it will be seen that the electric elasticity is represented by $4\pi/K$.

Again, in the case of a mass m moving with a velocity u, the kinetic energy is $\tfrac{1}{2}mu^2$. Now the energy per unit volume in a magnetic field is $H^2\mu/8\pi$, and if we take the case of the field inside an endless uniformly wound coil of n turns per unit length carrying a current I, the value of H is $4\pi nI$, and the energy per unit volume is $2\pi(nI)^2\mu$ or $\tfrac{1}{2}\,.\,4\pi\mu(nI)^2$. Now nI is the rate of displacement of electricity round unit length of the coil and represents electrical velocity in the same way as σ represents electric strain or displacement. Hence, comparing $\tfrac{1}{2}\,.\,4\pi\mu(nI)^2$ with $\tfrac{1}{2}mv^2$, the quantity $4\pi\mu$ evidently corresponds to m and measures the electric mass or inertia per unit volume. That is, $4\pi\mu$ is the electric density of the medium.

The velocity of electric waves in a medium for which $4\pi/K$ is the electric elasticity, and $4\pi\mu$ the electric density, is evidently given by

$$v = \sqrt{\frac{4\pi}{K}\Big/4\pi u} \quad \text{or} \quad v = \sqrt{\frac{1}{K\mu}}.$$

312. Electromagnetic Waves generated by an Oscillator.—We can now deal more exactly with the waves generated by the oscillators of Arts. 305 and 306.

Fig. 484 (*a*) depicts the Faraday tubes of the oscillator. When the oscillator begins to discharge the ends of the tubes move along the rods, thus constituting the electric current, and at the same time magnetic tubes appear as concentric circular curves with their centres on the rods as shown at M in Fig. 484 (*b*). Owing to the "mass" property of the Faraday tubes previously dealt with, and the rapidity of the oscillations, the tubes do not approach the rods as quickly as their opposite ends move along them, hence they take up the form shown in Fig. 484 (*c*). At a later stage the ends of the tubes will have "crossed over" as indicated in Fig. 484 (*d*), and a closed loop will be

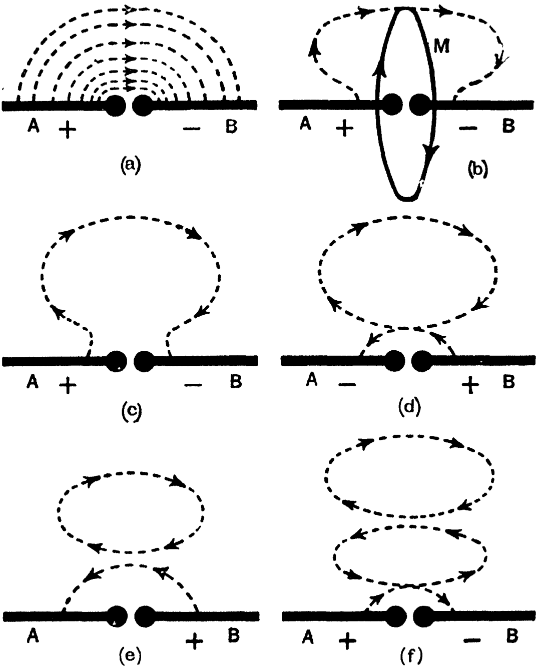


Fig. 484.

thrown off as shown in Fig. 484 (*e*). The next "surging" will give rise to another set of closed loops (Fig. 484 (*f*)), and so on. It is this detachment of loops which constitutes electric radiation, the loops travelling outwards with the velocity $1/\sqrt{K\mu}$. It will be noted that the direction of the electric strain is opposite in successive loops, and so also is the magnetic force due to the motion, the magnetic force being, as already mentioned, at right angles to the direction of the electric strain and the motion.

Instead of saying that the loops travel outwards, it would be more exact to say that a loop "dies" at one place and is "re-created" at another. Each tube shrinks, in so doing creating magnetic tubes, the rise and fall of which re-create loops of electric strain, and so on; hence the effect is equivalent to a progression through space of two sets of strain, electric and magnetic, which constitute the electromagnetic wave.

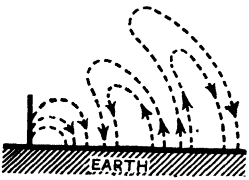


Fig. 485.

In the case of wireless telegraphy, since the lower end of the radiating aerial is earthed, the loops take the form shown in Fig. 485.

Consider now the two loops of Fig. 485 (*a*) in which *XY* is normal to the oscillator at its mid-point. At points such as *P* the magnetic force and the electric displacement or strain have their greatest values, at *Q* they are zero, at *R* they are again greatest but in the opposite direction to *P*, at *S* they are again zero and so on. Remembering that the magnetic force is at right angles to the electric force and the motion, Fig. 485 (*b*) shows how the three victors are related and depicts graphically the electromagnetic wave.

A brief explanation of the production of stationary waves by means of a metallic reflector referred to in Hertz's experiments may now be given. Considering the parts of the loops in the equatorial plane of the oscillator the tubes at *A*, *B*, and *C* (Fig. 485*c*) are positions of greatest electric displacement *B* being in the opposite direction to *A* and *C*. When *C* meets the reflector a tube *D* with reversed displacement sets off in the opposite direction. At the reflector *C* and *D* cancel each other and a "node" is formed there. If *t* be the period of the oscillator then in time *t*/4 seconds *D* will have moved back to *X*, a distance of a quarter wave length and *B* will have moved forward to

X through a distance of a quarter wave length, and as the electric displacements are in the same direction *D* and *B* help each other at *X* and an "antinode" is formed there (Fig. 485*d*). In another quarter period *A* moves forward a quarter wave length to *Y*, and *D* moves back through a quarter wave length to *Y* and as *D* and *A* have opposite electric displacements a "node" is formed there and so on. The distance between two consecutive nodes (or antinodes) is half a wave length ($\gamma/2$): hence knowing the frequency (n) the velocity is found, viz. $v = n\gamma$.

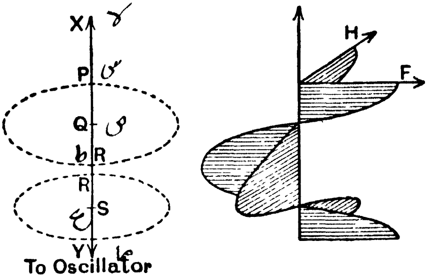
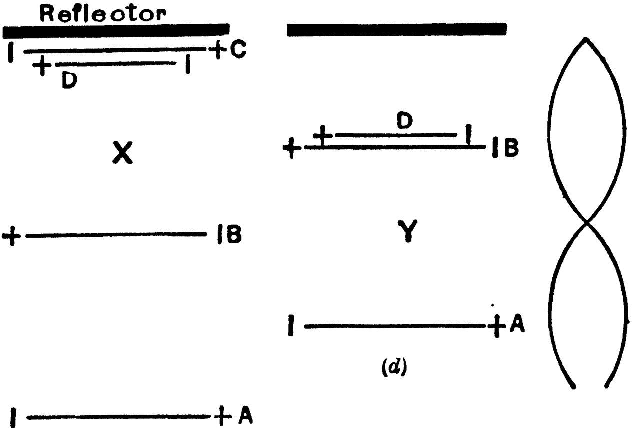


Fig. 485 (*a*) and (*b*).



(*c*) Fig. 485 (*c*) and (*d*).

313. Lecher's Wires, Blondlot's Experiment, and Fleming's Cymometer.—Lodge, Hertz, Sarasin, Lecher, and others have determined the velocity of propagation of the waves along wires and have found it to equal that of light. Lecher's arrangement is shown in Fig. 486. The two metal plates *AB* are fixed opposite and parallel to the plates of the oscillator; the parallel wires *CX* and *DY* are arranged as indicated. When the oscillator is in action stationary oscillations are set up in the wires, the ends of the wires being points of greatest variation of potential. The nodes and antinodes are found by placing a neon tube (a "vacuum" tube containing neon) across the wires: it glows at the antinodes, but does not do so at the nodes. In this way the wave length is found, and knowing the frequency the velocity is determined. If a metallic connection *EF* be put across the wires the variations at *XY* are greatest when *EF* is at a node.

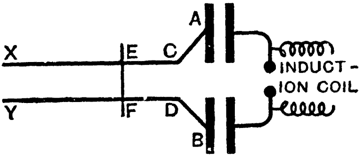


Fig. 486.

Blondlot used two cylindrical condensers, the inner coatings being tinfoil connected to the spark gap *S* (Fig. 487), the outer coatings of each consisting of two rings of tinfoil *AB* and *CD*. The upper rings *A* and *C* are joined direct to the spark gap S_1, whilst *B* is joined to one side of S_1 and *D* to the other side by wires each 1821·4 metres long. The rings are also joined by damp threads indicated by the dotted lines. When a spark passes at *S*, *A* and *C immediately* discharge by a spark at S_1, and *later B* and *D* discharge at S_1, the interval between the sparks at S_1 being the time taken for the electromagnetic wave to travel along 1821·4 metres of wire. This time interval is found by noting the distance between the images of the sparks upon a photographic plate produced by a rotating mirror. Blondlot's result was $2{\cdot}98 \times 10^{10}$ cm. per second.

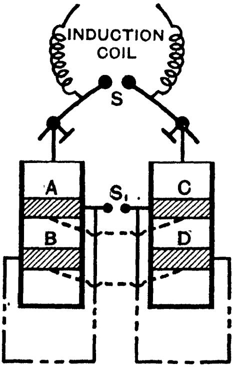


Fig. 487.

Fleming's cyclometer is an instrument devised for the measurement of the frequency of electrical oscillations. It consists (Fig. 488) of a cylindrical sliding condenser *IIOO* and a wire solenoid *LD*. The dielectric of the condenser is a cylinder of vulcanite; the inner coating of the condenser is joined to one end of the solenoid by the conductor *A'ABB'CD*, and the outer coating, which can slide along the vulcanite cylinder, makes contact with the solenoid by means of the connection *K*, the latter, of course, moving along the solenoid as the outer coating moves along the vulcanite cylinder. The "capacity" and "inductance" can therefore be varied by this

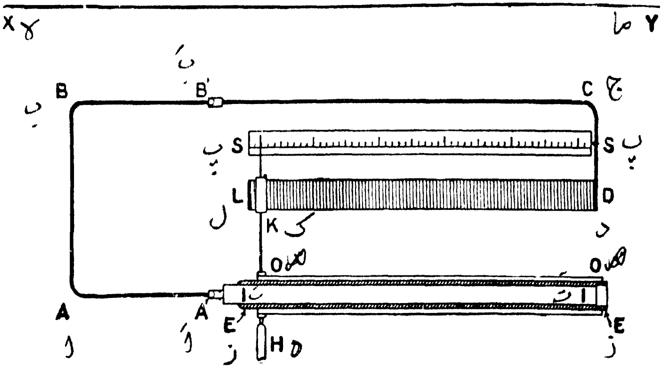


Fig. 488.

sliding device, which is operated by the handle *H*. If then oscillations occur in a neighbouring circuit, oscillations will be set up in the cymometer, and the effect will be a maximum (indicated by a neon tube attached to the condenser coatings) when the period of the cymometer $2\pi\sqrt{LC}$ is identical with that of the oscillations. The apparatus is adjusted for this maximum effect and the frequency calculated from the position of the slider pointer on the scale and the known constant of the instrument.

The cymometer may be used for the determination of capacity, specific inductive capacity, and inductance by oscillations. Thus imagine a *known* inductance and an unknown capacity in series with a spark gap. Oscillations can be set up in the circuit and the frequency n measured by the cymometer. Since $n = 1/2\pi\sqrt{LC}$ and L and n are known, C is determined.

Drude made use of the waves along a pair of parallel wires to determine the specific inductive capacity of a liquid and the *principle* of the method will be understood from Fig. 486. Suppose the wire bridge *EF* to be at a node and a second wire bridge, say *GH*, to be moved along the wires to the left until it occupies the position of the next node (a vacuum tube across the wires will indicate when *GH* reaches this position). The distance between the two bridges is $\frac{1}{2}\gamma_1$ so that the wave length γ_1 in air is found. A trough containing the liquid is now slid over the wires (the latter passing through holes in the ends) until the near end of the trough occupies the position of *GH*. A third bridge is now placed across the wires inside the trough and moved about as before until the distance between two nodes is again found. Thus γ_2 the wave length in the liquid corresponding to γ_1, the wave length in air is found.

Now if V_1 = velocity of electric waves in air and V_2 their velocity in the liquid then V_1/V_2 measures the index of refraction ρ. But $V_1 = n\gamma_1$ and $V_2 = n\gamma_2$ so that $\rho = \gamma_1/\gamma_2$. Again on the electromagnetic theory the dielectric constant K is equal to ρ^2. (See next section.) Hence :—

$$K = \left(\frac{\gamma_1}{\gamma_2}\right)^2,$$

thus K is determined.

314. The Electromagnetic Theory of Light. The Relation between the Index of Refraction and Specific Inductive Capacity. Fresnel and MacCullagh's Vibrations.

—The facts already dealt with in this chapter support the theory put forward by Maxwell that light waves are electromagnetic. In addition to the identity of velocity and various laws, both are propagated through a vacuum and both therefore require an "aether"; the "electrical" aether and the "optical" aether are identical.

Again, if v_1 and v_2 are the velocities of electric waves in two media, then

$$\left(\frac{v_1}{v_2}\right)^2 = \frac{K_2}{K_1} \cdot \frac{\mu_2}{\mu_1}.$$

Now for all transparent media μ_1 and μ_2 are practically equal and $v_1/v_2 = \rho$, the index of refraction from the first medium into the second. Hence for transparent media we have

$$\rho^2 = \frac{K_2}{K_1}.$$

If the first medium is air, for which $K_1 = 1$, then

$$\rho^2 = K_2.$$

That is, the specific inductive capacity of any medium (relative to air as unity) is equal to the square of the index of refraction of that medium. The index of refraction here involved is the index of refraction for electric waves of long wave length. If we take the optical index of refraction, however, we obtain the following; in the last two cases the discrepancy is very marked.

	K.	ρ^2.
Air	1·000590	1·000588
Hydrogen ...	1·000264	1·000276
Benzol	2·21	2·20
Paraffin... ...	2·29	2·02
Petroleum oil ...	2·07	2·07
Carbon bisulphide	2·67	2·67
Flint glass ...	10·12	2·92
Water	80-90	1·78

The agreement becomes nearer when ρ is measured for the slowest light vibrations and K for the most rapid electrical vibrations. In the case of water recent experiments with electric waves give $\rho^2 = 81$, which is in agreement.

In the case of polarised light Fresnel's theory is that the vibrations are perpendicular to the plane of polarisation, and MacCullagh's theory is that the vibrations are in the plane of polarisation. We have seen that in an electromagnetic wave we have both electric and magnetic forces at right angles. Theory shows the magnetic intensity is in the plane of polarisation, the electric intensity being perpendicular to that plane. These are both vibrating quantities, so that the former corresponds to MacCullagh's and the latter to Fresnel's vibration.

The energy is of course partly electric and partly magnetic; the former at any point is $2\pi D^2/K$ and the latter $\mu H^2/8\pi$ per unit volume. But $\mu H^2/8\pi = \mu\ (4\pi Dv)^2/8\pi = 2\pi\mu D^2v^2 = 2\pi\mu D^2/K\mu = 2\pi D^2/K$. The energy is therefore half electric and half magnetic.

Further details are dealt with in Chapter XXV.

In the preceding sections the theory of electromagnetic waves has been dealt with mainly from the point of view of Faraday tubes: we must now pass to a more purely mathematical treatment.

314a. Equations of a Field of Electric and Magnetic Forces referred to Rectangular Co-ordinates.—The four sets of equations of the electromagnetic field about to be established follow directly from important laws dealt with in previous chapters.

SET A.—*By Gauss's theorem the total normal induction* (*electric or magnetic*) *over a closed surface drawn in a field* (*electric or magnetic*) *is* 4π *times the total charge* (*electric or magnetic*) *inside* (Arts. 37*b*, 89).

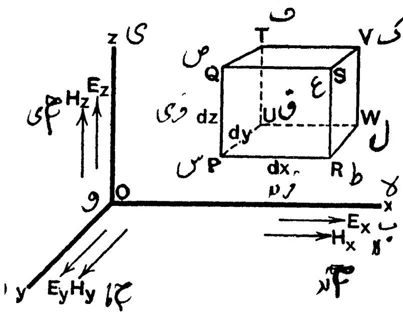


Fig. 488*a*.

Consider now the little parallelepiped drawn in the field (Fig. 488*a*), its sides being parallel to the three co-ordinate axes x, y, z. Let $PR = dx$, $PQ = dz$, $UP = dy$. Let E the electric intensity at P be resolved into rectangular components E_x, E_y, E_z, parallel to the axes x, y, z respectively.

Now the intensity at P in the x direction is E_x and at R it is

$$E_x + \frac{dE_x}{dx}dx, \text{ for } \frac{dE_x}{dx}$$

denotes the *rate of change* in this direction and dx is the distance The area of each of the faces $PQTU$ and $RSVW$ is $dy.dz$. Hence if K be the dielectric constant:—

$$\text{Normal Induction over } PQTU = KE_x dy\,.\,dz$$

$$\text{Normal Induction over } RSVW = K\left(E_x + \frac{dE_x}{dx}dx\right)dy\,.\,dz$$

and since the first is inwards and the second outwards, the total normal induction for these two faces is :—

$$K\left(E_x + \frac{dE_x}{dx}dx\right)dy\,.\,dz - KE_x dy\,.\,dz = K\frac{dE_x}{dx}dx\,.\,dy\,.\,dz.$$

Similarly it can be shown that the normal induction for the two faces $PQSR$ and $TUWV$ is $K\frac{dE_y}{dy}dx\,.\,dy\,.\,dz$, and for the remaining two faces $PUWR$ and $QTVS$ it is $K\frac{dE_z}{dz}dx\,.\,dy\,.\,dz$. Adding these we get the total normal induction for the whole closed surface, *i.e.*

$$\text{Total Normal Induction} = K\left(\frac{dE_x}{dx} + \frac{dE_y}{dy} + \frac{dE_z}{dz}\right)dx\,.\,dy\,.\,dz.$$

Again, if ρ be the volume density of the charge the total charge inside is $\rho dx\,.\,dy\,.\,dz$. Hence by Gauss's theorem :—

$$K\left(\frac{dE_x}{dx} + \frac{dE_y}{dy} + \frac{dE_z}{dz}\right)dx\,.\,dy\,.\,dz = 4\pi\rho dx\,.\,dy\,.\,dz,$$

$$\therefore \quad \frac{dE_x}{dx} + \frac{dE_y}{dy} + \frac{dE_z}{dz} = \frac{4\pi\rho}{K}.$$

By a similar treatment, if H be the magnetic field at P, and H_x, H_y, H_z the three rectangular components, we get:—

$$\frac{dH_x}{dx} + \frac{dH_y}{dy} + \frac{dH_z}{dz} = \frac{4\pi\delta}{\mu},$$

where δ is the volume density of magnetism and μ the permeability. If ρ and δ are zero the expression on the left is, in each case, zero. Collecting, we get our first set of equations:—

$$\left.\begin{aligned} \frac{dE_x}{dx} + \frac{dE_y}{dy} + \frac{dE_z}{dz} = \frac{4\pi\rho}{K} &= O \text{ if } \rho \text{ is zero} \\ \frac{dH_x}{dx} + \frac{dH_y}{dy} + \frac{dH_z}{dz} = \frac{4\pi\delta}{\mu} &= O \text{ if } \delta \text{ is zero} \end{aligned}\right\} \quad \ldots\ldots\ldots \text{(1)}$$

SET B.—In Art. 173 it is shown that *the work done in carrying a unit pole round a current is* 4π *times the strength of the current.* Now consider a *very small* area $PQTU$ parallel to the plane yz (Fig. 488b) and let $TQ = dy$ and $PQ = dz$. Let I_x be the component in the x direction of the current density so that the total current through $PQTU$ in this direction is $I_x dy.dz$. If H_y be the field intensity along UP, the value along TQ will be $H_y + \frac{dH_y}{dz}dz$ (see above); similarly, if H_z be the value along UT, the value along PQ will be $H_z + \frac{dH_z}{dy}dy$. Now:—

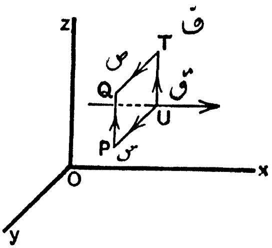


Fig. 488b.

Work *done on* unit pole going along $PU = H_y dy$

Work *done on* unit pole going along $UT = -H_z dz$

Work *done on* unit pole going along $TQ = -\left(H_y + \frac{dH_y}{dz}dz\right) dy$

Work *done on* unit pole going along $QP = \left(H_z + \frac{dH_z}{dy}dy\right) dz$

$\therefore$ Total work done on unit pole in going round the path

$$= H_y dy - H_z dz - \left(H_y + \frac{dH_y}{dz}dz\right) dy + \left(H_z + \frac{dH_z}{dy}dy\right) dz$$

$$= \left(\frac{dH_z}{dy} - \frac{dH_y}{dz}\right) dy.dz.$$

Equating this to 4π times the current, viz. $4\pi I_x dy.dz$ we get:—

$$\frac{dH_z}{dy} - \frac{dH_y}{dz} = 4\pi I_x.$$

The current components I_y and I_z in the y and z directions can be similarly dealt with by taking little areas parallel to xz and xy· hence we have our second set of equations :—

$$\left.\begin{aligned} \frac{dH_z}{dy} - \frac{dH_y}{dz} &= 4\pi I_x \\ \frac{dH_x}{dz} - \frac{dH_z}{dx} &= 4\pi I_y \\ \frac{dH_y}{dx} - \frac{dH_x}{dy} &= 4\pi I_z \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

SET C.—In Art. 239 it is proved that *the induced E.M.F. is equal to the rate of change of the number of unit tubes of induction threading a circuit*, i.e. that $e = - dF/dt$. Now consider again Fig. 488*b*. The *induction* through the area is $\mu H_x dydz$ where μ is the permeability, so that $e = - dF/dt = - \mu \frac{dH_x}{dt} dy \,.\, dz$. Further, E_y is the intensity (electric) along UP, and therefore $E_y + \frac{dE_y}{dz} dz$ is the intensity along TQ. Similarly E_z is the intensity along UT and $E_z + \frac{dE_z}{dy} dy$ the value along PQ. Now intensity is defined as numerically equal to the force on unit quantity, so that intensity multiplied by distance will give the work in moving unit quantity, *i.e.* it will measure the E.M.F. ; thus for the E.M.F. round $PUTQ$ we have :—

$$E_y dy - E_z dz - \left(E_y + \frac{dE_y}{dz} dz\right) dy + \left(E_z + \frac{dE_z}{dy} dy\right) dz,$$

$$\textit{i.e.} \left(\frac{dE_z}{dy} - \frac{dE_y}{dz}\right) dy \,.\, dz,$$

and equating this to $- \mu \frac{dH_x}{dt} dydz$ obtained above we get :—

$$\frac{dE_z}{dy} - \frac{dE_y}{dz} = - \mu \frac{dH_x}{dt}.$$

Dealing similarly with a *little area* parallel to the plane xy and another parallel to xz, we obtain similar expressions and complete our third set of equations, viz. :—

$$\left.\begin{aligned} \frac{dE_z}{dy} - \frac{dE_y}{dz} &= - \mu \frac{dH_x}{dt} \\ \frac{dE_x}{dz} - \frac{dE_z}{dx} &= - \mu \frac{dH_y}{dt} \\ \frac{dE_y}{dx} - \frac{dE_x}{dy} &= - \mu \frac{dH_z}{dt} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

SET D.—In Art. 88 it is shown that *the polarisation or displacement at any point in a medium is measured by the density* ρ *on a conducting surface placed at that point and by the number* D *of Faraday tubes per unit area at the point.* Now Maxwell's so called "displacement current" is measured by the rate of change of the electrical polarisation or displacement: in symbols $I = \frac{d}{dt}D$. Further, D is equal to $KE/4\pi$ (Art. 88), so that $I = \frac{K}{4\pi}\frac{dE}{dt}$. Now applying this to our I and E components we get:—

$$I_x = \frac{K}{4\pi}\frac{dE_x}{dt};\ I_y = \frac{K}{4\pi}\frac{dE_y}{dt};\ I_z = \frac{K}{4\pi}\frac{dE_z}{dt},$$

and substituting these values in (2) we get our fourth set of equations, viz.:—

$$\left.\begin{aligned} \frac{dH_z}{dy} - \frac{dH_y}{dz} &= K\frac{dE_x}{dt} \\ \frac{dH_x}{dz} - \frac{dH_z}{dx} &= K\frac{dE_y}{dt} \\ \frac{dH_y}{dx} - \frac{dH_x}{dy} &= K\frac{dE_z}{dt} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

314b. Wave Motion.—Taking the first equation in Set 4, viz.:—

$$K\frac{dE_x}{dt} = \frac{dH_z}{dy} - \frac{dH_y}{dz},$$

we get on differentiating:—

$$K\frac{d^2E_x}{dt^2} = \frac{d}{dy}\left(\frac{dH_z}{dt}\right) - \frac{d}{dz}\left(\frac{dH_y}{dt}\right).$$

Substitute from (3) the values of dH_z/dt and dH_y/dt, and we get:—

$$K\frac{d^2E_x}{dt^2} = \frac{d}{dy}\left\{\frac{1}{\mu}\left(-\frac{dE_y}{dx} + \frac{dE_x}{dy}\right)\right\} - \frac{d}{dz}\left\{\frac{1}{\mu}\left(-\frac{dE_x}{dz} + \frac{dE_z}{dx}\right)\right\}$$

$$= \frac{1}{\mu}\left\{-\frac{d^2E_y}{dx\,.\,dy} + \frac{d^2E_x}{dy^2} + \frac{d^2E_x}{dz^2} - \frac{d^2E_z}{dxdz}\right\}$$

$$i.e.\ K\mu\frac{d^2E_x}{dt^2} = \frac{d^2E_x}{dy^2} + \frac{d^2E_x}{dz^2} + \frac{d^2E_x}{dx^2} - \frac{d^2E_y}{dxdy} - \frac{d^2E_z}{dxdz} - \frac{d^2E_x}{dx^2}$$

on adding and subtracting d^2E_x/dx^2 on the right hand side. Hence:—

$$K\mu\frac{d^2E_x}{dt^2} = \frac{d^2E_x}{dx^2} + \frac{d^2E_x}{dy^2} + \frac{d^2E_x}{dz^2} - \frac{d}{dx}\left(\frac{dE_x}{dx} + \frac{dE_y}{dy} + \frac{dE_z}{dz}\right).$$

But from the first equation of Set 1 the last term in brackets is zero, hence:—

$$\frac{d^2E_x}{dx^2} + \frac{d^2E_x}{dy^2} + \frac{d^2E_x}{dz^2} = K\mu\frac{d^2E_x}{dt^2}.$$

Clearly then E_x satisfies the differential equation:—

$$\frac{d^2\theta}{dx^2} + \frac{d^2\theta}{dy^2} + \frac{d^2\theta}{dz^2} = \frac{1}{c^2}\frac{d^2\theta}{dt^2},$$

which, in works on higher mathematics, is shown to be *a general equation of wave motion, the velocity being c*. In this case c is evidently $1/\sqrt{K\mu}$. In the above we started with the first equation of Set 4, but similarly all the other components of electric and magnetic force can be shown to satisfy the same equation. **Thus electromagnetic actions are propagated through the aether with velocity $1/\sqrt{K\mu}$, which is, for example, equal to the ratio of the electrostatic to the electromagnetic unit charge, and which has been found equal to the velocity of light by various experiments.** The following simple case will fix ideas, lend definiteness to them, and bring out again the facts already proved in dealing with electromagnetic waves from the conception of Faraday tubes (Arts. 308-312).

314c. A Simpler Case of Wave Motion.—The facts dealt with in preceding pages will be emphasised by the consideration of a **plane wave**, *i.e.* a wave in which the intensity is the same over the whole plane at any instant. We will take the plane yz as the plane of the wave (the latter to travel to the right in the x direction). Hence, since the differential coefficient of a constant is zero, the differential coefficients with respect to y and z are zero since yz is the plane of the wave. The equations in Set 3 now become therefore:—

$$-\mu\frac{dH_x}{dt}=O;\ \mu\frac{dH_y}{dt}=\frac{dE_z}{dx};\ -\mu\frac{dH_z}{dt}=\frac{dE_y}{dx}\ldots(a)$$

These are written down from the equations of Set 3 by remembering that terms of the form dE/dy and dE/dz or dH/dy and dH/dz are zero. From the fact that $-\mu\frac{dH_x}{dt}=O$ it follows that H_x is a constant or zero: we must take it as zero for reasons into which we need not enter.

Similarly the equations in Set 4 become:—

$$K\frac{dE_x}{dt}=O;\ K\frac{dE_y}{dt}=-\frac{dH_z}{dx};\ K\frac{dE_z}{dt}=\frac{dH_y}{dx}\ldots\ldots\ (b)$$

and from the first one (as in the case above) E_x is zero.

Clearly then since $H_x=O$ and $E_x=O$ it follows that H and E, *i.e.* **the magnetic and electric intensities are both in the plane of the wave,** *i.e.* in the plane yz. Up to this point, then, we are left in the mathematics with H_y and E_y and H_z and E_z to be definitely settled.

Now let us take E the electric intensity as being parallel to the z direction: this means that E in the y direction is zero, *i.e.* $E_y=O$. From the relation $K\frac{dE_y}{dt}=-\frac{dH_z}{dx}$ above, if E_y is zero, H_z is zero. We now have therefore only E_z and H_y, *i.e.* the electric intensity in the z direction and the magnetic intensity in the y direction. Clearly then **the electric and magnetic intensities are at right angles to each other.** (Compare Fig. 485*b*)

Taking now equations (a) above, since $H_x=O$, and $H_z=O$ we are left with:—

$$\mu\frac{dH_y}{dt}=\frac{dE_z}{dx}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (c)$$

Taking also equations (b) above, since $E_x=O$ and $E_y=O$ we are left with:—

$$K\frac{dE_z}{dt}=\frac{dH_y}{dx}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (d)$$

From (*c*) and (*d*) by differentiating (*c*) with respect to x, and (*d*) with respect to t we get:—

$$\mu \frac{d^2H_y}{dx.dt} = \frac{d^2E_z}{dx^2} \text{ and } K\frac{d^2E_z}{dt^2} = \frac{d^2H_y}{dx.dt}$$

$$\therefore \frac{d^2E_z}{dt^2} = \frac{1}{K\mu}\frac{d^2E_z}{dx^2} \ldots\ldots\ldots\ldots\ldots\ldots \quad (e)$$

Again from (*c*) and (*d*) by differentiating (*c*) with respect to t, and (*d*) with respect to x we get:—

$$\mu \frac{d^2H_y}{dt^2} = \frac{d^2E_z}{dx.dt} \text{ and } K\frac{d^2E_z}{dx.dt} = \frac{d^2H_y}{dx^2}$$

$$\therefore \frac{d^2H_y}{dt^2} = \frac{1}{K\mu}\frac{d^2H_y}{dx^2} \ldots\ldots\ldots\ldots\ldots\ldots \quad (f)$$

Remembering that the electrical intensity is in the z direction, viz. E_z, and the magnetic intensity in the y direction, viz. H_y, we will throughout the remainder of this section write, for simplicity, H for H_y, and E for E_z, so that equations (*e*) and (*f*) become:—

$$\frac{d^2E}{dt^2} = \frac{1}{K\mu}\frac{d^2E}{dx^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (g)$$

$$\frac{d^2H}{dt^2} = \frac{1}{K\mu}\frac{d^2H}{dx^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (h)$$

Now these are the equations to plane waves parallel to Ox, the velocity c being $1/\sqrt{K\mu}$. (See Art. 314*b*.) The general solution to the first one is:—

$$E = \alpha\,(x - ct) + \beta\,(x + ct)$$

where α and β are any functions and $c = 1/\sqrt{K\mu}$. Further $\alpha\,(x - ct)$ denotes a wave motion in the direction Ox and $\beta\,(x + ct)$ one in the direction xO. We will first consider the former in the direction Ox and will take the case of E (really, of course, E_z) varying harmonically. Let

$$E = E_0 \sin \frac{2\pi}{l}(x - ct) \ldots\ldots\ldots\ldots\ldots \quad (m)$$

so that when $t = O$ we have:—

$$E = E_0 \sin \frac{2\pi}{l}x \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (n)$$

The curve E in Fig. 488c represents this. The maximum value of E is E_0. Further, if x be increased by an amount l we get:—

$$E = E_0 \sin \frac{2\pi}{l}(x + l) = E_0 \sin\left(\frac{2\pi}{l}x + 2\pi\right)$$

and the curve therefore begins to repeat: hence l is the wave length and is represented by the distance OS in Fig. 488c. At P, $x = l/4$ and substituting this in (n) we get $E = E_0$ (viz. Pp in Fig. 488c). At Q, $x = l/2$ and E is zero. At R, $x = 3l/4$ and $E = - E_0$ (viz. Rr in Fig. 488c) and so on.

An expression for H (really of course H_y) corresponding to the expression (m) for E, viz.:—

$$E = E_0 \sin \frac{2\pi}{l}(x - ct)$$

is readily obtained from the relation already established in (c) above, viz. $\mu \frac{dH_y}{dt} = \frac{dE_z}{dx}$. On working out the simple differentiation and integration we get:—

$$H = - H_0 \sin \frac{2\pi}{l}(x - ct) \quad \text{................} \quad (o)$$

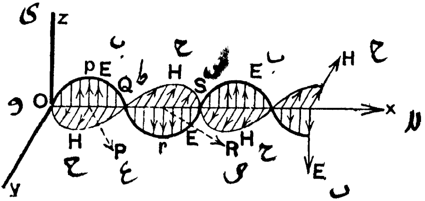


Fig. 488c.

Similarly, corresponding to (n), viz.

$$E = E_0 \sin \frac{2\pi}{l}x$$

we get

$$H = - H_0 \sin \frac{2\pi}{l}x \quad \text{.................} \quad (p)$$

This is represented by the curve H in Fig. 488*c*. The student will note that Fig. 488*c* is identical with Fig. 485*b* which we deduced from the idea of Faraday tubes. Incidentally, it is a matter of simple mathematics to show that the relation between the maximum values E_0 and H_0 is that $\sqrt{\mu}H_0 = \sqrt{K}E_0$.

314d. Reflection and the Production of Stationary Oscillations.—Consider now a plane wave such as the preceding travelling, however, *from right to left*, i.e. in the direction xO towards the plane $y.z$. From the preceding its E equation is evidently :—

$$E = E_0 \sin \frac{2\pi}{l}(x + ct) \dots\dots\dots\dots\dots\dots (q)$$

and the corresponding expression for H obtained as indicated above is in this case :—

$$H = H_0 \sin \frac{2\pi}{l}(x + ct) \dots\dots\dots\dots\dots\dots (r)$$

Imagine now that the plane $y.z$ is a metal sheet constituting a perfect conductor and in which therefore the electric intensity must be zero. The electric intensity at the sheet, due to the incident wave travelling from right to left is obtained by putting $x = O$ in (q), *i.e.* it is given by :—

$$E = E_0 \sin \frac{2\pi ct}{l}$$

and therefore for the intensity to be zero an equal and opposite intensity must be set up in the sheet, *i.e.* an intensity given by :—

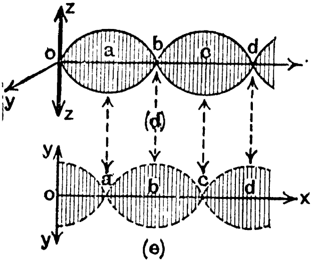
$$E = - E_0 \sin \frac{2\pi ct}{l}.$$

Now this gives rise to a reflected wave travelling back *from left to right*, and given by the equation :—

$$E = E_0 \sin \frac{2\pi}{l}(x - ct) \dots\dots\dots\dots\dots\dots (s)$$

for this latter evidently becomes at the sheet $(x = O)$

$E = -E_0 \sin \frac{2\pi ct}{l}$, *i.e.* the necessary value mentioned above for the electric intensity to be zero in the sheet.



Figs. 488*d*, 488*e*.

Consider now the resultant E due to both the incident and reflected waves. Evidently:—

$$E = E_0 \sin \frac{2\pi}{l}(x + ct) + E_0 \sin \frac{2\pi}{l}(x - ct)$$
$$= 2E_0 \sin \frac{2\pi}{l} x . \cos \frac{2\pi}{l} ct$$
$$= a \cos \frac{2\pi}{l} ct$$

which represents a steady oscillation of amplitude a where $a = 2E_0 \sin \frac{2\pi}{l} x$.

The variation in a is graphically represented in Fig. 488*d*. At the reflecting surface $x = O$, and $a = O$. At a, $x = l/4$ and a is a maximum, viz. $2E_0$. At b, $x = l/2$ and $a = O$. At d, $x = l$ and $a = O$ and so on. The student should note that this Fig. 488*d* is identical with Fig. 485*e* deduced from the view of Faraday tubes.

Turning now to the question of H: the H equation for the incident ray (viz. equation r) is:—

$$H = H_0 \sin \frac{2\pi}{l}(x + ct)$$

whilst the H equation corresponding to the E equation for the reflected ray, *i.e.* corresponding to equation (s) is obtained from (*s*) by simple differentiation and integration as indicated in the preceding section and is found to be:—

$$H = -H_0 \sin \frac{2\pi}{l}(x - ct).$$

Consider now the resultant H due to both the incident and reflected waves. Evidently:—

$$\begin{aligned} H &= H_0 \sin \frac{2\pi}{l}(x + ct) - H_0 \sin \frac{2\pi}{l}(x - ct) \\ &= 2H_0 \cos \frac{2\pi}{l} x . \sin \frac{2\pi}{l} ct \\ &= \beta \sin \frac{2\pi}{l} ct \end{aligned}$$

where as before $\beta = \text{amplitude} = 2H_0 \cos \frac{2\pi}{l} x$.

The variation in β is graphically represented in Fig. 488*e*. At the reflecting sheet $x = O$ and β is a maximum, viz. $2H_0$. At *a*, $x = l/4$ and $\beta = O$. At *b*, $x = l/2$ and β is again a maximum. At *c*, $x = 3l/4$ and $\beta = O$ and so on. The student should note carefully the difference between the two cases Fig. 488*d* and Fig. 488*e*: where E has its greatest variation H has its least, and where E has its least H has its greatest.

314e. Waves along Wires.—These can be dealt with by methods somewhat similar to those of the preceding sections. Without going into the mathematical details, the solution corresponding to a stationary wave in the case of waves along parallel wires (perfect conductors) may be approximately stated as:—

$$V = P \cos \frac{2\pi}{l} x . \cos \frac{2\pi}{l} ct$$

$$I = Q \sin \frac{2\pi}{l} x \sin \frac{2\pi}{l} ct$$

V denoting "potential" and I "current."

Now in the case of wires "free" at both ends it is evident I must $= O$ when $x = O$ and when $x = L$ where L is the length of the wires. The equation above for I satisfies the first condition, *i.e.* $I = O$ when $x = O$. Clearly it will satisfy the second condition if $l = 2L$ for if $l = 2L$ and $x = L$ it follows that $\sin \frac{2\pi}{l} x = O$ and

$I = O$. Clearly also it will satisfy the second condition if $l = L$, $l = \frac{2}{3} L$, etc. Thus in this case the possible wave lengths of the stationary waves are $l = 2L$, L, $\frac{2}{3} L$ etc. The general formula for this is in fact that l is equal to $2L/N$ where N is an integer.

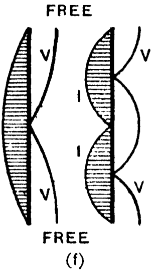


Fig. 488*f*.

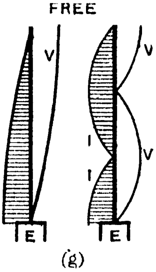


Fig. 488*g*.

Taking the V equation, at $x = O$ V is a maximum. At $x = L$ (taking the case of $l = 2L$) it is also a maximum. At $x = \frac{1}{2} L$, V is zero for $\cos \frac{2\pi}{2L} . \frac{L}{2} = \cos \frac{\pi}{2} = O$.

The variation of V and I for the cases where $l = 2L$ and $l = L$ (*i.e.* the "fundamental" and the "first harmonic") are graphically shown in Fig. 488*f*.

The case of the antenna used in Wireless Telegraphy may be dealt with somewhat similarly. The top end being free is a region of maximum variation for V and the bottom end being earthed is a node for V. The first two possible cases are shown in Fig. 488*g*. The student should work this and other cases out for himself.

Of course, in practice, wires are not "perfect conductors" and the capacity and inductance of wires become important when high frequency currents are being dealt with: thus there are several factors which render the more exact mathematics of waves along wires very complicated.

Exercises XXII.

Section A.

(1) Develop expressions for the frequency of the oscillations of a discharging condenser.

(2) Write a short essay on "Wireless Telegraphy."

(3) Explain any facts you are acquainted with which support the Electromagnetic Theory of Light.

Section B.

(1) Show that the discharge of a condenser is in general oscillatory, and describe one method by which the period of the oscillations may be measured. (B.E. Hons.)

(2) How has it been shown experimentally that the current in discharging a condenser may be alternating in character? Explain shortly how this fact has been utilised for telegraphing through space without wires. (B.E. Hons.)

Range of Aether Waves.

The following table will be interesting: the X-rays and Gamma rays are dealt with in the next chapter.

	Wave.	Approximate Frequencies (per second).
	Gamma rays	Hundreds of thousands of millions of millions
	X rays	↓
	Ultra violet	
Spectrum	Violet	800 millions of millions
	Indigo	↓
	Blue	
	Green	
	Yellow	
	Orange	
	Red	400 millions of millions
	Infra red	↓
	↓	
	Laboratory electric waves	30 thousand millions
	↓	↓
	Short wireless waves	3,000,000
	Broadcasting band	1,000,000—500,000
	Long wireless waves	170,000
	↓	↓
	Longer waves from an alternator in an alternating current power station.	

CHAPTER XXIII.

THE PASSAGE OF ELECTRICITY THROUGH A GAS.

315. Discharge at Low Pressure.—The phenomena of the electric discharge through gases at ordinary pressure have been dealt with in Chapter IX. Consider now the gas to be in a convenient tube provided with an electrode at each end, and, further, let the electrodes be connected to the secondary of an induction coil and the tube to a pump, so that the tube may be gradually exhausted. The following summarises the main results as exhaustion proceeds:—

(1) At pressures fairly low, say of the order of a centimetre of mercury, the discharge is a luminous column stretching from the anode almost to the kathode; it is known as the *positive column.*

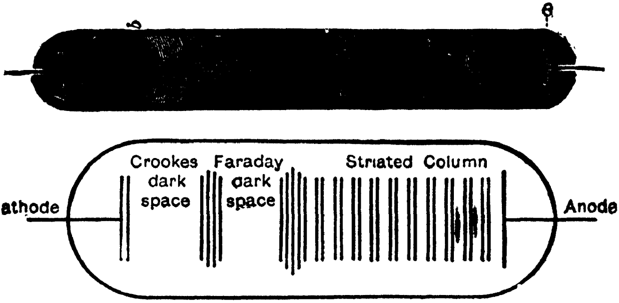


Fig. 489.

(2) At lower pressures the column breaks up into alternate bright shells and dark patches; there is a glow on the kathode, and between this and the first bright shell is a dark space known as the *Faraday dark space.*

(3) Most of the glow on the kathode moves from it, and another dark space appears between this and the kathode, known as the *Crookes dark space* (Fig. 489).

(4) At lower pressures the Crookes dark space extends until it practically fills the tube and the glass becomes phosphorescent, the colour being yellowish green for soda glass. It is in this condition that the tube is emitting aether pulses called **X rays,** and it will be seen presently that these are due to the bombardment of the walls, etc., by negatively charged particles *proceeding from the kathode*, and known as **kathode rays.**

(5) At still lower pressures the current diminishes, and at sufficiently high vacuum no current will pass.

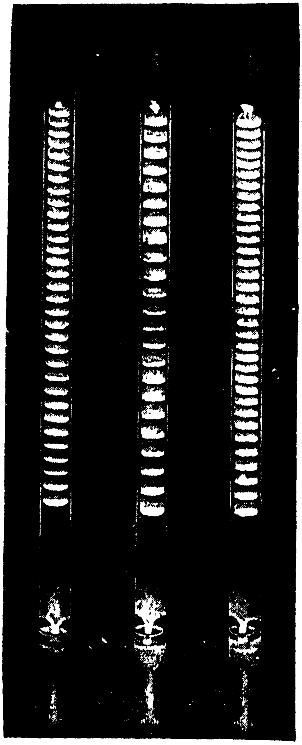


Fig. 490.

The above phenomenon must be seen to be fully realised; Fig. 490 (De La Rue and Muller) will, however, further assist the reader in grasping details.

316. Kathode Rays.—The kathode rays were discovered by Plücker in 1859, and the main properties may be briefly summarised as follows:—

(1) The rays are shot out normally from the kathode, their direction being in no way connected with the position of the anode.

(2) They travel in straight lines and cast shadows of objects placed in their path; in Crookes's experiment a hinged Maltese cross is placed opposite the kathode (Fig. 491) and a shadow appears, as shown, on the end of the tube. If the cross be lowered

the shadow disappears; its shape may perhaps still be seen (brighter than the rest) if the rest of the glass has become "fatigued" by phosphorescence.

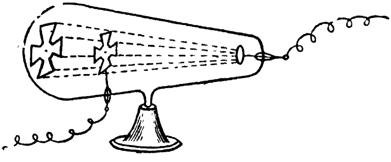


Fig. 491.

(3) A body placed in their path experiences a force tending to urge it away from the kathode. Thus rotation is produced in the case of the wheel fitted with vanes (Fig. 492), on which the rays fall.

(4) The rays produce heat when they fall upon matter. The motion in (3) is probably a radiometer effect caused by the heating of the surfaces on which the rays fall.

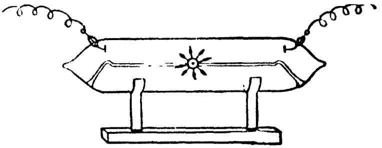


Fig. 492.

(5) They produce phosphorescence. The colour produced on the walls of the tube where struck by these rays depends on the chemical nature of the glass. Lead glass phosphoresces blue, soda glass yellowish green. Phosphorescence is also produced in barium platino-cyanide and the rare earths (cerium, lanthanum, etc.). Some bodies change colour, *e.g.* rock salt, which becomes violet, while in some cases chemical changes occur, though very likely this is partly due to the heating effect as well as to the phosphorescence. The kathode rays have a reducing effect.

(6) When the rays strike a solid obstacle the latter becomes a source of *aether pulses* known as **Röntgen** or **X rays.**

(7) The rays are deflected by a magnetic field, *the direction of deflection being that in which a stream of negatively charged particles would be deflected.* The deflection, under like conditions, is independent of the gas in the tube before exhaustion and of the kathode material.

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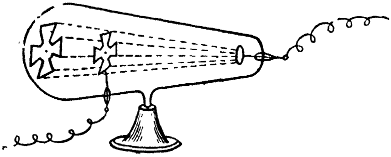

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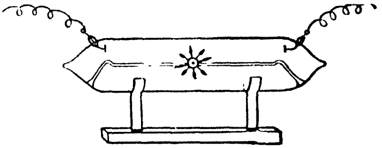

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(6) When the rays strike a solid obstacle the latter becomes a source of *aether pulses* known as **Röntgen** or **X rays.**

(7) The rays are deflected by a magnetic field, *the direction of deflection being that in which a stream of negatively charged particles would be deflected.* The deflection, under like conditions, is independent of the gas in the tube before exhaustion and of the kathode material.

(8) The rays "ionise" a gas and make it a conductor.

Goldstein (1876) regarded kathode rays as waves in the aether, a view long upheld in Germany, but in England Varley (1871) and Crookes (1876) contended that they were charged particles. Recent work has confirmed the latter view; the rays consist of negatively charged particles of mass about 1/2000 that of a hydrogen atom moving with a velocity about 1/10 that of light and carrying a charge equal to the smallest quantity or "atom of electricity" ($4{\cdot}65 \times 10^{-10}$ e.s. units). They are called "corpuscles" by J. J. Thomson and "electrons" by Johnstone Stoney. Further, they are probably not "matter" as ordinarily understood but simply "electricity" (Art. 338).

The fact that the kathode rays are negatively charged was proved by J. J. Thomson as follows. The apparatus (Fig. 493) consists of a vacuum tube provided with two bulbs. *C* is the kathode, and some of the rays leaving it pass through the slit *S* in a brass plug *A*, which is used as the anode, and strike the opposite wall of the larger bulb, giving a phosphorescent patch. Attached to this bulb is a side tube containing an earthed cylinder *M*, and within that, but carefully insulated from it, another small cylinder connected by a wire passing through the end of the tube to an electroscope or electrometer.

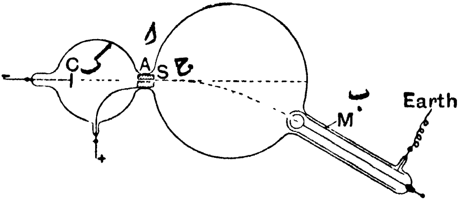


Fig. 493.

The cylinders are out of the direct line of fire, and when the rays are undeviated the electrometer shows no increase of charge. A magnet is now brought up to the bulb and the rays are deviated until—as shown by their phosphorescence—they pass through the slit of *M* into the insulated cylinder. The electrometer at once shows that the inner cylinder is being negatively charged, thus proving that the rays carry a negative charge. Of course *the direction of deflection* also confirms this. ✓

The charge on the inner cylinder will, however, not go on increasing indefinitely, for kathode rays render the gas through which they pass conducting, and so, as the potential of the insulated cylinder rises, more and more charge passes from it across the space to *M* and so to earth.

317. A Preliminary Note to the Newer Researches.—Before proceeding to the more recent experimental work on this subject the following points should be carefully noted:—

(*a*) *The effect of the motion of a body charged electrostatically is the same as that of a current.*—Maxwell proved theoretically that a charge of q electrostatic units moving with a velocity v is equivalent to a current of qv electrostatic units or $qv/(3 \times 10^{10})$ electromagnetic units; this was confirmed experimentally by Rowland.

(*b*) *Deflection of a stream of electrified particles shot across a magnetic field.*—Let n be the number of particles in unit length, m the mass of each particle, e its charge, v the velocity of propagation, and H the strength of the magnetic field supposed uniform and perpendicular to the direction of motion. Now by the last section the equivalent current due to the particles $= nev$. The force exerted on the particles in unit length by the field is, by Art. 170, equal to $nevH$, and is at right angles to both the direction of motion and the field. The force on each particle is therefore evH. The particle will therefore be deflected from its straight-line path, and will move along an arc of a circle in a plane *perpendicular to the lines of magnetic force.* Let R be the radius of this circle, then, since evH is the centripetal force on a particle of mass m,

$$\frac{1}{R} \cdot mv^2 = evH, \qquad \therefore \frac{e}{m} = \frac{v}{RH}.$$

If AB (Fig. 494) represents the line of moving particles before the field is on, and AC the line after the field is on,

$$AB^2 = BC\,(2R - BC),$$

from which, after measuring AB and BC, R can be found. If H is known, and if v can also be found, it is a matter of easy calculation to find e/m.

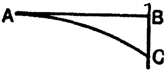


Fig. 494.

If the particle is projected obliquely to the magnetic field, it will move along a helix whose axis is parallel to the field.

(*c*) *Deflection of a stream of electrified particles shot across an* *electrostatic field.*—If the field is uniform and of strength X the force on each particle throughout its motion is Xe, and as before, if the field is perpendicular to the initial direction of motion, we get

$$\frac{1}{R'}mv^2 = Xe,$$

or

$$\frac{e}{m} = \frac{v^2}{XR'},$$

where R' is the radius of curvature of the path. The deflection in this case is perpendicular to the initial direction of propagation, and *in the direction of the electrostatic field.*

(*d*) *Case when both fields are in action.*—If the electrostatic field is perpendicular to the magnetic field the sign of the difference of potential may be altered so that the separate deflections due to the fields are in opposite directions. In this case, if the strengths of the fields are so adjusted as not to deviate the stream of particles, we must have

$$evH = Xe,$$

or

$$\frac{X}{H} = v,$$

so that by an experiment of this nature v can be found. If now either of the two foregoing experiments, (*b*) or (*c*), is performed the value of e/m may be found.

If the particles have different velocities, or there are different values of e/m, a dispersion effect will be observed in both the magnetic and electrostatic experiments.

318. Determination of e/m and v for the Kathode Rays. Thomson's Method.

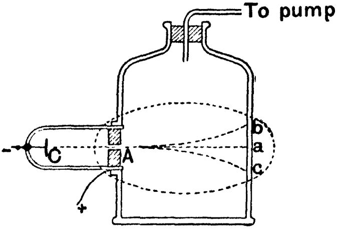


Fig. 495.

—J. J. Thomson used the apparatus of Fig. 495 (C = kathode, A = anode). It was placed between the poles of a large magnet so that a uniform magnetic field could be established over the space indicated by the dotted oval area, this field being perpendicular to the path of the kathode rays from C, *i.e.* perpendicular to the plane of the paper. Before the magnetic field is established the kathode rays produce a patch of phosphorescence at a. When the field is put on in one direction the phosphorescent path moves to b and when the field is reversed it moves to c. The actual deflection of the rays due to the field is half bc and from this and the known horizontal distance Aa, the radius of curvature R of the path of the rays can be determined as indicated in Art. 317.

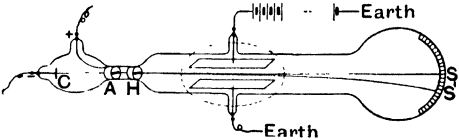


Fig. 496.

To determine v, J. J. Thomson used the apparatus of Fig. 496. The kathode rays from C pass through small slits in the anode A and in the metal plug H and this narrow beam of rays produces phosphorescence at S. A magnetic field uniform over the dotted oval area is then applied perpendicular to the plane of the paper and the phosphorescent patch moves to S'. By means of the two horizontal plates shown in the figure an electrostatic field is then put on in the necessary direction and its strength adjusted until it exactly neutralises the effect of the magnetic field and the phosphorescent patch moves back to S. If X denotes the known intensity of the electrostatic field which just neutralises the magnetic field of known strength H, then from Art. 317:—

$$v = \frac{X}{H}$$

and v is therefore determined. Turning now to the preceding experiment (Fig. 495) we have from Art. 317:—

$$\frac{e}{m} = \frac{v}{RH}$$

so that knowing v, R, and H the ratio e/m is determined. *The value of e/m was found to be independent of the material of the kathode and independent of the nature of the gas in the tube before exhaustion.*

The value of e/m was found to be $7{\cdot}7 \times 10^6$ electromagnetic C.G.S. units per gramme, v was found to vary between $2{\cdot}2 \times 10^9$ and $3{\cdot}6 \times 10^9$ cm. per sec. More recent results give e/m to be $1{\cdot}772 \times 10^7$ e.m. units per gramme. Simon and Kaufmann give the value $1{\cdot}86 \times 10^7$ e.m. units or $5{\cdot}6 \times 10^{17}$ e.s. units per gramme.

319. Determination of e/m and v for the Kathode Rays. Another Thomson Method.—A second method due to J. J. Thomson is interesting and may be briefly dealt with: it is however not so reliable and accurate as the method of the preceding section.

The apparatus is similar to that of Fig. 493. The kathode rays from C pass through the slit in A and then by means of a magnet are deflected so as to fall into the small insulated cylinder which is joined to the electrometer. If N electrons enter the cylinder in one second, Ne will be the charge given to the cylinder in one second, and this is therefore known from the known capacity of the system and the change of potential per second indicated by the electrometer. Let Ne be denoted by Q.

Now the rays on entering the small cylinder fall upon one junction of a thermo-electric couple. This thermo-electric circuit includes a galvanometer from the indications of which the rise in temperature of the junction in one second due to bombardment by the electrons can be determined. Knowing the capacity for heat of the couple and the rise in temperature per second, the energy w imparted to the junction in one second is known.

Now if v be the velocity of the kathode particles the kinetic energy of each is $\frac{1}{2}mv^2$, and since N particles enter per second, the energy imparted to the junction per second is $\frac{1}{2}Nmv^2$. Thus $\frac{1}{2}Nmv^2 = w$, and since $Ne = Q$, i.e. $N = Q/e$, we get

$$\tfrac{1}{2}Q\frac{m}{e}v^2 = w.$$

But from the known deflection by a known magnetic field

$$\frac{e}{m} = \frac{v}{RH} \text{ or } \frac{m}{e} = \frac{RH}{v}.$$

Substituting this for m/e in the above we get

$$v = \frac{2w}{QRH},$$

and
$$\therefore \frac{e}{m} = \frac{2w}{QH^2R^2}.$$

The mean value obtained by this method was $e/m = 1{\cdot}2 \times 10^7$ electromagnetic C.G.S. units per gramme. The values of v were of the order $2{\cdot}4$ to $3{\cdot}2 \times 10^9$ cms. per second. Uncertainty in the measurement of Q and w renders the method less reliable than that of Art. 318.

319a. Kaufmann's Method of Determining *e/m* for the Kathode Rays.—In Kaufmann's apparatus the kathode K is an aluminium plate (Fig. 496*a*), and the anode A an earthed platinum wire. The chamber C behind the anode is arranged between two solenoids, the latter giving a uniform field H over practically the whole chamber. G is a glass plate so prepared that it becomes fluorescent under the action of the kathode rays from K. A shadow of the anode A is of course thrown upon G. By means of an electrometer the P.D. between K and A was measured. Kaufmann used a Wimshurst machine instead of an induction coil. The chamber C was screened from electrostatic action. The experiment consists in starting the rays and noting the deflection d of the rays on G when the field H is established.

If V be the velocity of the electron in the kathode rays on reaching A and entering C, v its velocity at K, and m its mass, then :—

$$\tfrac{1}{2} m(V^2 - v^2) = Ee$$

where E is the P.D. between A and K indicated by the electrometer and e is the charge. Kaufmann assumed that v could be neglected in comparison with V and therefore :—

$$\tfrac{1}{2} mV^2 = Ee \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Let l be the distance the electron travels in the magnetic field before reaching G and let d be the deflection on G

due to the field. Then l corresponds to AB and d to BC in Fig. 494. From the relation $AB^2 = BC(2R - BC)$ we have (neglecting BC^2 for BC is small) $R = l^2/2d$. The expression $e/m = v/RH$ of Art. 317 becomes on substitution for R and writing V for v:—

$$\frac{e}{m} = \frac{V.2d}{Hl^2} \quad \text{(2)}$$

Substituting from (1) the value of V, viz. $V = \sqrt{\frac{2Ee}{m}}$ we get:—

$$\frac{e}{m} = \sqrt{\frac{2Ee}{m}} \cdot \frac{2d}{Hl^2} = 2\frac{d}{Hl^2} \cdot \sqrt{2E} \cdot \sqrt{\frac{e}{m}}$$

$$\therefore \sqrt{\frac{e}{m}} = 2\frac{d}{Hl^2} \cdot \sqrt{2E}$$

$$i.e.\ \frac{e}{m} = 8E\left(\frac{d}{Hl^2}\right)^2$$

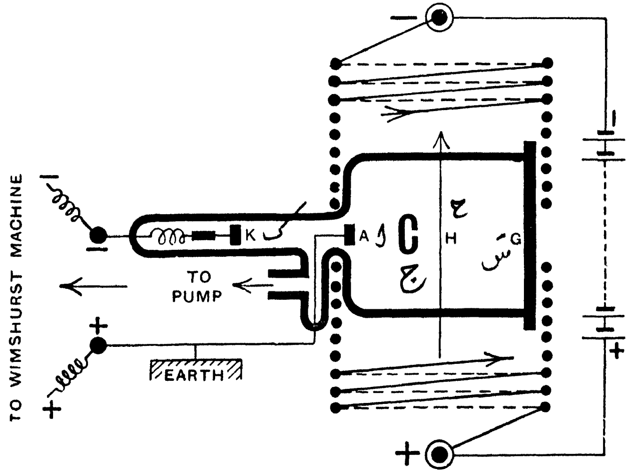


Fig. 496*a*.

All the factors on the right are known, hence e/m is determined. As already mentioned his result is $5{\cdot}6 \times 10^{17}$

e.s. units per gramme. In all this new work care must be taken not to "mix up" the "units."

Now in the case of a hydrogen ion in electrolysis it is known that e/m is $9{\cdot}6 \times 10^3$ electromagnetic C.G.S. units per gramme; hence the ratio of the mass of the hydrogen atom in electrolysis to the mass of the electron in the kathode rays is

$$= \frac{e/m \text{ for electron}}{e/m \text{ for hydrogen ion}} = \frac{1{\cdot}86 \times 10^7}{9{\cdot}6 \times 10^3} = 1937,$$

so that the mass of an electron is roughly 1/2000 of that of a hydrogen atom. It is assumed that e is the same for the hydrogen ion and for the electron; this is shown later.

Radio-active substances give out penetrating rays which consist of negatively charged corpuscles, and e/m for these is the same as for kathode rays. Röntgen, kathode, Lenard, and Becquerel rays, as will be seen later, all ionise a gas by detaching from the gas atoms negative corpuscles, and e/m for these *isolated* corpuscles is the same as for kathode rays. Ultra-violet light liberates the same corpuscles from zinc plates. The Zeeman and other effects can be explained by assuming vibrating negative corpuscles in the atoms with this same value of e/m. Besides having the same value of e/m all negative corpuscles, however produced, have the same value for e and m, the e being the "atom of electricity." It will be seen later that the atoms of matter are looked upon as consisting of systems of negative corpuscles or electrons; this is the Corpuscular or Electronic Theory of Matter.

We will now leave the electron of the kathode rays and pass on to an examination of this same electron as it makes its appearance in other phenomena. Other points about the vacuum tube of Art. 315, *e.g.* X-rays, etc., will be dealt with later.

320. Determination of *e* and *m* for the Electron. Condensation Experiments.—It is well known that if air saturated with water-vapour is subjected to a sudden (*i.e.* adiabatic) expansion condensation occurs and the space is filled with a cloud. Aitken and Kelvin have shown that the formation of such a cloud is impossible unless nuclei are provided on which the water may condense. In ordinary air dust particles provide most of the nuclei, but

C. T. R. Wilson showed that condensation occurred in dust-free gas provided that a stream of kathode, Röntgen, or ultra-violet rays, or a stream of rays from radio-active matter, was allowed to pass through the gas. These rays *ionise* the gas, *i.e.* produce in it charged particles—some positive, some negative—called *ions*. With a certain expansion the condensation is only formed on the negative ions[1]; with a larger expansion condensation occurs on both positive and negative ions. In fact if the expansion exceeds 1 : 1·25 in volume, condensation takes place on the negative ion whilst if it exceeds 1 : 1·3 condensation occurs on both positive and negative ions. When such small spheres fall in a gas they soon take a steady velocity due to the upward force of viscosity balancing the downward force of gravitation and Sir. G. G. Stokes has shown that:—

$$v = \frac{2}{9} \cdot \frac{\rho g a^2}{\mu}$$

where v is the steady velocity of a sphere of radius a, density ρ, falling in a medium of viscosity μ.

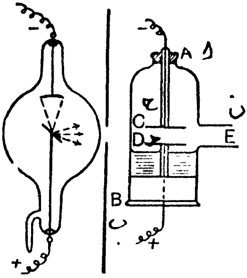


Fig. 497.

J. J. Thomson in 1898 and H. A. Wilson in 1903 used this property *to find the charge on the negative ion* and as a negative ion is really an electron loaded up by having attached to it one or more *neutral* atoms or molecules, **the charge on the negative ion is, of course the electronic charge,** the same, for example, as that on the kathode rays. Wilson's method is briefly as follows:—The lower half of the vessel AB (Fig. 497) contains water so that the space above, for example the space between the horizontal plates C and D, is saturated. By means of the tube E the vessel AB communicates with an expansion chamber and a manometer. X-rays from the bulb on the left can be sent along the space between C and D to ionise the gas there. In the first place a few expansions are made in order to remove all dust particles, and the expansion apparatus is arranged so that, in the experiment to follow, the expansion will be such that condensation will only take place on the negative ions. Next the plates C and D are joined so that they are at the same potential and the X-rays are turned on for a short time to ionise the gas. Then the rays are turned off, the expansion produced thus forming a cloud, and the velocity v_1 with which the top of the cloud between C and D settles down is noted. C is now connected to the negative and D to the positive pole of a battery so that the electric field hastens the fall of the cloud and the velocity v_2 with which the top of the cloud now falls is noted. From the data of the experiment and assuming that each drop has one electron for its nucleus, the value of e the electronic charge is readily determined.

---

[1] A negative ion is an electron loaded up by having attached to it one or more neutral atoms or molecules; at low pressures the electron throws off its attached neutral atoms or molecules and its e/m value is then as stated in Art. 318. A positive ion is an atom which has lost one electron, either alone or loaded up by having attached to it one or more neutral atoms or molecules. The charge carried by a negative ion is equal and opposite to the charge carried by a positive ion.

Before the electric field is put on, the downward force on a particle is

$$Mg$$

where M is its mass and g is the acceleration due to gravity.

When the field is put on, the downward force on a particle is

$$Xe + Mg$$

since the field X is in such a direction as to hasten the fall: hence

$$\frac{Xe + Mg}{Mg} = \frac{v_2}{v_1}$$

$$Xe = Mg\,\frac{v_2 - v_1}{v_1}$$

$$e = \frac{Mg}{X} \cdot \frac{v_2 - v_1}{v_1}.$$

Now $M = \frac{4}{3}\pi a^3 \rho$ where a is the radius of a drop and ρ the density, and from Stokes' formula $a = \sqrt{\frac{9\mu v_1}{2\rho g}}$. Substituting we get:—

$$e = 9\pi \sqrt{\frac{2\mu^3}{\rho . g}} . \frac{(v_2 - v_1)\sqrt{v_1}}{X}$$

and since X, v_1 and v_2 are known whilst $\rho = 1$ grm. per c.c. $g = 981$ cm. per sec. per sec. and μ for air $= 1{\cdot}8 \times 10^{-4}$, the value of e is determined.

Another method is as follows. Before the field is put on, the velocity of fall v_1 of the top of the cloud between C and D is noted and from Stokes' formula the radius a of the drop is determined. From this the mass M is calculated. The field is then put on *but with C positive and D negative* so that *the field opposes the action of gravity* and the former is adjusted until the two balance and the cloud is stationary. When this condition is realised we have

$$Xe = Mg$$
$$\therefore\ e = \frac{Mg}{X}$$

from which e is determined since M, g, and X are known.

The early condensation experiments of Thomson on the value of e were somewhat different from the above experiments of Wilson. In Thomson's experiments the lower part of a spherical glass vessel contained water and the vessel was closed at the top by an aluminium plate. This plate was exposed to X-rays and the air in the vessel ionised. A sudden expansion was produced, the cloud formed, its velocity of fall noted and the radius of each drop calculated from Stokes' formula. The total quantity of condensed moisture was calculated from the fall in temperature. Knowing the radius, and therefore the volume, of each drop and the total quantity of vapour condensed the number N of drops was found.

Now imagine that there are at a particular instant N negative ions in the space between the surface of the water and the aluminium plate. Imagine the latter joined to an electrometer and the water surface suddenly charged negatively so as to drive all the negative ions to the plate. The charge given to the plate would be Ne and could be measured by the electrometer. Knowing Ne and N, e would be determined. *This merely indicates the principle of the method* which however is not so satisfactory as Wilson's method.

Superposed on the chief cloud he found two minor clouds from whose velocity of descent he deduced that some drops carried a charge $2e$ and others a charge $3e$.

Wilson's mean value of e was $3{\cdot}1 \times 10^{-10}$ e.s.u.; more recent results give

$$e = 4{\cdot}65 \times 10^{-10} \text{ e.s.u.} = 1{\cdot}55 \times 10^{-20} \text{ e.m.u.}$$

Combining this value of e with the mean value of e/m found for the kathode rays, viz. $e/m = 1{\cdot}86 \times 10^7$ electromagnetic C.G.S. units per gramme, we get

$$m = 8{\cdot}4 \times 10^{-28} \text{ grammes}$$

as the mass of an electron.

There are indications that the mass of an electron is purely electrical, *i.e.* that the kathode rays, for example, are independent particles of negative electricity, the amount of charge being the smallest possible quantity or "atom of electricity"; it is now usual to speak of "the atom of electricity" as "the electron."

In electrolysis 1 e.m. unit liberates 1·16 c.c. of hydrogen at 0° C. and 760 mm. of mercury. If N be the number of molecules of hydrogen in 1 c.c., then $2N$ will be the number of atoms, and therefore there will be $2{\cdot}32N$ atoms of hydrogen in 1·16 c.c. If *E be the charge on a hydrogen atom in electrolysis in e.m. units*

$$2{\cdot}32NE = 1, \qquad \therefore E = \frac{1}{2{\cdot}32N}.$$

Now the electronic charge e is $1{\cdot}55 \times 10^{-20}$ e.m. units; *hence, if E and e be equal in magnitude,*

$$1{\cdot}55 \times 10^{-20} = \frac{1}{2{\cdot}32N}, \qquad \therefore N = 2{\cdot}77 \times 10^{19},$$

which is in close agreement with the value of N deduced from the Kinetic Theory of Gases; hence $E = e$, both being the smallest charge or "atom of electricity" or "electron."

Again, considerations of velocity and diffusion of ions in gases lead to the result that $Ne = {\cdot}41$, where e is the charge on a gaseous ion; putting $e = 1{\cdot}55 \times 10^{-20}$, this gives $N = 2{\cdot}6 \times 10^{19}$ (approx.) —again a close agreement. (See later).

We have stated that the charge e measured above in the case of the negative ion is the same as the charge for the kathode rays. The connection between the two is shown as follows. If ordinary ultra-violet light falls on a zinc plate negative corpuscles are expelled If the experiment be in air at ordinary pressure these are identical with ions produced by X-rays and the value of e is that given above.

If the experiment be in air at low pressure these same corpuscles have all the features of the kathode ray particles and a similar e/m value. We may therefore conclude from all these results that the charges carried by the kathode particle, the gaseous ion, and the hydrogen ion in electrolysis are the same, each being the "atom of electricity," and further, that the electronic theory of matter must fairly well represent the facts.

320a. Determination of e/m, e and m for the Electron. The Photo-electric Effect.—Hertz in 1887 showed that when ultra-violet light fell on the negative terminal of a spark gap the passage of the spark was facilitated. Later, Hallwachs and others showed that negatively charged metals, particularly zinc and aluminium, lost their charge when the ultra-violet rays fell on them, but that positively charged metals did not. This "photo-electric effect" is due to the light detaching electrons from the surface, their repulsion by the negatively charged plate constituting the loss which experiment shows: if the plate is positive they are not repelled away from the plate and no loss is indicated. The explanation of the detachment of the electrons is that the light sets the electrons of the metal into forced vibration until finally the amplitude becomes so great that an electron is ejected from the atom: ultra-violet light is possibly most effective in this because its period is nearer to the vibration period of the electrons in the atoms of the metal. If the experiment be in air the ejected electrons may attach themselves to neutral atoms and molecules of the gas to form "ions": if in a vacuum the ejected electrons resemble the electrons of the kathode stream.

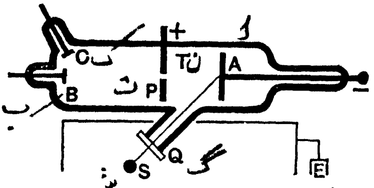


Fig. 497a.

Lenard's method of carrying out the quantitative measurements on these electrons is as follows: Light from a spark gap S (Fig. 497a) passes through a plate of quartz Q into the tube T, which is very highly exhausted, and falls upon an aluminium plate *A*, the latter being maintained at a *negative* potential (about 600 volts). *P* is an earthed aluminium plate provided with an aperture, whilst *B* and *C* are electrodes which can be connected to an electrometer. The electrons pass through the opening in *P* and fall upon *B*. A magnetic field is then established perpendicular to the plane of the paper, the field being uniform over the space between *P* and *B*, and produced by a current as in Kaufmann's experiment. Two curves are drawn giving the deflections at *B* and *C* for various currents in the magnetising coils: the difference in the positions of the maxima indicates the current to deflect the electrons from *B* to *C*. Knowing this and the potential of *A*, and the dimensions, e/m can be calculated just as in Kaufmann's determination of e/m for the kathode rays. Lenard's result was $e/m = 3{\cdot}5 \times 10^{17}$ electrostatic units per gramme, but later work gave $e/m = 5{\cdot}28 \times 10^{17}$ e.s. units per gramme, which is in close agreement with the value found for the kathode rays. The velocity v is of the order 10^8 cm. per second.

Another method due to J. J. Thomson is instructive. *Z* (Fig. 497*b*) is a zinc plate negatively charged and ultra-violet light is falling on the face of it, which is towards the right. To simplify matters we will assume the experiment to be in a vacuum. If *E* be the intensity at the surface, the force on an electron will be *Ee* towards the right. The electrons will be driven to the right, so that a parallel plate *P* placed as indicated will receive them, and an electrometer joined to *P* will indicate their arrival.

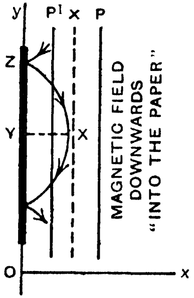


Fig. 497*b*.

Let now a magnetic field be established in front of *Z* and perpendicular to the plane of the paper—say "into the paper." The force on an electron due to this field will be directed *downwards* and of magnitude *Hev* if *H* be the magnetic field and *v* the velocity of the electron. Taking the x and y axes as indicated we have:—

$$\text{Velocity of electron in } x \text{ direction} = \frac{dx}{dt}$$

$$\text{Velocity of electron in } y \text{ direction} = \frac{dy}{dt}$$

$$\therefore \text{ Acceleration parallel to } Ox = \frac{d^2x}{dt^2}$$

$$\text{Acceleration parallel to } Oy = \frac{d^2y}{dt^2}$$

Again, if m be the mass of an electron, the force on it parallel to Ox is $m\frac{d^2x}{dt^2}$. But the force on it parallel to Ox is Ee due to the plate and $He\frac{dy}{dt}$ due to the field. An application of the left hand rule will show that an electron (negative) moving *downwards* has a force on it (due to the magnetic field) *towards the left*. Hence for the x direction we have:—

$$Ee - He\frac{dy}{dt} = m\frac{d^2x}{dt^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Similarly, for the y direction, remembering that the plate does not exert a force in this direction, we get:—

$$He\frac{dx}{dt} = m\frac{d^2y}{dt^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

The solution to (1) and (2) is:—

$$x = \frac{mE}{eH^2}(1 - \cos\theta t)$$

$$y = \frac{mE}{eH^2}(\theta t - \sin\theta t)$$

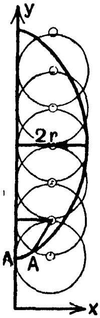


Fig. 497c.

where $\theta = He/m$.

Now imagine a circle of radius r rolling along the y axis. The locus of a point A on the circumference is called a *cycloid* (thick curve in Fig. 497c). The equations are:—

$$x = r(1 - \cos a)$$
$$y = r(a - \sin a)$$

where $\alpha = \omega t$ the angular velocity being ω. It will be clear from the figure that the greatest value of x, i.e. the greatest distance of A from the y axis as the circle rolls is $2r$.

Comparing the above equations, it will be clear that the path of the electron when the magnetic field is on is a cycloid (see Fig. 497*b*), *i.e.* the electron returns to the plate and that its maximum distance from the plate, *i.e.* XY in Fig. 497*b* is given by :—

$$XY = [2r] = 2\frac{mE}{eH^2}.$$

The plate P will receive electrons if there is no magnetic field, but no electrons will reach it when this field is on. If P be placed at P^1, it will receive electrons whether the magnetic field is on or not. P is moved forward (right to left) until the limiting position is found (XX), at which the magnetic field affects the rate at which P is charged. Thus XY is found, and knowing E and H the ratio e/m is found. The result obtained was $7{\cdot}3 \times 10^6$ electromagnetic units per gramme.

To determine e and m it is only necessary to repeat the Thomson and Wilson cloud experiment (Art. 320), using, to produce the nuclei, not X-rays but a negatively charged zinc plate on which ultra-violet light is falling. The ejected electrons form ions, as already mentioned, which constitute the nuclei. As in other cases e comes out to be the "atom of electricity," viz. $1{\cdot}55 \times 10^{-20}$ e.m. units.

320b. Determination of *e* and *m* for the Electron. Millikan's Balanced Oil Drops.—The cloud experiments of Art. 320 have been modified by Millikan in order to avoid the assumption that the drops are of one size and do not change in any way by evaporation. He used oil drops instead of water drops, the drops passing in between the plates (say C and D of Fig. 497) through a hole in the upper one; and by a suitable adjustment of the potential difference between the plates, and arranging that the electric field *opposed* the motion of the drops, a drop could be kept in view for some considerable time and its motion observed through a telescope provided with a graduated scale in the eye-piece.

The calculations involved in his early experiments are those outlined in Art. 320. To fix ideas, imagine the space to be ionised and a single oil drop of mass m and charge e (gathered by collision) to be under observation. The downward force on the drop is mg and the upward force is Xe, where X is the intensity of the field. If X be adjusted so that the drop is stationary

$$Xe = mg = \tfrac{4}{3}\pi a^3 \rho g,$$

a being the radius and ρ the density of the drop.

Now let the field be cut off and the velocity of fall (v) of the drop be observed. By Stokes' formula $v = 2\rho g a^2/9\mu$ (Art. 320); hence

$$\begin{aligned} e &= \frac{mg}{X} = \frac{4}{3} \cdot \frac{\pi \rho g}{X} \cdot a^3 \\ &= \frac{4}{3} \cdot \frac{\pi \rho g}{X} \cdot \left(\frac{9}{2} \cdot \frac{\mu v}{\rho g}\right)^{\frac{3}{2}} \\ &= \frac{18\pi\mu v}{X} \cdot \sqrt{\frac{\mu v}{2\rho g}}. \end{aligned}$$

Millikan also varied the procedure outlined above and worked to a higher degree of accuracy than has been indicated, correcting for buoyancy and using a modified form of Stokes' formula. Theory indicates that in the case of a sphere moving with velocity v in a medium of viscosity μ the force acting is best given by the expression $f = K\mu v a$, where a is the radius and $K = 6\pi\left(1 + \frac{\alpha}{aP}\right)$, P being the pressure and α a constant ($\alpha = 6{\cdot}25/10^5$). Thus if v be the velocity of fall of the drop when an opposing field X is on, V the velocity of fall when the field is off, and d the density of air—

$$\begin{aligned} \tfrac{4}{3}\pi a^3(\rho - d)g - Xe &= K\mu v a, \\ \tfrac{4}{3}\pi a^3(\rho - d)g &= K\mu V a, \end{aligned}$$

from which equations e is determined. The expression $\frac{4}{3}\pi a^3(\rho - d)g$ is the term mg corrected for buoyancy.

For simplicity we have assumed a drop to have the single charge e, but they frequently carry more than one of these "natural unit charges," and the "handing on" of these charges from molecule of gas to drop of oil and *vice versa* can be noted with the telescope. Thus imagine a drop quite stationary, the electric and gravitational forces balancing, and suppose that a collision results in the drop taking up another electron; the electric force will then exceed the gravitational force and the drop will move upwards. Similarly, if the collision results in the drop losing an electron the electric force will be less and the drop will begin to fall. These effects are frequently observed.

Millikan's apparatus is shown in Fig. 497*d*, some of the smaller details being omitted. *C* and *D* are two parallel

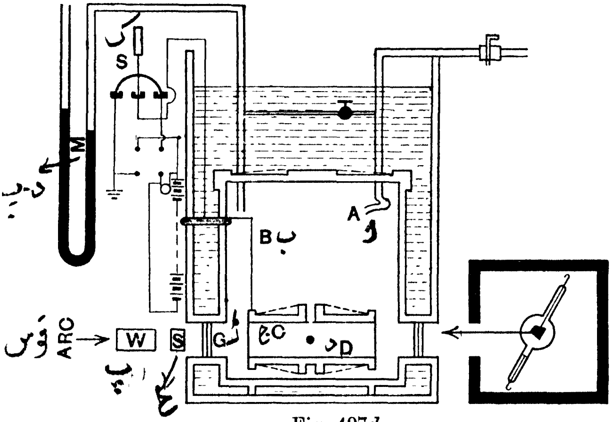


Fig. 497*d*.

plates fixed 16 mm. apart. By means of the switch *S* these plates can be charged to the P.D. of a 10,000 volt battery, or they can be short circuited and the field between them reduced to zero. The brass vessel *B* was completely surrounded by a constant temperature bath of gas engine oil. *A* is an atomizer through which the oil spray is blown into the vessel *B*. The very minute droplets of oil forming the spray fall slowly, and occasionally one of them finds its way through the pinhole in the middle of the upper plate *C*. The droplet in between *C* and *D* was strongly illuminated by light from an arc, the rays passing through a water cell *W* and a cupric chloride cell *S*, thence through the window *G*. The cells *W* and *S* are used for the purpose of absorbing the heat rays. The air about the droplet can be ionised if desired, or electrons discharged directly from the drop by means of X-rays as shown. Through a window (not shown) the droplet can be viewed by a telescope. *M* is a manometer to indicate the pressure. Millikan's result is $e = (4{\cdot}774 \pm {\cdot}005) \times 10^{-10}$ e.s. unit.

We will now return to the vacuum tube of Art. 315, and to the consideration of other phenomena connected therewith.

321. Lenard Rays.—In 1894 Lenard made the piece of apparatus shown in Fig. 498. The kathode *K* was of aluminium, and the anode a brass tube *A* lining the tube behind it. The anode was earthed. The end of the tube was closed by an earthed metal plate, out of which a central hole had been cut. This hole was closed by a thin bit of aluminium foil *F*, $\frac{1}{400}$ mm. thick. The tube was exhausted and a strong kathode stream obtained. The room was then darkened, when the air outside *F* was observed to be glowing and bodies placed just beyond *F* phosphoresced. The issuing rays were found to be deflectable, and by putting on another vacuum tube in the position indicated by the dotted lines Lenard showed, by methods similar to those described above for the rays inside the tube, that these rays were identically the same as kathode rays. The rays before reaching the foil are called kathode rays, but to these rays which have penetrated the foil the name Lenard Rays is usually given.

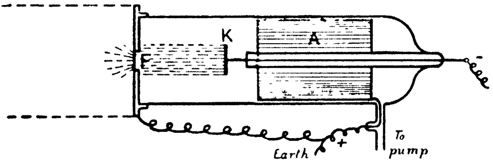


Fig. 498.

322. Röntgen or X Rays.—Soon after the exhaustion of a vacuum tube reaches the point at which the dark space surrounding the kathode exends to the anode a marked change takes place, and Röntgen found that a photographic plate lying near the apparatus was affected; he attributed the result to some unknown form of radiation emanating from the tube, and to this he applied the term X rays. It is now considered that the X rays are aether pulses which originate at a solid substance when it is struck by kathode rays. An X ray tube is shown in Fig. 499; the kathode on the left is concave and a platinum plate (which may be the anode) is placed at the centre of curvature with its plane at an angle of 45° to the axis; the X rays are given off as indicated.

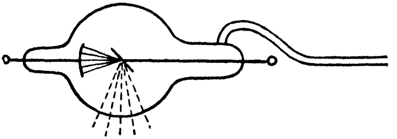


Fig. 499.

The main points about these rays may be briefly summarised as follows:—

(1) The rays are *not* deflected by a magnetic or electric field. This differentiates them from kathode rays; they are probably not charged particles.

(2) They can penetrate a layer of air several feet thick and can pass through many solid substances. The transparency of substances to X rays depends upon the density of the substances; thus lead is practically opaque to the rays, but aluminium is transparent. Soda glass is transparent to them, but lead glass is opaque. Flesh is much more transparent to the rays than bones; hence their use in surgery to examine the bones, to detect fractures and foreign bodies, etc. (See "radiograph" facing page 414.)

(3) They excite fluorescence in many substances, *e.g.* barium platino-cyanide. If the rays fall on a screen coated with this and the hand be interposed, a shadow of the bones appears on the screen, the fluorescence being less here than at other parts owing to the rays being more absorbed by the bones.

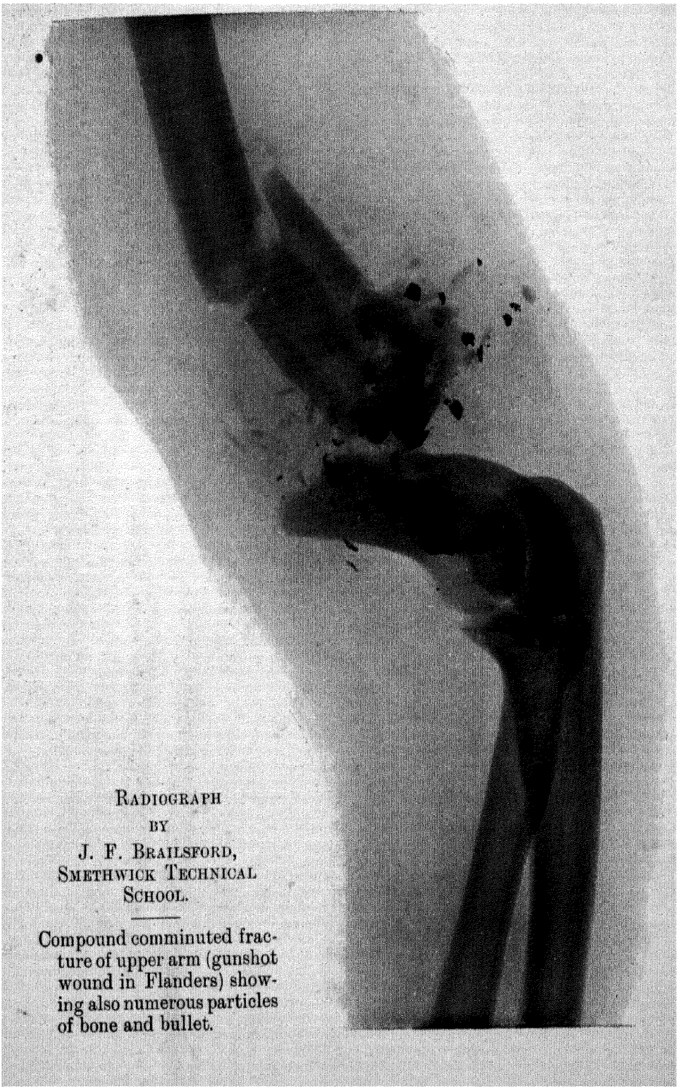
(4) The rays are not refracted. Very little trace of

regular reflection has been noted. Quite recently, and with difficulty, signs of polarisation have been detected. Sharp shadows due to them indicate rectilinear propagation, and investigations seem to indicate a velocity equal to that of light.

(5) They ionise a gas through which they pass.

(6) When they fall upon any material they give rise to other rays of a somewhat similar character known as **Secondary X-Rays.** In the case of gases and other substances of low atomic weight the secondary rays are of similar penetrating power to the primary rays and the action consists, most probably, of a *scattering* of these, although it is likely that some analogous radiation is excited in the obstacle, *i.e.* that the characteristic radiation of the obstacle is also present. In the case of metals the secondary rays are less penetrating than the primary, most of them being homogeneous and characteristic of the radiator whilst Curie and Sagnac showed that in the case of *heavy* metals the secondary rays were really of the nature of kathode rays, *i.e.* they had a negative charge. The homogeneous secondary X-rays are usually classified into "**Series K**" and "**Series L**," the former being the "harder" and more penetrating. Barkla's latest work on secondary X-rays shows that all elements emit their characteristic radiation, the lighter ones giving the "K" series (which increases in penetrating power with the atomic weight) and the heavier ones emitting *in addition* the "L" series. An atom probably consists of a positive nucleus surrounded by electrons revolving in orbits round it and the K radiation (highest frequency) is due to perturbations of the innermost, most rapid, ring whilst the L radiation of lower frequency is from the next outer ring. Lately an M radiation has been noted from a ring outside the latter and there is talk of a J radiation of extra high frequency from regions close to the nucleus. Barkla recently confirms that an element bombarded by X-rays may emit both corpuscular and X radiation.

It has been stated that X rays are aether pulses produced when the corpuscles of the kathode rays strike an obstacle. Consider now a negative corpuscle moving forward along AB (Fig. 500) with a small velocity v. The Faraday tubes



Radiograph
by
J. F. Brailsford,
Smethwick Technical
School.

Compound comminuted fracture of upper arm (gunshot wound in Flanders) showing also numerous particles of bone and bullet.

are distributed uniformly all around it. Suppose that when the corpuscle reaches O a force acts upon it which quickly brings it to rest in a small time δt, the final position of the particle being not sensibly different from O. To find the position of the Faraday tubes a time t after the first application of the force; measure along OB a distance OO' equal to vt, with O as centre describe a sphere with radius $V(t-\delta t)$, and with O' as centre describe a sphere of radius Vt, where V is the velocity of light (see Art. 310). Then if no force had acted on the corpuscle the tubes would be all radiating from O'. As it is, however, only the tubes outside the larger sphere are radiating from O'; the disturbance having passed over the tubes inside the inner sphere they radiate from O, while the portions of the tubes within the spherical shell are in the transition state, the disturbance shifting them from the radiating centre O' to that of O. They, of course, must join up the interior tubes to the exterior tubes, so that a tube inclined at an angle θ to the direction of motion appears as in Fig. 500, PQ being nearly straight if δt is very small. PQ has a tangential component; thus within the shell there is a tangential electric force, and J. J. Thomson has calculated that the electric and magnetic forces brought into existence by this tangential shearing of the Faraday tubes are greater than the forces due to the radial tubes simply moving forward. The pulse due to this tangential shearing travels outwards along the tube with a velocity V (V being equal to the velocity of light), and this, in J. J. Thomson's opinion, constitutes the Röntgen rays produced when the negative carriers of the kathode stream strike a solid obstacle.

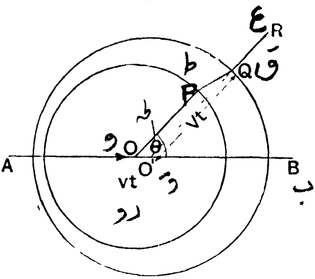


Fig. 500

The energy in the pulse is found thus (N. R. Campbell). The electric polarisation in PQ (Fig. 500a) may be regarded as compounded of one in the direction PL and one along QL. The former is $e/4\pi r^2$, and if the latter be D we have

$$D \Big/ \frac{e}{4\pi r^2} = \frac{LQ}{LP}, \qquad \therefore D = \frac{e}{4\pi r^2} \cdot \frac{LQ}{LP}.$$

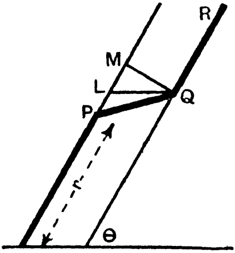


Fig. 500a.

Now the polarisation to be considered in the propagation of the disturbance along the tube is the one perpendicular to the tube, *i.e.* along QM; if this be D', then $D' = D \sin\theta$.

Again, $r = V(t - \delta t) = Vt$ (say), $\therefore t = r/V$. Further $LQ = OO' = vt$; hence $LQ = vr/V$, and $PL = d$ (say),

$$\therefore D' = \frac{e}{4\pi r^2} \cdot \frac{LQ}{LP} \sin\theta$$
$$= \frac{ev \sin\theta}{4\pi rd \, . \, V}.$$

Now

$$\text{Electrostatic energy} = \frac{2\pi(D^1)^2}{K} = \frac{e^2 v^2 \sin^2\theta}{8\pi r^2 d^2 V^2}, \text{ since } K = 1.$$

$$\text{Magnetic energy} = \frac{\mu H^2}{8\pi}$$
$$= \frac{e^2 v^2 \sin^2\theta}{8\pi r^2 d^2 V^2} \text{ since } H = 4\pi D' V \text{ and } \mu = \frac{1}{V^2},$$

$$\therefore \text{ Total energy} = \frac{e^2 v^2 \sin^2\theta}{4\pi r^2 d^2 V^2} \text{ per unit volume.}$$

So far we have considered only one tube; for the whole energy of the pulse we must sum this for all the tubes; the integration gives for the energy E—

$$E = \frac{2}{3} \cdot \frac{e^2 v^2}{d\, V^2}.$$

We may therefore look upon Röntgen rays as transverse pulses in the aether of very short wave length. The energy in the pulse depends upon the suddenness with which the stoppage occurs. If the stoppage is very sudden the shell is thin, the output of energy large, and the Röntgen rays produced are very penetrating, or in technical language "hard," while if the stoppage is relatively slow the rays carry very little energy and are easily absorbed by matter. In this case they are termed "soft."

Another theory, proposed by Professor Bragg, suggests that the constitution of a Röntgen ray is that of a closely associated pair of electrically charged particles moving with great velocity, one particle being a corpuscle or negative electron, the other a positive electron. It is not easy to see how such a neutral pair can be ejected from an antikathode struck by a negative electron.

Recently considerable progress has been made on the "**crystalline reflection" of X rays,** *i.e.* their reflection by crystals, and with associated problems—the nature of atomic structure, the arrangement of atoms in the molecule, etc. (see Chapter XXV.); but for details the student must consult the original papers.

Barkla and Dunlop have just completed experiments on the scattering of X-rays. From these it appears that (1) the scattering of X-rays of very short wave length by equal masses of different substances increases only slightly with the atomic weight of the scatterer; (2) when the radiation is of longer wave length the scattering increases very much with the atomic weight of the scatterer: these observations have an important bearing on atomic structure (Chapter XXV.).

323. Positive Rays or Canal Rays.—If the anode in a vacuum tube be placed facing the kathode and the latter is perforated, streamers may be observed behind the kathode if the pressure in the tube is within certain limits. These rays produce phosphorescence, penetrate thin aluminium foil, and consist of *positively charged particles of mass about the same as an ordinary gas atom.* The main results of investigations on these "positive" or "canal" rays may be briefly summarised as follows:—

(*a*) *Earlier Investigations.*

(1) Wein found e/m for these rays to be 10^4 e.m. units per gramme (approximately), which is very nearly the same as that of a hydrogen ion in electrolysis; the value of v was $3{\cdot}6 \times 10^7$ cm. per second, about 1/100 of that of the kathode rays.

(2) Thomson, working at very low pressures, found that the rays consisted of two sets, for one of which e/m had the value 10^4, and for the other e/m had one half of this value, viz. 5×10^3; this latter is the same as for the α particles from radio-active bodies.

(3) For the gases hydrogen, air, carbon dioxide, and neon the above two groups only were present, but for helium a third group was also present, for which e/m had one quarter the first value given above, viz. $\cdot 25 \times 10^4 = 2{\cdot}5 \times 10^3$.

(4) It was, therefore, suspected that these rays or carriers of positive charges in gases were bodies identical with atoms or molecules.

As to their actual nature nothing could be definitely stated at first. It was suggested that the particle for which e/m was 10^4 was an atom of hydrogen; that for which e/m was one half of this, a molecule of hydrogen carrying the charge e, or an atom of helium (atomic weight 4) carrying a charge $2e$; that for which e/m was one quarter, an atom of helium carrying a charge e. To account for hydrogen it was suggested that the particles were derived from hydrogen contained as an impurity, or that there might be a transmutation resulting in the production of an atom of hydrogen from another element; it is believed that the a particle in radio-activity is an atom of helium with a charge $2e$.

(b) *Thomson's Later Investigations.*

The apparatus is shown in Fig. 501. The tube in which the rays are formed is partly shown on the left, *K* being the kathode. The latter is of aluminium and is pierced by a copper tube of ·1 mm. diameter; protecting shields of iron *I*, *I*, are provided as shown. When the rays pass through the opening and appear behind the kathode they travel between two blocks of soft iron *A*, *B*, and fall upon the plate *P*, which may be either coated with willemite or covered by a photographic plate. *A* and *B* constitute the pole-pieces of an electro-magnet *EM* (from which, however, they are insulated by mica, *m*), and they are also joined to the poles of a battery and charged to different potentials. Thus the rays are under the combined influence of a magnetic field and an electric field, and, as the fields are parallel, *the deflections they produce will be at right angles to each other.* The tube *t* leads to another containing charcoal immersed in liquid air, the object being to absorb, as much as possible, the gases in the chamber on the right. There are many other details.

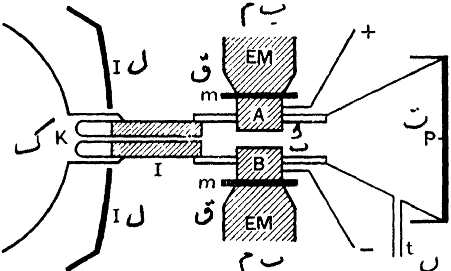


Fig. 501.

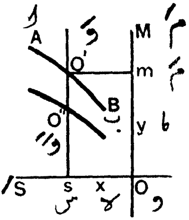


Fig. 502.

Now let *O* (Fig. 502) be the point of impact on *P* of the undeflected particle, *Om* the deflection due to the magnetic field (H) alone, and *Os* that due to the electric field (X) alone; the actual impact will be at O'. As similar particles will be moving with different velocities the actual result is a curve (Art. 317) such as $AO'B$, *i.e.* the curve $AO'B$ is formed by particles *all of which have the same value of* e/m but possess different velocities. In practice several such curves may be obtained, e/m being the same for any one curve but different for different curves (Fig. 502). The curves are portions of parabolas, and the values of e/m and v may be obtained from them by simple measurement (see below).

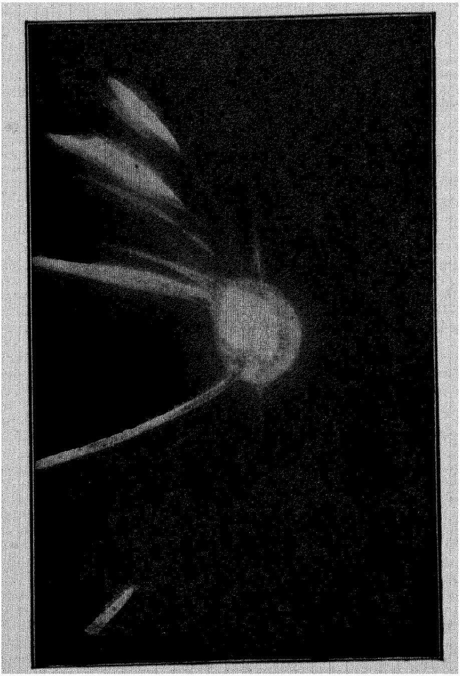


Fig. 502*a*.

NOTE.—It is usual to reverse the field half way through the experiment, thus obtaining curves on the other side of OS. Certain curves quite clear on the negative do not come out well in the figure.

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Fig. 502*a* depicts an actual experiment by J. J. Thomson. In the table the second column gives the heights of the various parabolas measured from the horizontal OS along a common perpendicular such as $SO''O'$ (Fig. 502). For convenience the value m/e (instead of e/m) for each curve is estimated and referred to a maximum of 200; these values are given in the third column. The gas in the tube before exhaustion was air freed from oxygen; in such a case traces of nitrogen, oxygen, argon, mercury vapour (mercury is used in the pumps), carbon (as impurities), etc., might be expected.

1	2	3	1	2	3	1	2	3
	mm.	m/e		mm.	m/e		mm.	m/e
a	7·2	200	*e*	16·5	39	*i*	27·6	14
b	10·3	100	*f*	19·4	28	*j*	30	12
c	12·4	67	*g*	23·1	20	*k*	38·7	7
d	15·4	44	*h*	25·6	15·9			

It is likely that (*a*) is a mercury atom (at. wt. 200) carrying a single positive charge, (*b*) a mercury atom carrying two charges, (*c*) a mercury atom carrying three charges, (*d*) a molecule of carbon dioxide (m. wt. = 44) carrying a single charge, (*e*) an argon atom (at. wt. = 40) carrying a single charge, (*f*) a nitrogen molecule (m. wt. = 28) with a single charge, (*g*) a neon atom (at. wt. = 20) with a single charge, (*h*) an oxygen atom (at. wt. = 16) with a single charge, (*i*) a nitrogen atom (at. wt. = 14) with a single charge, (*j*) a carbon atom (at. wt. = 12) with a single charge, and (*k*) a nitrogen atom carrying two charges. Another line—the hydrogen line—is deflected off the record.

As indicated below, the actual values of m/e and v may be estimated from the curves. The least value found for m/e is 10^{-4} ($e/m = 10^4$), which agrees with hydrogen. One particle for which m/e is 3×10^{-4} ($e/m = \frac{1}{3} \times 10^4$) has been noted; it is suggested that it may be a form of hydrogen (H_3) or a new element of atomic weight 3.

It is matter of simple proof that the magnetic deflection Om is given by $Om = y = LlHe/mv$, and the electric deflection Os by $Os = x = LlXe/mv^2$, where L = distance of P from centre of field and l = length of path acted on by the field; hence

$$\frac{y}{x} = \frac{Hv}{X}, \qquad \therefore v = \frac{y}{x} \cdot \frac{X}{H};$$

$$\frac{y^2}{x} = \frac{LlH^2e}{Xm}, \qquad \therefore \frac{e}{m} = \frac{y^2}{x} \cdot \frac{X}{LlH^2}.$$

Clearly these later experiments support the early investigations, and indicate that *the positive rays are really atoms and molecules of the substances present in the discharge tube, such atoms and molecules having lost one or more electrons*; some carry the charge $+e$, others an integral multiple of $+e$ (up to $+8e$ in the case of mercury). Further work on positive rays may probably throw light on the nature of positive electricity.

324. General Ideas on Gas Conduction—(1) Ionisation.—The student will have noted that under ordinary conditions gases are very poor conductors of electricity. Thus, if very careful attention be paid to insulation, a charged electroscope will keep its charge for some considerable time. The leaves do, however, fall gradually, which indicates that there is a leak, although very small, through the gas. Recent work shows that this leak can be enormously increased, and, in fact, the gas made a good conductor by exposing it to kathode rays, Lenard rays, X-rays, hot bodies, flames, ultra-violet light or radioactive substances. The conductivity persists for a time after the agents which have produced the conductivity are removed, but it ceases in time.

In Fig. 50~~4~~, *E* is a charged electroscope, *TB* a tube which terminates in a funnel, below which is an X-ray bulb. The bulb is in a lead box provided with an opening just below *B*, the lead box being necessary to screen *E* from the direct action of the X-rays. The tube on the left of *E* communicates with a water-pump, so that air can be drawn through *BT* and *E*. When the bulb is on and the air drawn along *BT* is therefore exposed to the rays, the electroscope collapses, whether the charge on it is positive or negative, thus showing that the X-rays have made the gas a conductor.

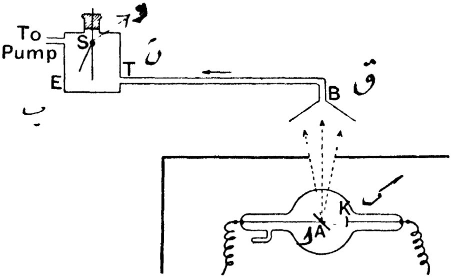


Fig. 503.

If a plug of cotton wool be placed in the tube *BT* the leak in *E* is *not* altered when the bulb is put on, *i.e.* the conductivity seems to be "filtered out." The same happens if the conducting air is bubbled through water. If *TB* is a metallic tube with a wire stretched along its axis, and the tube and wire be maintained at a high potential difference, the same thing happens, *i.e.* the air loses its conductivity.

These experiments indicate that the agents mentioned above produce in the gas "charged particles" which, when they enter the electroscope, neutralise the charge on the gold leaf. Since the gas, as a whole, is neutral, the charged particles must be of two kinds, positive and negative, the amount of charge carried by the positive particles being equal numerically to the amount of charge carried by the negative particles. These electrified particles are called **ions,** and the process of producing them in a gas is called the **ionisation of the gas.** In an electric field the positive ions move in the direction of the field, and the negative ions move in the opposite direction: their movement constitutes a current through the gas, for when they reach the electrodes their charges are given up. The number of ions in existence in an ionised space does not increase indefinitely with time, as positive and negative ions are constantly re-combining. We have seen that a gas does conduct a little even when not purposely exposed to any of the ionising agents mentioned above: this is spoken of as **spontaneous ionisation,** and is due to some form of radiation always present.

The act of ionisation in gases really consists in the detachment of an electron from a neutral atom of the gas. The residue of the atom is, of course, positively charged, and it consists practically of the whole mass of the original atom: it constitutes the "positive ion." The detached electron soon attaches itself in a gas to a neutral atom, and it constitutes the "negative ion": it is thus that the mobilities and diffusion coefficients of negative ions are somewhat of the same order of magnitude as those of the positive ions. In fact, both the electron and the positive residue attach themselves in a gas at ordinary pressure to neutral atoms and molecules of the gas, but as the pressure is reduced the negative ion throws off its attendants; hence it is that in the vacuum tube the electron travels "free." The "charge" on the negative ion is, of course, the electronic charge: the "charge" on the positive ion—the atom which has had one electron knocked out of it—is numerically equal to this. In most cases of ionisation only one electron is ejected from an atom, but Thomson's positive ray experiments seem to indicate that the slow moving positive rays may detach several electrons from certain atoms (*e.g.* the ionisation of a mercury atom in this way may consist in the ejection of eight electrons from the atom).

A gas may be put into the conducting state ~~either~~ by the extraction of electrons from its own atoms, as indicated above, or from the atoms of the containing vessel, etc. The following summarises the chief methods :—

(*a*) By passing X-rays, Lenard rays, Canal rays, or rays from radio-active substances (α, β, and γ rays) through them.

(*b*) By ultra-violet light (Art. 320*a*).

(*c*) By raising the temperature. Gases near flames and hot metals are found to conduct. Platinum heated in a vacuum was found to give off both positive and negative particles, the latter being electrons and the former the residue of the atom of the metal : if in a gas they condense molecules of the gas round them to form "ions" (Art. 328*d*).

(*d*) By the electric discharge: after the first spark the discharge takes place more readily, due to ionisation.

Millikan has done much work on the mechanism of ionisation in gases, and his conclusions seem to be :—

(1) Ionisation by β rays (electrons from radio-active bodies) consists in the shaking off, without any appreciable energy, of *one* electron from an occasional molecule, through which the β ray passes. The faster the β ray the less frequently does it ionise.

(2) Ionisation by X-rays and light consists in the throwing out of *one* electron from an occasional molecule over which the wave passes. The energy of ejection depends on the frequency of the incident wave, and may be great.

(3) Ionisation by α rays (atoms of helium from radio-active bodies) consists in the shaking off of *one* electron from the molecule through which the α ray passes.

(4) A slow moving positive ray may be able to detach several electrons from an atom. It is quite conceivable that a comparatively massive positive ray moving slowly—but not too slowly—may cause more trouble to an atom through which it passes than either an α particle or an electron, just as a brick thrown at a window will do more damage to the glass than a small but very rapid moving bullet from a rifle.

325. Methods of Measurement.—The current through a gas, unless strongly ionised by radium salts, is too weak to be measured by a galvanometer. A quadrant electrometer or gold-leaf electroscope must be used instead. If C is the capacity of the electrometer or electroscope and V the rise of potential per second, CV is the current flowing into it.

The form of quadrant electrometer usually employed is that known as the Dolezalek instrument (Fig. 237).

The electroscope usually employed was invented by C. T. R. Wilson. It consists of a small, nearly cubical brass box provided with holes at C and D (Fig. 504). Through C there projects an insulated rod carrying a brass plate P which nearly fills up the end of the electroscope, while through D projects a well-insulated metal rod carrying a very narrow gold-leaf L. The leaf is about a millimetre wide and should be just long enough not to reach across to P. W is a window through which its motion is observed. The insulation at C may be ebonite, that at D must be sulphur or quartz. When in use P is kept charged to (say) 160 volts, and the instrument is placed in an oblique position (30°, about) to increase the sensitiveness. The case A is earthed, and L is joined to the insulated system into which the current is flowing. As the voltage of this system rises L is attracted towards P, and its movement is measured by a microscope directed to look through the window. The capacity is about 1 electrostatic unit, and in a sensitive position the leaf will move about 1 mm. for one tenth of a volt, and a small fraction of this can be easily read with a good microscope furnished with an eye-piece scale.

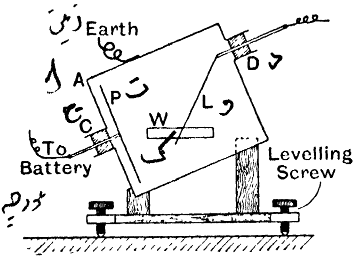


Fig. 504.

326. General Ideas on Gas Conduction—(2) Saturation Current.

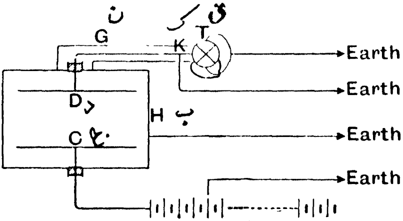


Fig. 505.

—Let *C* and *D* (Fig. 505) represent two insulated parallel discs placed inside an earthed metal vessel *H*. *C* is attached to a battery *B* of many cells. By means of an earthed wire making contact with the plates of the cells the voltage on *C* can be varied within wide limits. *D* is well insulated and attached to one pair of quadrants of an electrometer or electroscope *T*, the other pair of quadrants being earthed. *K* is a key by which *D* may readily be earthed. *G* is an earthed tube-guard to protect the wire from *D* to *T* from external induction effects. The air in *H* being ionised by some ionising agent, *C* is charged successively to different voltages and the current flowing from *C* to *D* observed. If the plate *C* is connected to the negative pole of the battery it repels the negative ions to *D*, which thus receives a negative charge. If *C* is positive *D* receives a positive charge. The results being plotted, a curve like Fig. 506 is obtained.

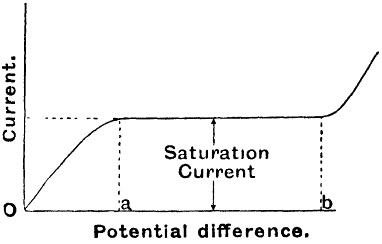


Fig. 506.

At first the current increases almost according to Ohm's law, *i.e.* as the P.D. between the plates increases the current increases: this is shown by the rising part of the curve in Fig. 506. When the P.D. reaches a value represented by *Oa* a certain current is flowing. When the P.D. increases beyond *Oa* however the current remains constant and it continues to be constant for a big increase in P.D., until in fact a P.D. represented by *Ob* is applied: this constant current is shown by the horizontal part of the curve. When the P.D. increases beyond the value *Ob* the current again increases: this is shown by the rising part of the curve on the right. The practically constant current represented by the horizontal part of the curve is called the **Saturation current.**

When the P.D. is low only a few ions are able to reach the plates before recombining with opposite ions and the current is small. As the P.D. increases more and more ions reach the plates before recombination and the current increases. Between the P.D.'s *Oa* and *Ob* all the available ions are driven to the plates and the current is steady. Beyond *Ob* the strong P.D. causes the ions to have such a velocity that *first* the negative ions *and then* the positive ions produce fresh ions by colliding with molecules and detaching electrons; thus the current again increases.

If the ionisation is merely spontaneous ionisation, a P.D. of about 10 volts per cm. produces the ionisation current, but with agents such as X-rays, radio-active bodies, etc., the ionisation is so great that a much greater voltage is necessary for saturation. The same happens if the distance between the plates is increased for more ions are produced in the greater space. Thus for the same P.D. between the plates *the current increases as the plates are drawn further apart.* The student should note this difference between the laws of conduction through gases and that of conduction through metals, viz. Ohm's law.

If the ionising agent is removed the current drops down to the normal air current in a few seconds. This is due to recombination occurring between the positive and negative ions.

If Q ions of each kind be produced per unit volume per second by the ionising agent, the total number produced per second in the space between the plates will be QAl, where A is the area of the plates and l the distance apart. If all these ions are driven to the plates before any recombinations take place we evidently have the saturation current I; hence, if e be the charge on an ion,

$$I = QAle \quad \text{......................} \quad (1)$$

The saturation current is, therefore, proportional to l, the distance between the plates as indicated above.

When a positive ion and a negative one collide they may combine to form a neutral. If at any instant there are n ions of each kind per unit volume the number of collisions per second will be proportional to n^2, and if a certain fraction $\triangle$ of these collisions results in the formation of a neutral, the number of ions of each kind disappearing per second per unit volume will be $\triangle n^2$. The resultant total rate of increase of ions of each kind will be the difference between the number produced per second and the number disappearing per second, *i.e.*

$$\frac{dn}{dt} = Q - \triangle n^2 \quad \text{.....................} \quad (2)$$

and if the ionisation be constant dn/dt will be zero, and

$$Q = \triangle n^2 \quad \text{..........................} \quad (3)$$

Again, when a P.D. E exists between the plates the current is given by the product of the number of ions crossing a section normal to the motion per second, the charge on an ion (e), and the potential gradient (E/l); that is

$$i = Ane\,(v_+ + v_-)\,\frac{E}{l} \quad \text{..................} \quad (4)$$

where v_+ and v_- are the mobilities of the positive and negative ions respectively, *i.e.* their velocities under unit potential gradient; hence

$$\frac{i}{I} = \frac{n\,(v_+ + v_-)\,E}{Ql^2} \quad \text{.....................} \quad (5)$$

or, since $Q = \triangle n^2$,

$$\frac{i}{I} = \frac{(v_+ + v_-)\,E}{l^2\sqrt{\triangle Q}} \quad \text{.......................} \quad (6)$$

Hence in the early stages the current i is proportional to E (as experiment shows), for all the other terms on the right hand side are constant.

327. General Ideas on Gas Conduction—(3) Facts about the Ions.—Experiments have shown that the negative ion consists of a negative corpuscle either alone or loaded up by attracted gas molecules, while the positive ion consists of an atom of gas which has lost a corpuscle also either alone or loaded up by attendant molecules. The masses of positive and negative ions are thus comparable with the molecules and atoms of the gas, but while the mass of the positive ion is approximately the same at all pressures, that of the negative ion decreases greatly as the pressure is decreased, showing that at low pressures the negative corpuscle throws off its attendant molecules.

Condensation experiments (Art. 320) have shown that if the expansion is sufficient to bring down all the positive and negative ions, the number of positive ions is equal to the number of negative ions. From this it follows that since the gas was electrically neutral to begin with, the charges carried by positive and negative ions are numerically equal. The charge carried by an ion is independent of the nature of the gas, and the gas ion thus differs very much from the ion of electrolysis (see also Art. 323). Townsend has shown that even with the strong ionisations produced by radium only one molecule of gas out of a hundred millions is ionised per second. In weak electric fields only the negative ion produces ions by collision, in strong fields the positive ion is also able to produce ions by collision; this explains the rise of the current curve for large voltages (Fig. 506).

With regard to the *negative ions* in dry air the following points may be noted:—If V be the electric force in volts per centimetre, and P the pressure in millimetres of mercury, then:—(1) If V/P be less than ·01 the ion is a negative corpuscle loaded up by a group of molecules. (2) As V/P increases from ·01 to ·2 the mass of the ion diminishes. (3) If V/P exceeds ·2 the ion throws off the molecules and is a free negative corpuscle or electron. In the case of the *positive ion* the only change is perhaps from a *small* group of molecules to a single molecule or atom.

328. Determination of the Velocities of Gaseous Ions.—Rutherford made use of the relation (4) of Art. 326, to find $(v_+ + v_-)$. The current i was obtained from the rate of deflection of the electrometer needle; the ionising rays were cut off, a very high P.D. then suddenly applied, thus driving all the ions to the plates, and the charge $neAl$ was, therefore, known from the electrometer deflection. Now,

$$(v_+ + v_-) = \frac{il}{AneE} = \frac{il^2}{(neAl)E};$$

thus $(v_+ + v_-)$ is known. Zeleny found v_-/v_+ by noting the electric force required to push an ion against a gas moving with a certain velocity in the opposite direction. From the two results v_+ and v_- are determined.

Another method is as follows. Imagine two parallel plates P and Q maintained at a fixed potential difference, and let ions be produced near P; ions of one sign will give up their charge to P, and the others will move towards Q. Just as they are about to reach Q imagine the field to be reversed; the ions will be driven back towards P and a fresh set of opposite sign will move towards Q. Just as they are about to reach Q imagine the field again reversed, and so on. By a careful arrangement of an experiment of this kind and a noting of the time between reversals, for which Q is just on the point of taking a charge, we obtain the time required for the ions to travel the distance between the plates, and, therefore, the velocity of the ions. Many other methods have been used.

Experiment shows that the plate Q takes a negative charge sooner than a positive one; the velocity of the negative ion is, therefore, the greater; the following numbers may be noted:—

	v_+	v_-
Air	1·36	1·87
Oxygen	1 36	1·80
Hydrogen	6·70	7·95
Helium	1·42	2·03

(These values are for ions produced by Röntgen rays. The velocities are in cm. per sec., the potential gradient being one volt per cm)

Recent experiments have brought out the following points in connection with the mobilities of ions in gases:—

(*a*) The mobility of a positive ion depends not on the nature of the gas out of which it is formed, but on the gas through which it is passing.

(*b*) The mobility is independent of the temperature if the density of the gas is constant.

(*c*) The mobility is approximately inversely as the square root of the density.

(*d*) There is a considerable increase in the mobility of a negative ion when the pressure is reduced below a certain value ; at *very low* pressures the positive ion also shows an increase in mobility.

Two theories have been put forward : (1) that the forces between ions and molecules vary inversely as the fifth power of the distance between them, (2) that the action between ions and molecules is similar to impacts between hard elastic spheres.

Thomson has shown that (*a*) follows from the first theory if an ion is a cluster having a mass much greater than that of a molecule of the gas through which it is passing; that (*b*) follows from the first theory ; that (*c*) follows from the second theory if the ions in the various gases have the same size, and it follows from the first theory if all the molecules in the gases exert the same force on a point charge at the same distance ; and that (*d*) follows on either theory if the ions dissociate at low pressures so that free corpuscles are brought into existence.

Erikson's latest results seem to indicate that the mobility at constant gas density does vary with the temperature, the positive ion having a maximum mobility at about 0° C., the negative ion showing a maximum also, but being more uncertain in its behaviour. No definite conclusions have yet been drawn from this.

328a. Ionisation by Collision—(1) The Negative Ion.—Reference has been made to the fact that the rise of the curve (Fig. 506) when the voltage becomes high is due to the fact that under the influence of the intense field *first* the negative ions *and then* the positive ions acquire such a velocity that they are able to detach electrons from the molecules of the gas by colliding with them, and in this way the number of ions (and therefore the current through the gas) is increased. We will consider first the case of the negative ion, assuming that the initial ionisation is produced by ultra-violet light falling upon a zinc plate.

Let n_0 be the number of electrons emitted in one second by the negative plate A (Fig. 506*a*) these negative ions moving immediately across the field with the appropriate velocity. Let a be the average number of new ions formed by one of these negative ions in moving through one centimetre towards the positive plate B. Imagine that, when ionisation by collision has set in, there are n negative ions in the space between A and dx. In going through dx these will produce $andx$ new ions. Calling this dn we have :—

$$dn = andx \quad or \quad dn/n = adx$$

Integrating this

$$\log n = ax + K$$

where K is a constant. But $n = n_0$ when $x = O$; hence on substituting:—$\log n_0 = K$

$$\therefore\ \log n = ax + \log n_0$$

$$\log \frac{n}{n_0} = ax$$

$$\frac{n}{n_0} = e^{ax}$$

$$\therefore\ n = n_0 e^{ax}$$

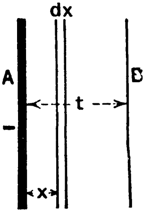


Fig. 506*a*.

Thus the number of ions reaching the plate B will be given by:—

$$n = n_0 e^{at}$$

where t is the distance between the plates.

The formula above fits in well with experiments. With air at 4 mm. pressure and a field of 700 volts per cm. $a = 8{\cdot}16$.

328b. Ionisation by Collision—(2) The Positive Ion.—The theory of the preceding section may now be extended to include the positive ion. Let n_0 be as before the number of electrons emitted at the zinc plate A per second (Fig. 506*b*), and n the number reaching B. Let Q_1 and Q_2 be the numbers produced by collision in the two spaces as shown. Clearly

$$n = n_0 + Q_1 + Q_2 \quad \ldots\ldots\ldots\ldots (1)$$

Again $n_0 + Q_1$ electrons (negative ions) pass dx towards the right in one second and Q_2 positive ions pass the other way. Let a have the same meaning as before and let β be the number of electrons ejected (by collision) by a positive ion in travelling one centimetre in the direction of the field. Hence:—

$$dQ_1 = (n_0 + Q_1)\,adx + Q_2\beta dx$$

for both sides denote the number of electrons formed in dx.

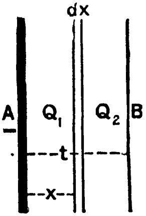


Fig. 506*b*.

$$\frac{dQ_1}{dx} = (n_0 + Q_1)\,a + Q_2\beta$$

$$= n_0 a + Q_1 a + Q_2\beta$$

$$= n_0 a + Q_1 a + \beta(n - n_0 - Q_1)$$

since $Q_2 = n - n_0 - Q_1$ from (1)

$$i.e.\ \frac{dQ_1}{dx} = (n_0 + Q_1)(a - \beta) + n\beta$$

$$or\ \frac{d}{dx}(n_0 + Q_1) = (a - \beta)(n_0 + Q_1) + n\beta$$

the solution to which is:—

$$n_0 + Q_1 = Ze^{(a-\beta)x} - \frac{n\beta}{a - \beta} \quad \ldots\ldots\ldots\ldots (2)$$

where Z is constant. When $x = O$, $Q_1 = O$, hence :—

$$n_0 = Z - \frac{n\beta}{\alpha - \beta} \qquad i.e.\ Z = n_0 + \frac{n\beta}{\alpha - \beta}$$

When $x = t$, $n_0 + Q_1 = n$. Hence substituting in (2)

$$n = \left(n_0 + \frac{n\beta}{\alpha - \beta}\right)e^{(\alpha - \beta)t} - \frac{n\beta}{\alpha - \beta}$$

$$\therefore\ n = n_0 \frac{(\alpha - \beta)e^{(\alpha - \beta)t}}{\alpha - \beta e^{(\alpha - \beta)t}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

If $\beta = O$, this reduces to :—

$$n = n_0 e^{\alpha t}$$

as obtained in the preceding section. Positive ions do not ionise to the extent that negative ions do owing to their smaller velocity: the positive α particle, however, ejected from radio-active bodies travels with a velocity of 2×10^9 cms. per second and has greater ionising power even than an electron.

The formula developed above fits in well with experiment. For air at 4 mm. pressure, electric force 700 volts per cm. $\alpha = 8{\cdot}16$ and $\beta = {\cdot}0067$.

328c. Decay of Ionisation when the Ionising Agent is Cut off.—It has been indicated that the ionisation of a gas exists for a time after the ionising agent is cut off but that it finally disappears. The law governing the rate of decay may be determined as follows :—

By (2) of Art. 326 :—

$$\frac{dn}{dt} = Q - \Delta n^2$$

where Q denotes the number of ions of each kind *produced* per unit volume per second, n the number in existence per unit volume at any instant and Δn^2 the number disappearing per second from unit volume.

If the ionising agent be cut off, $Q = O$ and :—

$$\frac{dn}{dt} = - \Delta n^2$$

$$\therefore\ \frac{1}{n^2} dn = - \Delta\, dt.$$

Integrating :—

$$- \frac{1}{n} = - \Delta t + K.$$

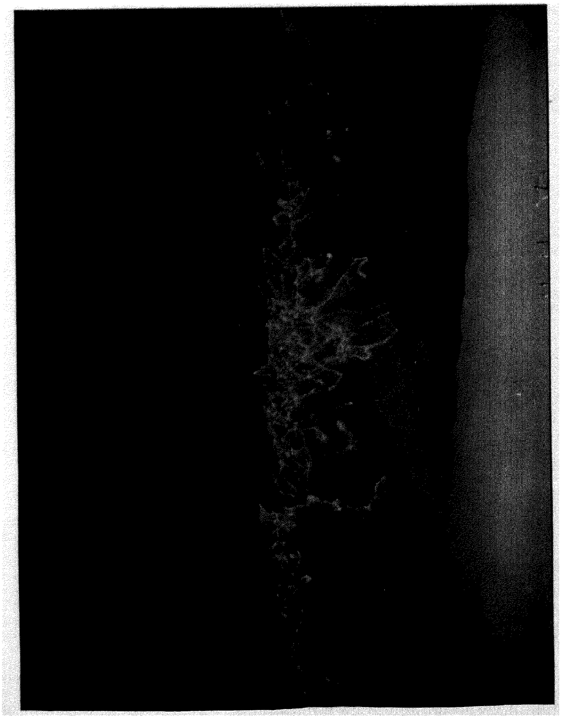


Fig. 506*d*.

Now when $t = O$, *i.e.* at the instant the agent is cut off let the ions per unit volume $= n_0$. Putting $t = O$ above

$$K = -\frac{1}{n_0}$$

$$\therefore \quad \frac{1}{n} = \Delta t + \frac{1}{n_0}$$

$$i.e. \quad n = \frac{n_0}{\Delta n_0 t + 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

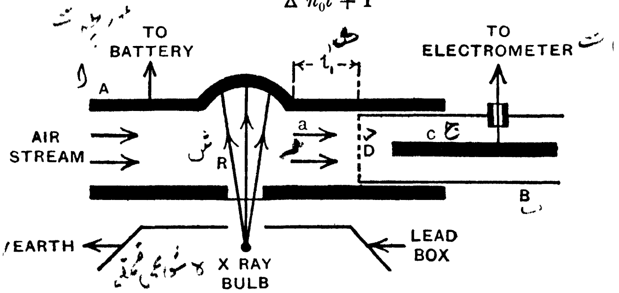


Fig. 506*c*.

Thus in time $t = 1/\Delta n_0$ the value of n will be $n_0/2$, *i.e.* the ionisation will have fallen to half value. Rutherford verified the above experimentally. The ionising agent was cut off and, after a known time, the ions were driven to the plates by a strong field and the charge ($\propto n$) noted. This was repeated for various time intervals.

The constant Δ is called the **recombination coefficient** and one method of determining it is as follows:—A wide metallic tube B (Fig. 506*c*) fitted with a gauze screen D is capable of sliding in the tube A, these tubes being kept strongly charged by means of a battery. A stream of gas is caused to pass through the tubes, the gas being ionised at R. These ions pass through D and are, under the action of the field, collected by C so that the electrometer is deflected. The charge received per second is noted (1) when the distance aD is l_1 and (2) when the distance aD is l_2.

Now let v be the velocity of the current of gas along the tubes, S the sectional area of the tubes, n_1 the ions per c.cm. at D when aD is l_1, n_2 the ions when aD is l_2, and e the ionic charge. If Q_1 be the charge per second received by C when the distance is l_1 and Q_2 when it is l_2:—

$$Q_1 = Sn_1ev \quad \text{and} \quad Q_2 = Sn_2ev$$

and the time t taken by the ions to move through the length $(l_2 - l_1)$ is given by $t = (l_2 - l_1)/v$. Further $n_1 = Q_1/Sev$ and $n_2 = Q_2/Sev$.

Again from (1) above $\Delta t = \frac{1}{n} - \frac{1}{n_0}$ so that if t be the time to fall from n_1 to n_2 :—

$$\begin{aligned} \Delta &= \frac{1}{t}\left(\frac{1}{n_2} - \frac{1}{n_1}\right) \\ &= \frac{v}{l_2 - l_1}\left(\frac{Sev}{Q_2} - \frac{Sev}{Q_1}\right) \\ &= \frac{Sev^2}{l_2 - l_1} \cdot \frac{Q_1 - Q_2}{Q_1 Q_2} \quad \text{..................... (2)} \end{aligned}$$

Rutherford obtains Δ for air $= 1{\cdot}58 \times 10^{-6}$, for hydrogen $1{\cdot}42 \times 10^{-6}$, and similar values for other gases (the pressure is atmospheric). If we take Δ in round figures to be 10^{-6} we see from (1) above that if there were 10^6 ions per c.cm. half of them would re-combine in one second.

328d. Ions from Hot Bodies.—It has been known for nearly 200 years that gases in the neighbourhood of hot bodies become conductors. Richardson and others have shown that if a platinum plate be heated in a vacuum then at a dull red heat it gives off streams of positive electricity, but that as the temperature continues to rise this stream becomes less, vanishes and changes sign (any positive at high temperature is masked by the considerably larger negative): these negative particles are electrons.

Wehnelt found that this emission by hot metals was increased by coating the surface with calcium oxide. In the **Wehnelt-Braun Tube** the kathode is a piece of platinum coated with calcium oxide, and it is heated by the current from a battery. The electrons pass through a hole in the anode and are then subjected to the influence of an electric and a magnetic field. By the method outlined in previous pages the value *e/m* is found. The result is rather low but near enough to show that they are electrons.

If the hot body be in air the electrons condense molecules of the gas round them to form ions.

Carbon is very efficient in the phenomena we are considering. The negative carbon of an arc lamp emits streams of electrons, which falling on the positive carbon keep it very hot. If a metal plate be placed between the

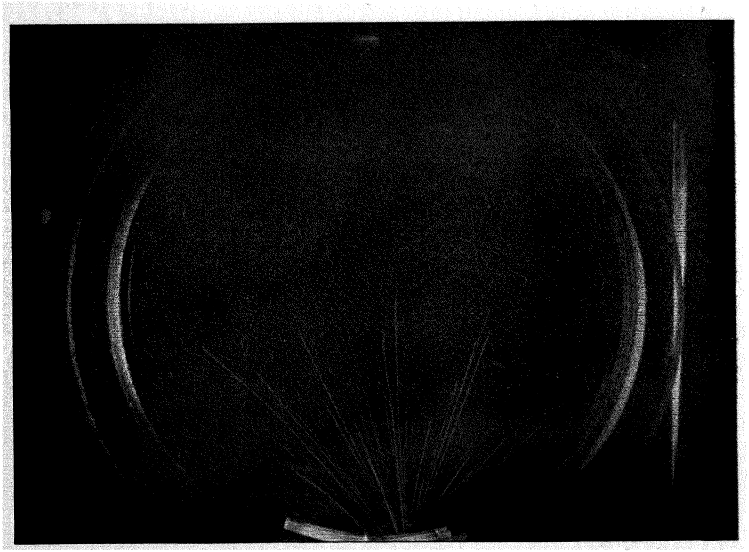


Fig. 506*e*.

two limbs of a carbon filament lamp and the plate be joined to a galvanometer, the latter will be deflected when the other terminal of it is joined to the positive terminal of the lamp, but not when it is joined to the negative terminal of the lamp. This is used in **Fleming's Oscillation Valve** employed as a detector of electromagnetic waves on the same principle as the carborundum crystal (Chapter 22).

Owing to the emission of electrons by an incandescent wire carrying a current the effective area of the wire is increased, *i.e.* a part of the current will pass outside the wire. The emission of electrons by the sun has been suggested as an explanation of well known magnetic and electrical phenomena, *e.g.* the Northern Lights, etc.

328e. C. T. R. Wilson's Ionisation Photographs.—Recently Mr. C. T. R. Wilson has produced some remarkable photographs on the tracks of α particles, etc., which not only throw light on many of the facts mentioned in connection with ionisation but also on atomic structure. The particles ionise some of the molecules through which they pass, these ions immediately condense water vapour around themselves forming droplets, and these are finally illuminated for the short time necessary for a photograph to be taken by virtue of the light which they reflect.

Fig. 506*d* shows the effect when X-rays pass through air. The X-rays hurl electrons from certain atoms with great energy and it is the paths of these electrons ionising as they go which are shown by the little wavy lines in the figure. It has been stated that ionisation by X-rays consists in the hurling of *one* electron from the atom and this seems to be upheld by the photograph, for if two or three electrons were hurled out, each bright dot would be the starting point of two or three wavy lines and this is not shown. Further, the fact that electrons do not eject other electrons from atoms with *very great energy* is also indicated in the figure, for if the energy of these secondary ejected electrons were great, each dot along the path of an ionising electron would be the starting point of another wavy line and so on. This coincides with Braggs' conclusion, viz. that the bulk of the ionisation is due to the high speed electrons hurled out by the X-rays, and not so much to the electrons shaken loose by these high speed ones as they continue to ionise. The curl towards the end of the track shows that the electron as it slows down is less able to resist any deflection from its straight course due to the constituents of the atom through which it is passing.

Fig. 506*e* shows the tracks of α particles (*i.e.* atoms of helium from radium). An α particle shoots straight through about 7 centimetres of air going therefore *right through* (not pushing to one side) nearly 500,000 atoms without approaching near enough to the central nucleus of an atom to suffer deflection at the speed with which it is travelling. It ionises a good deal, hence the path in the photograph is continuous. Towards the end there is a distinct deflection showing that as the particle slows down the nuclei of the atoms begin to deflect it from its straight line track. This is just what we would expect, for an atom is a kind of solar system with a small positive nucleus and certain electrons revolving in orbits, but, nevertheless, mainly we might say, empty space, and it is quite possible for another atom to shoot straight through this atom without so to speak hitting anything or being deflected from its course *if only it is endowed with sufficient speed.* Sometimes, however, it approaches an obstacle and as the speed falls the time they are in each other's vicinity is greater and deflection ensues.

Fig. 506*f* shows the track through air of a β particle (electron) from a radio-active body. The track is dotted showing that ions are produced spread out along the track. The track shows a kind of continuous curvature due to a number of smaller deflections. Measurements indicate that this electron went through about 10,000 atoms before it came sufficiently near an electron in an atom to eject it and form an ion—another indication of the "empty space" character of an atom.

The mechanism of ionisation by γ rays is similar to that of X-rays; electrons are hurled from the atoms, and these produce other ions as they travel along.

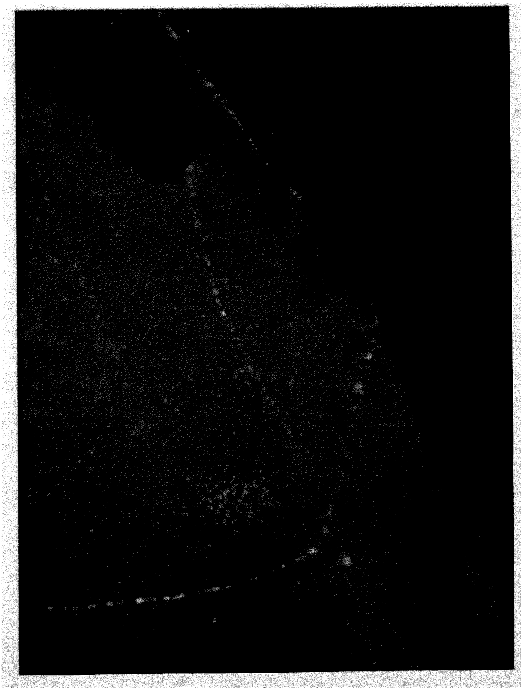


Fig. 506*f*.

Note.—In the following worked examples only approximate data are employed.

Examples.

1. *If e/m for the kathode rays be 6×10^{17} e.s. units per gramme, and v be $1/10$ that of light, find the electrostatic field which will deflect a stream 10 cm. long through a distance of 1 mm.*

In Fig. 494 $AB = 10$, $BC = 1/10$.

Now $$AB^2 = BC(2R - BC),$$

$$\therefore\ R = \frac{AB^2 + BC^2}{2BC} = \frac{10001}{20} = 500 \text{ nearly.}$$

From Art. 317 $$X = \frac{v^2}{R} \cdot \frac{m}{\epsilon} = \frac{(3 \times 10^9)^2}{500} \cdot \frac{1}{6 \times 10^{17}},$$

$$\therefore\ X = \boldsymbol{\cdot 03} \textbf{ e.s unit,}$$

and, since $X = dV/dx$ (neglecting sign), and one electrostatic unit of potential = 300 volts, a potential gradient of 9 volts per cm. will be necessary.

2. *The number of molecules per cubic centimetre of a gas at N.T.P. is 3×10^{19}. Find* (1) *the mass of a hydrogen atom,* (2) *the mass of a corpuscle,* (3) *the charge on a corpuscle* (i.e. *the electron*).

$$1 \text{ c.cm. of hydrogen at N.T.P.} = 9 \times 10^{-5} \text{ grm.,}$$

$$\therefore\ 1 \text{ molecule of hydrogen at N.T.P.} = \frac{9 \times 10^{-5}}{3 \times 10^{19}} = 3 \times 10^{-24} \text{grm.,}$$

i.e. $$\textbf{1 atom of hydrogen} = \frac{3 \times 10^{-24}}{2} = \mathbf{1{\cdot}5 \times 10^{-24}} \textbf{ grm.}$$

Taking the mass of a corpuscle = 1/1900 of that of a hydrogen atom,

$$\textbf{1 corpuscle} = \frac{1{\cdot}5 \times 10^{\ 24}}{1900} = \mathbf{8 \times 10^{\ 28}} \textbf{ grm.}$$

Again, from electrolysis, 1 grm. of hydrogen carries a charge of $9650 \times 3 \times 10^{10}$ e.s. units.

$\therefore$ 1 atom of hydrogen carries $9650 \times 3 \times 10^{10} \times 1{\cdot}5 \times 10^{-24}$ e.s. units, or $4{\cdot}3 \times 10^{-10}$ e.s. units.

But this is the electronic charge ; hence

Electronic charge = $\mathbf{4{\cdot}3 \times 10^{-10}}$ e.s. units.

3. *The coefficient of viscosity of air is $\cdot 00015$. Find the radius and mass of a water drop in Wilson's experiment which falls at the rate of $\cdot 02$ cm. per sec., and the electric field which will keep the drop steady if the charge on it is the electronic charge, say 3×10^{-10} e.s. unit.*

Here (in Stokes' formula, Art. 320) $v = \cdot 02$, $\rho = 1$, $g = 981$, $\mu = \cdot 00015$. On substituting,

$$\textbf{Radius} = a = \mathbf{\cdot 000117},$$

$$\textbf{Mass} = \text{Volume} \times \text{Density} = \tfrac{4}{3}\pi a^3 = \mathbf{6 \cdot 7 \times 10^{-12}}.$$

If X be the field required to keep the drop stationary,

$$\begin{aligned} \text{Upward force} &= \text{Downward force} \\ eX &= mg \\ (3 \times 10^{-10})X &= (6 \cdot 7 \times 10^{-12})\,981, \\ \therefore X &= \mathbf{21 \cdot 9 \text{ e.s. units.}} \end{aligned}$$

Exercises XXII.

Section C.

(1) Describe the general character of the discharge in a partially exhausted tube. (B.Sc.)

(2) An electrified particle traverses an electric field, the intensity of the field being normal to the original direction of motion of the particle. Find an expression for the deflection of the particle.

What other experiments must be made in order to determine the ratio of the mass of the particle to its electric charge? (B.Sc.)

(3) How may the electrical conductivity of an ionised gas be determined? What is meant by the saturation current? (B.Sc.)

(4) Two metallic plates, separated by a layer of air, are connected with the opposite poles of a battery, in which the number of cells may be increased indefinitely. Trace the change in the relation between potential difference and the current, explaining those changes in terms of the ionisation theory.

How would you measure experimentally the currents in such a case? (B.Sc. Hons.)

CHAPTER XXIV.

RADIO-ACTIVITY.

329. Radio-active Elements. Three Types of Radiation.—In 1896 Becquerel discovered that **uranium** and its compounds gave out rays which affected a photographic plate and ionised a gas just like kathode and X rays do, and in 1898 the same thing was discovered for **thorium**; substances which act in this way are said to be *radio-active.* Just after these discoveries M. and Mme Curie made a systematic study of the radio-activity of many minerals, and having discovered that radio-activity is an atomic property, being in no wise dependent on the chemical combination entered in by the element, found that some of these minerals, pitchblende and chalcolite in particular, were more active than the amounts of uranium and thorium they contained would justify. They concluded from this that these minerals *must contain another active element*, and from a ton of uranium residues from pitchblende they succeeded in isolating ·3 gramme of a very active element to which the name **radium** was given. Incidentally a less active element which they called **polonium** was also discovered. Mme Curie determined the atomic weight of radium and found it to be 226, which places it in the periodic table in the same column as the metals of the alkaline earths and in the same row as uranium and thorium, whilst Demarcay examined its spectrum and found it gave new lines, thus confirming that it was a new element. A third strongly radio-active body was identified in pitchblende by M. Debierne, who named it **actinium.** Radium, polonium, and actinium have as yet been obtained only in very small quantities.

Three methods have been employed in measurements connected with the radio-activity of bodies :—

(*a*) The action of the rays on a photographic plate.

(*b*) The fluorescence produced by the rays on a screen of zinc sulphide, barium platino-cyanide, willemite, etc.

(*c*) The ionising action of the rays on the surrounding gas.

Exps. (1) In 1899 Rutherford examined the radiation from uranium by an electrical method, the apparatus being similar to that shown in Fig. 505. The uranium compound was placed in a recess in the plate *C*. The space *CD* was electrically saturated and the saturation current obtained first with the uranium uncovered and then with the uranium covered with different thickness of thin foil. From the variation of current with the number of layers of foil he deduced that the radiation from uranium must consist of three kinds : (i) a part easily absorbed by thin foil, (ii) a part more difficultly absorbed by thin foil, (iii) a part hardly absorbed at all by thin foil. To distinguish these rays from each other he called them α, β, and γ rays respectively. From his results he calculated that the α rays are reduced to half their initial value by passing through ·0005 cm. of aluminium foil, the β rays to half value by ·05 cm. of aluminium foil, and the γ rays to half value by at least 8 cm. of foil, so that *the relative powers of penetration of the* α, β, *and* γ *rays are as* 1 : 100 : 10000. He also showed that for a thin layer of unscreened radio-active matter *the relative ionising-powers are as* 10000 : 100 : 1.

(2) Rutherford showed that **the α rays are positively charged particles** by deflecting them by a magnetic field. The rays from radium passed upwards between a series of vertical parallel plates each provided at the top with a projecting ridge at one side, then through an extremely thin aluminium window into an electroscope thereby "ionising" and causing a collapse of the leaves. A magnetic field was then applied in such a direction as to deflect the α particles to the side of the plates carrying the ridge, in which case they did not enter the electroscope. From the known direction of the field and the direction of deflection it was proved that the particles were positively charged. A stream of gas passed downwards through the apparatus to carry off the "emanation" (Art. 335). It is mentioned above that ionisation is mainly due to α rays.

(3) M. and Mme Curie showed that **the β rays are negatively charged particles.** They embedded a thick lead plate in a block of paraffin enclosed in an earthed metal casing. The casing over one face of the block was very thin. The lead plate was connected to an electroscope and the thin part of the casing was held over some barium-radium chloride. The lead became increasingly charged with negative electricity. Since all the α rays are absorbed by the

thin casing the negative charge must be due to the absorption of the β rays. Since radium is originally electrically neutral, if we enclose it in a glass vessel of sufficient thinness to let the β rays escape but to absorb the α rays, the inside of this vessel should become positively charged. This has been experimentally confirmed by Mme Curie.

The behaviour of the rays in a magnetic field is shown in Fig. 507. *L* is a lead block containing a narrow hole at the bottom of which is placed some radium. The α, β, γ rays from the radium emerge from the hole as a narrow vertical pencil. On the application of a strong magnetic field the α rays behave like positively charged bodies and are slightly deviated to one side. The β rays, acting like negatively charged particles, are deviated to a much greater extent on the other side of the vertical, and if the field is strong the β particles describe circles of variable sizes, depending on the velocity with which they leave the radium and the strength of the field. The γ rays are not deviated ; **they are aether pulses, not charged particles.** The deflection of the α rays is exaggerated in the figure.

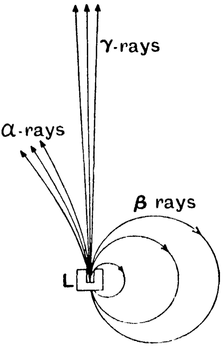


Fig. 507.

More recent work has elucidated the following information about the rays :—

330. The α Rays.—These are the least penetrating rays from radio-active bodies, being completely absorbed by ·01 cm. of aluminium foil. They are only slightly deviated in a strong magnetic field, and consist of a stream of positively charged particles, projected from the active matter with a velocity of from $1{\cdot}55 \times 10^9$ to $2{\cdot}25 \times 10^9$ cm. per sec. Their value of e/m is $5{\cdot}1 \times 10^3$ electromagnetic C.G.S. units per gramme. By direct counting, electrically and optically, Rutherford and Geiger found that one gramme of radium itself (*i.e.* the products excluded) gives off $3{\cdot}4 \times 10^{10}$ α particles per second. They also measured the charge carried off by the α particles, and from the knowledge of the number deduced that the charge borne by each α particle is $9{\cdot}3 \times 10^{-10}$ e.s.u., *i.e.* $3{\cdot}1 \times 10^{-20}$ e.m.u. Combining this with the value of e/m they found that the mass of an α particle is $6{\cdot}2 \times 10^{-24}$ grm. This is equal to the mass of a helium atom. The charge on an α particle is twice the electronic charge. Since helium is always found in company with radio-active bodies, Rutherford concludes that the α particle is an atom of helium, or rather that the α particle, after it has lost its positive charge, is an atom of helium. He has confirmed this with the spectroscope. Most of the energy emitted by radio-active bodies is carried by the α rays. During the flight of an α particle through a gas it knocks off negative electrons from many of the atoms and is thus an effective ioniser (200,000 ions per α particle), rapidly spending its energy in the process. The expulsion of an α particle from an atom being the result of an internal explosion, the residual atom "recoils." The existence of these recoil atoms has been shown in many ways. Further, taking the α particles to be helium, one gramme of radium in equilibrium would emit ·175 c.c. of helium in one year; the amount actually observed is ·170 c.c.

331. The β Rays.—These are more penetrating than the α rays. A thickness of aluminium foil equal to ·5 cm. is required for complete absorption. Owing to their penetrating power they are easily detected by photography, and Becquerel by photographic methods measured their deflection in a magnetic field and found that e/m was very nearly 10^7 electromagnetic C.G.S. units per gm., that v was from $1{\cdot}6 \times 10^{10}$ to $2{\cdot}8 \times 10^{10}$ cm. per second, which is nearly equal to the velocity of light, and that their charge was negative. They are therefore identical with the electrons of the kathode stream. The β particles from radium have, on the whole, higher velocities than those from uranium, but these latter are more uniform in velocity. The "mass" of the β particle increases with the velocity (and therefore e/m decreases); this is a consequence of the electromagnetic theory, as will be seen later.

332. The γ Rays.—These are extremely penetrating rays; Rutherford has detected them after they have passed through a foot of iron. They readily excite luminosity in various platino-cyanides and in willemite (zinc silicate) after passing through half an inch of lead. They cannot be deflected in a magnetic field, so that it seems they are not a stream of electrified particles. From the similarity of their action to penetrating Röntgen rays it has been concluded that **they consist of pulses in the aether of very short wave length generated when an electron is expelled from an atom.** This view is borne out by the fact that these γ rays accompany β rays and are proportional to them. Recent investigations on the absorption in aluminium and lead of the γ rays from radium *B* indicate that several series of characteristic radiations are present and that the γ rays correspond to the natural modes of vibration of the internal structure of the radio-active atoms.

333. The δ Rays.—Slowly moving negative corpuscles have been detected by Thomson which are very similar to the β rays (electrons), but which do not ionise owing to their low velocity; these have been called δ rays. The least velocity for ionisation is $3{\cdot}6 \times 10^8$ cm. per sec., whilst the velocity of the δ rays is $3{\cdot}2 \times 10^8$ cm. per sec.

334. Determination of e/m and v for the α and β Rays.—(1) *α Rays.* In Fig. 508 *W* is a groove containing a wire which has been rendered active by exposure to radium emanation. The α rays pass through the slit *S* and fall on the photographic plate *P*; the vessel is exhausted. A magnetic field is applied parallel to the wire and slit, thus deflecting the rays to one side. The field is then reversed, thus deflecting the rays to the other side. If *d* be half the distance between the bands on the plate *P*, *r* the radius of the circle described by the rays, *p* the distance between *P* and *S*, *w* the distance between *W* and *S*, and *H* the intensity of the field, then, *d* being small—

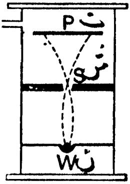


Fig. 508.

$$2rd = p(p+w) \qquad \text{and} \qquad \frac{mv}{e} = Hr,$$

$$\therefore \frac{e}{mv} = \frac{2d}{Hp(p+w)} \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

The apparatus is somewhat modified for the electrostatic deflection. The rays from the wire pass up between two vertical parallel plates (·21 mm. apart) kept at a P.D. X by means of a battery. In passing between the plates they describe a parabolic path and on leaving proceed in a straight line to the photographic plate P. The field is reversed as in the case above. If p_1 be the distance between P and the end of the parallel plates, q the distance between the parallel plates, and D the distance between the extreme edges of the deflected band, it is readily established that

$$\frac{e}{mv^2} = \frac{(D-q)^2}{8p_1^{\,2}qX} \quad \dots\dots\dots\dots\dots\dots \quad (2)$$

From (1) and (2) e/m and v are determined.

(2) β *Rays.*—The principle of the method is as follows The rays are subjected to superimposed magnetic and electrostatic fields, the fields being parallel so that the deflections they produce are at right angles, and the rays are received on a photographic plate. Imagine the source to be L cm. in front of the plate and let O (Fig. 502) be the point of impact when both fields are absent, OS the direction of the electrostatic deflection, and OM the direction of the magnetic deflection. When both fields are on, the point of impact will be (say) O', in which case Os is the electrostatic and Om the magnetic deflection, both of which can therefore be measured. The calculations of e/m and v are then those outlined in Art. 323.

The value of v varies, and as it increases, approaching that of light, e/m decreases, or, assuming e constant, *the mass m therefore increases with the velocity* (see Art. 338).

As already indicated the value of e/m for the α particle is $5{\cdot}1 \times 10^3$ *e.m.* units per gramme, and for the β particle about $\cdot 63 \times 10^7$ to $1{\cdot}71 \times 10^7$ *e.m.* units per gramme depending on the velocity. Little further need be said about the β rays in this chapter for the β particles are *electrons* (Chapters XXIII. and XXV.).

334a. Determination of *e* and *m* for the *α* Particle. Identity with Helium.—The action of individual α particles was first made visible by Sir W. Crookes' spinthariscope. Near one end of a tube is fitted a screen coated with zinc sulphide, whilst just in front of it is fixed a needle which has been dipped in radium solution. The screen can be viewed through a lens fitted into the other end of the tube. If the eye be first made sensitive by being in the dark for some few minutes, then, on examining the screen, the latter will be seen to be twinkling with points of greenish light. These scintillations are due to the impact of α rays for if a suitable sheet of mica be put between the needle and screen so as to cut off the α rays the action ceases. It may be due to the crystals of zinc sulphide being fractured under the bombardment of the α particles. Now assuming that each α particle produces a scintillation a suitable experiment of this nature might be arranged, the number of α particles counted, the total charge measured, and thus the charge e on one α particle determined; this method was adopted by Regener.

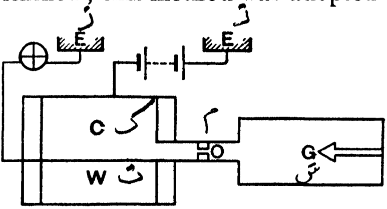


Fig. 509.

Rutherford and Geiger worked as follows:—The glass point G (Fig. 509) carries the active material which is ejecting α rays *in all directions*, those falling on the small opening O passing through into C. The potential difference between the wire W and the walls of C is such that the saturation current is flowing and the rising part of the curve is almost beginning. The conditions are such therefore that the advent of an additional α particle increases the conductivity and produces a distinct throw of the electrometer. By noting these, the number n of α particles which enters the chamber in one minute can be determined. From this the total number N given out by G is obtained by the relation $N/n = 4\pi/\omega$, where ω is the solid angle subtended at G by the opening O. With full attention to details it was found that the number of α particles emitted by Radium C from one gramme of radium was $3{\cdot}4 \times 10^{10}$ per second. Finally, the total charge given per second to a conductor was determined, the β rays being deflected away from the conductor by a magnetic field. Knowing the charge per second and the number of α particles per second, the charge e on the α particle was found. This as already stated is $9{\cdot}3 \times 10^{-10}$ *e.s.* unit or $3{\cdot}1 \times 10^{-20}$ *e.m.* unit, *i.e.* **twice the electronic charge.** Combining this with the e/m value above, viz. $5{\cdot}1 \times 10^{3}$, we get $m = 6 \times 10^{-24}$ gramme which is **the mass of a helium atom.**

That the α particle is an atom of helium has also been demonstrated as follows :—Rutherford and Royds obtained some very thin glass bulbs (·01 mm. thick) which would allow α particles to pass through them and at the same time would stand a satisfactory pressure. Radium emanation was placed in a bulb, the whole was put in an exhausted tube and the spectrum was examined when a current passed through the tube. For twenty-four hours there were no signs of helium, in four days helium lines appeared, and in six days the full helium spectrum made its appearance. *As nothing but α particles could get into the outer tube* the conclusion was obvious that the α particles were helium atoms. Finally, the experiment was repeated but with helium itself under pressure in the inner bulb: in this case there were no signs of helium in the outer tube *thus confirming the above.*

335. Radio-active Changes. (1) General.—It has been conclusively shown by Rutherford and others that the radio-active elements are elements in a more or less rapid state of disintegration. When a corpuscle, *i.e.* a β particle, gets loose and escapes and when an α particle is expelled the remainder become a new form of atom. As soon as an atom of one kind disintegrates into an atom of another kind, this new atom also begins disintegrating, and so on. Rutherford has shown that the rate of disintegration follows a definite law, which may be expressed as follows:—Let M_0 be the mass of active material initially present, and M_t the mass t seconds afterwards, then:—

$$M_t = M_0 e^{-\gamma t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where e is the base of the Napierian logarithms, and γ is a constant for the particular substance (radio-active constant). Hence the time T for half the substance to disintegrate will be:—

$$\tfrac{1}{2}M_0 = M_0 e^{-\gamma T}, \quad i.e.\ T = \frac{\log_e 2}{\gamma},$$

$$\text{or} \qquad T = \frac{\cdot 693}{\gamma} \quad \text{and} \quad \gamma = \frac{\cdot 693}{T} \ldots\ldots\ldots\ldots \quad (2)$$

To take an example: in the case of **radium emanation**—one of the disintegration products of radium—the time T for half value is 3·86 days, and therefore γ for this particular disintegration product is ·693/(3·86 × 24 × 60 × 60) *i.e.* 1/480000.

Again differentiating (1) we get:—

$$\frac{dM_t}{dt} = -\gamma M_0 e^{-\gamma t} = -\gamma M_t,$$

$$i.e.\ \frac{dM}{dt} = \gamma M \text{ (numerically)} \ldots\ldots\ldots\ldots \quad (3)$$

from which it follows that in the example taken—radium emanation—only 1/480000 of the number of existing atoms disintegrate per second. As further examples it may be mentioned that the half value time T for **thorium emanation**—a disintegration product of thorium—is 54 seconds, so that $\gamma = 1/76$. The half value time for radium is 1730 years.

Evidently, if we keep a quantity of the primitive substance and allow nothing to escape, we shall have in it a collection of atoms of all the products of disintegration, and if we keep it long enough a steady state will be obtained in which the number of atoms of any one product changing per second into the next product will be equal to the number of atoms of the product formed by disintegration of the previous product. The products are then said to be in a state of **radio-active equilibrium**.

In the sections which follow the successive disintegration products or "**metabolons**," as they are called, which occur in the disintegration of uranium, radium, actinium, and thorium are dealt with. The disintegration products are all solids except the "emanations" from radium, thorium, and actinium. These are gases which have many of the properties of an ordinary gas. The three "emanations" are very similar in their general properties, but they differ greatly in their rates of production and decay. As radium exists in the earth's crust radium emanation exists in the atmosphere.

The curve, of which (1) above is the equation, is called "exponential": it appeared in Chapter XVII. when dealing with the decay, etc., of a current in a wire, and it will appear again in dealing with the radio-active changes in the sections which follow.

335a.—Radio-active Changes. (2) Uranium X and Thorium X.—Crookes in 1900 added ammonium carbonate to a solution of uranium nitrate, and dissolved the precipitate by excess of the reagent. He found that the precipitate did not totally redissolve, and that the slight precipitate left was *photographically* as active as the original uranium, while the uranium in the filtrate was photographically inactive. This precipitate he called Uranium X. The two were kept apart for some time, and it was found that the solution wholly regained its activity, while the precipitate completely lost its activity.

More recent experiments have shown that the decay of photographic activity of the Ur. X follows the exponential law $I_t = I_0 e^{-\lambda t}$, and that the rate of recovery of the photographic activity of uranium follows the complementary law $I_t = I_0 - I_0 e^{-\lambda t}$, where λ is the radio-active constant of Ur. X; this is indicated in Fig. 510. From the curves it is clear that Ur. X falls to half value in 22 days, whilst Uranium recovers to this value in the same time:

at any instant the sum of the values for the two curves is equal to the original value.

The explanation of the above is simple on the disintegration theory. When the Ur. X is separated from the uranium it begins to lose its activity. The uranium, however, goes on producing new Ur. X *at the same rate,* and thus begins to increase in activity. Of course this new Ur. X will begin to lose activity, just like the Ur. X which was separated, but the constant production of fresh Ur. X causes the activity of the uranium to increase until the equilibrium condition is reached, when the loss of activity per second of the Ur. X present equals the gain in activity per second due to new Ur. X: the uranium will then have regained its normal activity.

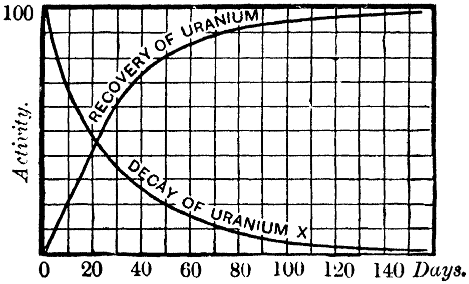


Fig. 510.

Electrical measurements showed that uranium lost little or none of its *ionisation* activity on the precipitation of the Ur. X, and that the Ur. X was almost inactive.

Rutherford has explained the above on the assumption that uranium itself only gives off α rays, while the Ur. X into which it disintegrates only gives out β rays. The α ray activity, and therefore nearly all the ionisation activity, is thus unaltered by the removal of the Ur. X, while this removal takes all the β ray activity, and therefore nearly all the photographic activity, with it.

Rutherford and Soddy precipitated thorium from solution by means of ammonia, and found that the solution which is free from thorium possessed the greater part of the activity, whilst the precipitated thorium had lost the greater part of its activity. The new substance was called Thorium X. After the lapse of a month the Thorium X had lost its activity and the thorium had recovered. Neglecting certain irregularities in the early stages (which, however, are quite capable of explanation), the decay and recovery curves are similar to those of Fig. 510 for uranium and Uranium X. The half value time for Thorium X is about 4 days.

Actinium X, a transformation product of actinium has a half value time of about 10 days.

Modern work indicates that the changes from thorium to Thorium X, and actinium to Actinium X, *are not direct changes*, but that there are intermediate products and various complications: these, however, do not interfere with the general principles and theory, and will be introduced later.

335b. Radio-active Changes. (3) The Emanations.—Radium, actinium, and thorium differ from uranium and polonium in that each constantly gives off a heavy gas which is strongly radio-active. This gas is called an *emanation*, and it has many of the properties of an ordinary gas. It can be carried along pipes by a current of air, it diffuses through porous bodies, it can be condensed, etc. The emanations from radium, actinium, and thorium are very similar in their general properties. They differ, however, greatly in their rates of production and decay, and in some other properties. The first emanation discovered was that of thorium. Radium exists very widely distributed throughout the earth's crust, and therefore radium emanation exists (to a very small extent) in the atmosphere. It may be absorbed from atmospheric air by drawing the air through a tube packed with coco-nut charcoal. If the charcoal is afterwards heated to redness the emanation is driven off and may be collected and tested.

To study the life of thorium emanation the apparatus of Fig. 511 has been employed.

Exp. A water pump draws air through cotton-wool (to filter out dust particles) in a tube D, and then over a layer of thorium oxide or hydroxide in a tube T. Charged with the emanation the air current enters an insulated metal can C through a tap at a, and after passing through the can and mixing with the contents leaves it by a tap at b. The mouth of the can is sealed by a sulphur plug in a short tube G, G being fitted into an ebonite plug which fits the mouth of C. A wire penetrates the sulphur plug and hangs axially within the can; its upper end is connected to an electroscope or electrometer. The can is connected to a battery of sufficient voltage to saturate the space between the wire and the walls of the can, the other terminal of the battery being earthed. The guard ring G is earthed, thus all possibility of a leak from the can to the wire is checked, and the good insulation of the sulphur combined with the low potential difference which usually exists between the wire and G during an experiment stops any leakage from the wire.

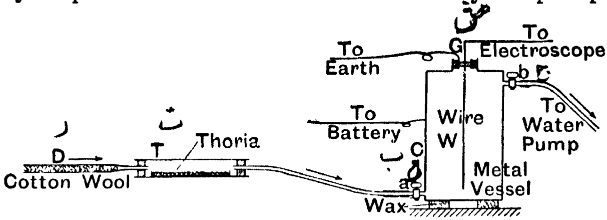


Fig. 511.

The air leak being first taken the air current is started. It is observed that the leak in C gradually increases owing to the flow of emanation into it. The leak, however, is not entirely due to the emanation, for, as will be explained later, the emanation causes the deposition of active matter on the walls of C and on the wire. The leak due to this active matter gradually increases, but in about ten minutes attains a maximum which is not very large compared with the emanation leak, and keeps practically constant during the rest of the experiment. As soon as the leak is practically steady, the taps are turned off and the pump stopped. The emanation in C is thus isolated, and it will be found that the current through C gradually decreases. The readings usually taken in such an experiment are the deflection of a gold-leaf of an electroscope, or of a needle of an electrometer. The former deflection must be translated into voltage, and the latter one is already proportional to voltage. The voltage-time curve must be first plotted, and then by the method of differences $\left(\frac{d\,(\text{voltage})}{d\,(\text{time})} \propto \text{current}\right)$ a current-time curve constructed. If accurate measurements are taken the current-time curve will be very similar to Fig. 512. The ordinates of

AB represent the total current. After ten minutes or so the leak will be steady at the value given by *OC*. *OC* will be greater than the normal air leak *OE* owing to the active deposit mentioned above. Through *C* draw a line *CD* parallel to the axis of time.

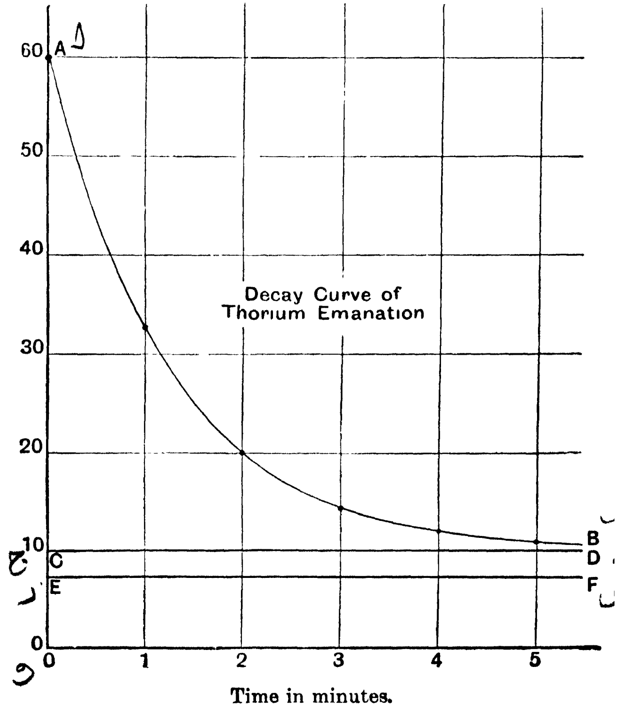


Fig. 512.

The ordinates of *AB* measured from *CD* will give the current due to the emanation, which is proportional to the amount of emanation present.

A casual glance at the curve shows that the intensity of the ionisation due to the emanation decays to half value in about a minute, *i.e.* any ordinate reckoned from *CD* is half the value of the ordinate drawn about a minute earlier. As before, the curve is "exponential," and its equation is

$$I_t = I_0 e^{-\lambda t},$$

where I_0 is the value of the ordinate when the time is begun, and I_t is the value t seconds afterwards. In the case under consideration I_0 and I_t are the intensities of the ionisation due to the emanation, and are proportional to the amount of the emanation. λ is of course the constant of radio-active change of the emanation.

A close study of the curve shows that the half value time is 54 seconds; hence from Art. 335 $\gamma = {\cdot}693/54 = {\cdot}013 = 1/76$. This means that, for thorium emanation, in one second 1/76 of the amount of emanation in existence at the beginning of that second has decayed into other matter.

Radium emanation can be studied in a somewhat similar manner. It is a much more stable gas than the above, its half value time being 3·86 days and $\gamma = 1/480000$.

Actinium emanation has a very short life: its half value time is only 3·9 seconds.

Radium emanation is produced direct from radium itself. In the case of thorium emanation and actinium emanation, there are intervening disintegration products between the emanations and the thorium and actinium respectively (see later).

335c. Radio-active Changes. (4) The Active Deposits.—It was mentioned above that thorium emanation deposited a radio-active substance upon the walls of the can. To study the rate of change of this product it is best to cause the deposit to be made on a wire by inserting the wire over some thoria in a metal tube. After a time the wire is removed and inserted in a testing vessel; the rate of change of current in the vessel with time then gives the rate of change of the deposit. Although nothing can be seen on the wire the deposit is intensely active. The deposit may be removed from the wire by rubbing it with glass-paper; in this case the glass-paper becomes active. Radium and actinium emanations also produce active deposits.

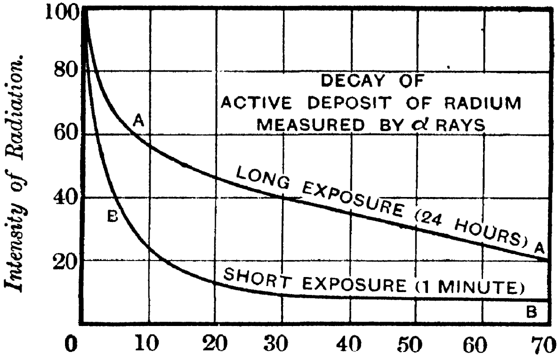
Radio-active changes are by no means finished when the "active deposit" is reached but the nature of the deposit, the complicated changes which follow, the vast amount of work which has been done in sorting out these changes, and the methods adopted are subjects too lengthy to be discussed in detail here: we can only briefly *indicate*, and for details the student must refer to specialised works on the subject.

Taking, in illustration, the case of radium, if a radium solution be placed in a flask and a platinum wire kept at a negative potential be inserted through a rubber stopper, the emanation with the emission of an α particle forms the active deposit on the wire, the emanation itself decaying to half value, as we have seen, in roughly four days

The wire can be removed at any stage and the activity of the active deposit investigated. It is found that whilst the activity of the deposit is very much reduced in twelve hours and the greater part is gone in twenty-six hours there is always a part remaining which persists in activity for many years, decaying of course in the meantime and small in amount compared with the preceding. Hence it becomes necessary to divide the active deposit into two parts, viz. **the active deposit of quick decay** and **the active deposit of slow decay.**

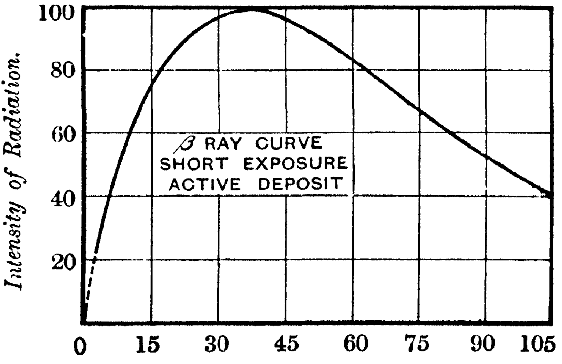
Taking first the activity of quick decay. Experiment shows that quite different decay curves are obtained if the activity is tested by the α rays and if it is tested by the β rays. Further, different curves are obtained if the wire be exposed for a few seconds before testing or say for ten hours.

Fig. 513*a* depicts the curves for an α ray examination in the case of a long and a short exposure to the emanation. Taking the case of the short exposure of one minute (curve *BB*) the activity at first falls exponentially, the half value time being three minutes. After twenty minutes the activity remains nearly constant for a time and then gradually falls. After several hours the decay sets in exponentially with a half value time of about twenty-eight minutes. In the case of the longer exposure of twenty-four hours (curve *AA*) the results are somewhat similar; there is first a decay with a half value time of three minutes,



Time in Minutes.

Fig. 513*a*.



Time in Minutes.

Fig. 513*b*.

then a gradual fall and finally after some hours an exponential fall with a half value time of about twenty-eight minutes. The middle portion in each case seems to indicate a battle between a substance decaying exponentially and the fresh production of that substance at an uneven rate.

Fig. 513*b* depicts the curve for a β ray examination of the same short exposure. The β ray activity is small at first, then increases to a maximum in thirty-five minutes and then falls: several hours later it is falling exponentially with a half value time of about twenty-eight minutes. Fig. 513*c* gives the β ray examination for the longer exposure: it falls from the start and finally exponentially with a half value time of about twenty-eight minutes.

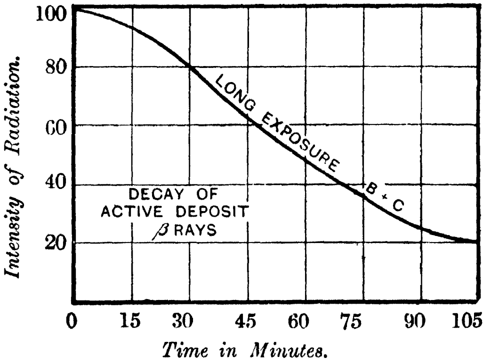


Fig. 513*c*.

A careful consideration of all these changes together with a mathematical treatment of them shows that they can be satisfactorily explained on assuming three successive changes as follows:—

(1) The emanation is transformed into a substance called Radium *A* which gives out α rays and falls rapidly, its half value time being three minutes.

(2) The Radium *A* is transformed into a substance

called Radium *B* which falls to half value in about twenty-eight minutes.

(3) The Radium *B* is transformed into a substance called Radium *C* which gives out α and β rays and falls to half value in about twenty-one minutes.

Later investigations *whilst not altering the general theory and explanation* introduce slight alterations in the numbers and put additional products into the transformations. Thus the half value time for Radium *B* is now considered to be not twenty-eight minutes, but 26·7 minutes, and this is followed by Radium C_1 19·5 minutes, Radium C_2 1·4 minutes, and Radium C^1 which is very rapid the half time value being only a fraction of a second. These more minute details may, however, be neglected at this stage.

Similarly the activity of slow decay has been investigated and classified as follows:—

(1) Radium *C* is transformed to Radium *D*, the half value time of which is perhaps fifteen years.

(2) Radium *D* is transformed to Radium *E*, the half value time of which is about five days.

(3) Radium *E* is transformed to Radium *F*, the half value time of which is about 136 days. Radium *F* is identical with *polonium*.

Modern work introduces certain extra products into the preceding: these are dealt with in the next section.

336. Radio-active Changes. (5) The Metabolons.

(1) *Uranium.*—Until quite recently the uranium transformation was considered to be as follows:—

$$\textbf{Ur.} \xrightarrow{2\alpha} \textbf{Ur. X} \xrightarrow{\beta} \textbf{Ionium} \xrightarrow{\alpha} \textbf{Ra.}$$

The atomic weight of uranium is 238·5. One atom of uranium gives off two α particles in becoming one atom of Ur. X; the uncharged α particle is an atom of helium (atomic weight 4), so that the atomic weight of Ur. X is 238·5 − 8 or 230·5. Ur. X with the expulsion of β and γ rays disintegrates into ionium; an atom of this gives out one α particle, and then radium is reached, *i.e.* radium is really a disintegration product of uranium ; if a uranium atom loses, as thus indicated, three α particles the atomic weight of the resulting product will be 238·5 – 12 or 226·5 ; the atomic weight of radium is 226·4.

It should be noted that the loss of a β particle does not alter the atomic weight. Further, γ rays nearly always accompany β rays.

It was early suggested that uranium really broke up in two ways, Ur. X and Ur. Y, and latest work indicates that the full uranium transformation is probably as follows :—

$$\mathbf{Ur._1} \xrightarrow{\alpha} \mathbf{Ur.\ X_1} \xrightarrow{\beta} \mathbf{Ur.\ X_2} \xrightarrow{\beta} \mathbf{Ur._2} \xrightarrow{\alpha} \mathbf{Ur.\ Y} \xrightarrow{\beta} \mathbf{Io.} \xrightarrow{\alpha} \mathbf{Ra.}$$

The new element—Uranium X_2—finds a place in the fifth group of the last line of the periodic system ; it has been called *brevium.*

(2) *Radium.*—Until quite recently the radium transformation was considered to be as follows :—

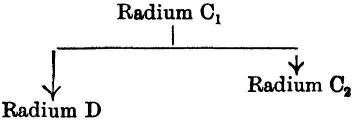
$$\overset{\to\alpha\beta}{\mathbf{Ra.}} \to \overset{\to\alpha}{\mathbf{Ra.\ Em.}} \to \overset{\to\alpha}{\mathbf{Ra.\ A}} \to \overset{\to\beta}{\mathbf{Ra.\ B}} \to \overset{\to\alpha\beta}{\mathbf{Ra.\ C}} \to \overset{\to\beta}{\mathbf{Ra.\ D}} \to$$

$$\overset{\to\beta}{\mathbf{Ra.\ E}} \to \overset{\to\alpha}{\mathbf{Ra.\ F}} \to$$

The disintegration of an atom of radium consists of the expulsion of α and β particles, the result being an atom of emanation. This change is slow, a quantity of radium decaying to half value in about 2,000 years. The emanation atoms change by the expulsion of α particles and slowly moving β particles into Radium A, which constitutes the first product of the active deposit. Only one α particle but more than two β particles are expelled from the emanation atom in changing into Radium A. The emanation being previously neutral, this leaves the Radium A atom positively charged, and hence in an electric field it at once goes to the kathode. The whole of the remaining changes now occur in the active deposit. Radium A is very short-lived. In three minutes only half the Radium A atoms are left, the remainder having changed by the expulsion of an α particle per atom into Radium B. Radium B has been called a "rayless" change, but it is not really so. Slow-moving β particles are expelled, and Radium C results. Radium B decays to half value in 27 minutes, Radium C in 19 minutes ; these, together with Radium A, constitute the rapidly changing active deposit. An atom of Radium C gives out α and β particles and γ rays, meanwhile becoming an atom of Radium D. Radium D slowly changes into Radium E, which quickly changes on the expulsion of β and γ rays into Radium F, which is identical with Polonium. The activity of a solid due to the A, B, and C deposits is much greater than that due to the D, E, F deposits into which they disintegrate. The properties of the next atom to Polonium have not been definitely worked out, but it is extremely probable that this atom is lead. (See below.)

A radium atom gives out four α particles in becoming an atom of polonium; this gives the atomic weight of polonium as 226·4 – 16 or 210·4, which suits its position in the periodic table very well. The expulsion of an α particle from polonium would leave a remainder of atomic weight 206·4, which fits in well with lead (207). Campbell has found that nearly all ordinary metals are slightly active, and of these lead is one of the most active.

It was noticed that the break up of Radium C was somewhat unusual, and the suggestion was put forward that there were certain successive products or that the atom broke up in two different ways thus:—



or that both conditions existed. Latest work indicates that the full radium transformation is probably as follows:—

$$\textbf{Ra.} \xrightarrow{\alpha\beta} \textbf{Ra. Em.} \xrightarrow{\alpha} \textbf{Ra. A} \xrightarrow{\alpha} \textbf{Ra. B} \xrightarrow{\beta} \textbf{Ra. C}_1 \xrightarrow{\alpha(?)\beta} \textbf{Ra. C}_2 \xrightarrow{\beta}$$

$$\textbf{Ra. C}^1 \xrightarrow{\alpha} \textbf{Ra. D} \xrightarrow{\beta} \textbf{Ra. E} \xrightarrow{\beta} \textbf{Ra. F (Polonium).}$$

Radium emanation has been given the name *niton*. A recent examination of lead ores from Joachimsthal, Morogoro, and Norway has yielded leads of apparently 206·405, 206·046, and 206·063 atomic weights respectively (common lead = 207); these appear to agree with Ra. G (uranium lead) from Ra. F by the expulsion of an α particle. Soddy and others have shown that in that part of the periodic table where evolution of elements is still going on there may be more than one element varying somewhat in atomic weight. The conclusion certainly is that Ra. G is an isotope of lead. (The word *isotope* means "the same position" in the periodic table). Modern work on isotopes probably indicates that an element may have more than one atomic weight due to "atomic nuclei" of the same element and total charge being built up in slightly different ways (Chapter XXV.).

The following summarises the uranium—radium group of radioactive changes. The first number below each is the atomic weight obtained by subtracting from the atomic weight of uranium (238·5) the atomic weight (4) of each helium atom emitted. The second number is the half value time (y = years, d = days, h = hours, m = minutes, s = seconds) :—

$\longrightarrow \alpha$	$\longrightarrow \beta$	$\longrightarrow \beta$	$\longrightarrow \alpha$
Ur. 1 $\longrightarrow$	**Ur. X$_1$** $\longrightarrow$	**Ur. X$_2$** $\longrightarrow$	**Ur. 2** $\longrightarrow$
238·5	234·5	234·5	234·5
$5 \times 10^9 y$.	24·6d.	1·15m.	$4 \times 10^6 y$.

$\longrightarrow \beta$	$\longrightarrow \alpha$	$\longrightarrow \alpha\beta$	$\longrightarrow \alpha$
Ur. Y $\longrightarrow$	**Io.** $\longrightarrow$	**Ra.** $\longrightarrow$	**Ra.Em.** $\longrightarrow$
230·5	230·5	226·5	222·5
25·5h.	$10^5 y$.	1730y.	3·85d.

$\longrightarrow \alpha$	$\longrightarrow \beta$	$\longrightarrow \beta$	$\longrightarrow \beta$
Ra. A $\longrightarrow$	**Ra. B** $\longrightarrow$	**Ra. C$_1$** $\longrightarrow$	**Ra. C$_2$** $\longrightarrow$
218·5	214·5	214·5	214·5
3m.	26·7m.	19·5m.	1·4m.

$\longrightarrow \alpha$	$\longrightarrow \beta$	$\longrightarrow \beta$	$\longrightarrow \alpha$
Ra. C^1 $\longrightarrow$	**Ra. D** $\longrightarrow$	**Ra. E** $\longrightarrow$	**Ra. F** (Polonium) $\longrightarrow$
214·5	210·5	210·5	210·5
$10^{-6} s$.	15·83y.	4·85d.	136d.

$\longrightarrow$ **Pb**
206·5

(3) *Thorium* (Fig. 514).—The relation between Th. X and thorium (or perhaps radiothorium) is very similar to that between Ur. X and uranium. It has been found that thorium emanation gives off

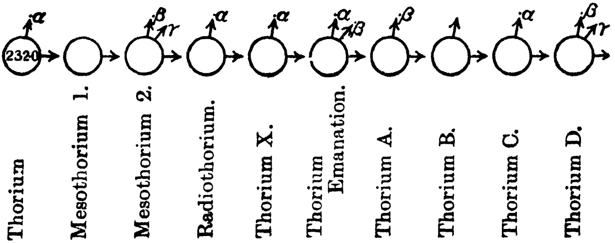


Fig. 514.

slow-moving β particles as well as α particles, and possibly Thorium A also emits these slow β particles. It has been suggested that the final product in the case of thorium is bismuth. (For latest work see page 466.)

(4) *Actinium.*—The changes are very similar to those of thorium. Actinium emanation has a very short life ("half value" in 4 sec., *i.e.* $\gamma = \cdot 17$). (For latest work see page 466.)

The radio-active changes referred to in the preceding are tabulated on page 466.

336a. Radio-active Changes. (6) The Mathematics of the Changes.—We have seen (Art. 335) that if there are M atoms of a radio-active substance present at any instant the number disintegrating in one second is γM where γ is the radio-active constant.

Consider now a substance W disintegrating and producing a substance X. Let $\gamma_o M_o$ be the rate of production of X due to the decay of W. Let γ_1 be the radio-active constant of X and let M be the number of atoms of X at any particular instant. In a small interval of time dt the number of atoms of X which decay will be $\gamma_1 M dt$, and the number which will be produced by W will be $\gamma_o M_o dt$; hence the increase dM is given by

$$dM = \gamma_o M_o dt - \gamma_1 M dt$$

$$\therefore \frac{dM}{dt} = \gamma_o M_o - \gamma_1 M \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Integrating we get:—

$$M = \frac{\gamma_o M_o}{\gamma_1} + K e^{-\gamma_1 t}$$

K being a constant. Now at time $t = O$ there are no atoms of X yet formed, *i.e.* $M = O$ when $t = O$. Substituting, $K = -\gamma_o M_o / \gamma_1$ and we get:—

$$M = \frac{\gamma_o M_o}{\gamma_1}\left(1 - e^{-\gamma_1 t}\right) \ldots\ldots\ldots\ldots\ldots (2)$$

The case of three substances may be dealt with similarly. Thus if W disintegrates to X and X to Y it can be shown that:—

$$\left.\begin{aligned} M_1 &= M_o x \\ M_2 &= M_o \cdot \frac{\gamma_1}{\gamma_1 - \gamma_2} \cdot y \\ M_3 &= M_o \cdot \frac{\gamma_1 \gamma_2}{(\gamma_1 - \gamma_2)(\gamma_1 - \gamma_3)(\gamma_2 - \gamma_3)} \cdot z \end{aligned}\right\} \ldots\ldots\ldots\ldots (3)$$

$$\text{where } x = e^{-\gamma_1 t}, \quad y = e^{-\gamma_1 t} - e^{-\gamma_2 t}$$

$$z = \left[(\gamma_2 - \gamma_3)e^{-\gamma_1 t} + (\gamma_1 - \gamma_2)e^{-\gamma_3 t} - (\gamma_1 - \gamma_3)e^{-\gamma_2 t} \right]$$

In the above $M_1 M_2 M_3$ are the number of atoms of $W X Y$ present at any particular instant, $\gamma_1 \gamma_2 \gamma_3$ are the three radio-active constants and M_o the number of atoms of W initially present.

336b. Radio-active Changes. (7) Explanation.—The disintegration theory previously outlined is universally accepted, but a satisfactory *explanation* will only be possible when much more is known about atomic structure. It has been incidentally mentioned that the rings of electrons within the atom are in rotation, and for stability the velocity must be above a certain value. Owing to radiation and consequent loss of energy the velocity may fall below this value: when this happens an electron leaves its ring, starting an orbit of its own. Now the radiation from a single electron is very much greater than from a ring of electrons, so that the energy loss is now much greater, the stability is less, and an "explosion" occurs, a helium atom (α particle) being apparently torn from the atom and expelled. The ejection of this is sometimes accompanied by the ejection of the electron (β particle) which started the trouble: sometimes more than one helium atom is ejected before this electron is finally expelled. Of course, after these ejections the remainder is a new form of atom, and the process continues, the rate of disintegration, however, being invariably different—sometimes faster, sometimes slower, but frequently slower for a time once the disorderly electron has been expelled. The present stage of our knowledge certainly provides some explanation as to why the electron is the "rowdy boy" in the atomic household: it provides no definite explanation as yet as to *why* the ejected particles apparently torn from the atom are atoms of helium (Art. 339).

337. Miscellaneous. The atom is a huge storehouse of energy. The electrons arranged within the atom are in extremely rapid rotation. When an electron (*i.e.* a β particle) gets loose and escapes some of the energy is made manifest. When an α particle becomes detached much more energy is lost by the atom. With most bodies it is very rare that either an α or a β particle gets loose, but it is common enough with uranium, radium, thorium, actinium, and polonium. Hence their radio-activity. Most of the energy set free is due to the α particles. The escaping α particles from the interior of a mass of radium are absorbed by the outer layers with evolution of heat, and hence the radium becomes warm. Curie and Laborde showed experimentally that radium keeps itself about 2° C. hotter than its surroundings, energy being emitted at the rate of 118 grm.-calories per hour per gramme of radium. During the disintegration of one gramme of radium energy is given out equal in amount to that obtained by burning 500,000 grammes of coal, but it takes the radium somewhere in the order of 3,000 years to undergo the change.

Radium exists very minutely but very widely distributed throughout the crust of the earth. Strutt has calculated that the heat generated by disintegration of the radium present in 40 miles of crust is sufficient to keep up the temperature of the earth to its present value. Incidentally, therefore, all former calculations on the age of the earth have been falsified.

The velocity of an α particle can be calculated from the distance it will travel in air before its ionisation and photographic powers have dwindled to zero. The following table gives in the first column the greatest distance the α particles from the metabolons enumerated travel in air, in the second column the initial velocity of these same particles, and in the third column the relative amounts of energy these same particles carry:—

	cm.	cm. per sec.	
Radium	3·5	$1{\cdot}56 \times 10^9$	24
Radium emanation	4·33	$1{\cdot}70 \times 10^9$	29
Radium A	4·83	$1{\cdot}76 \times 10^9$	31
Radium C	7·08	$2{\cdot}06 \times 10^9$	42
Radium F	3·86	$1{\cdot}62 \times 10^9$	26

Considerations of the "K" and "L" series of characteristic radiations (Art. 322) led Rutherford and Richardson to believe that the γ rays from radio-active matter must consist of the characteristic radiations of these heavy elements similar to the corresponding radiations observed in ordinary elements when bombarded by X-rays. To investigate this the γ rays were analysed by means of their absorption by aluminium and lead. In the latter case the rays from Radium B could be divided into four types, and similar results were obtained for all elements emitting γ rays; in some cases the γ rays corresponded to the characteristic radiation of the "K" series and in others to the "L" series. From these and other considerations it became evident that the γ rays corresponded to the natural modes of vibration of the inner structure of the radio-active atoms.

Fajans has recently indicated that:—(1) A radio-active element ejects either α particles or β particles but not both. (2) The ejection of an α particle involves a shift of two places in the periodic table. (3) The ejection of a β particle involves a shift of one place in the opposite direction to the above. This seems to be borne out pretty well. At the same time it is worthy of note that since the initial uranium is neutral and the final lead is neutral, at some stages more than one electron per atom must be expelled; thus in the disintegration of uranium there are eight helium atoms ejected each with a charge $+ 2e$ and sixteen ejected electrons are necessitated each with a charge $-e$: intermediate products of short life may of course be assumed to account for it all. The readers of this book, the rising generation of physicists, will have ample opportunity of sorting out these problems which are of absorbing interest but—to-day—only partially solved.

In the study of radio-activity we have witnessed the gradual "breaking down" of the heavy radio-active elements into lighter ones with the ejection of helium atoms and electrons, and as there are indications that possibly all substances are "active" to some extent (page 459) the suggestion may be made that, on this earth, all matter is possibly gradually breaking down into helium (the cautious physics student will add "and probably hydrogen") with the emission of electrons. Now in the study of astronomy it is found that the newest (hottest) stars are made up of helium and hydrogen (and two other unknown elements), whilst as older and still older stars are examined, heavier and still heavier elements make their appearance. Is it possible therefore that in the "earth's beginnings" the elements as we know them now were gradually built up from these light elements—helium and hydrogen—to undergo again a gradual breaking down into these light elements on the earth as we know it now? To quote Mr. J. A. Crowther (Molecular Physics): "The question suggests itself—are the elements merely a part of a great cycle of growth and decay? Is the atom born, to grow old, decay and die? Are new atoms being formed in the secret places of the universe to take the place of those that have passed away?"

Time will probably show.

Exercises XXIII.

Section C.

(1) Give an account of the various kinds of radiation emitted by a solid compound of radium, explaining how the properties of the various rays may be investigated experimentally. (B.Sc. Hons.)

(2) Describe a method of determining the quotient of the mass by the charge of an electron.

Show that the value of this quotient may depend on the velocity of the electron, and describe experimental work dealing with this effect. (B.Sc. Hons.)

(3) Give an account of the principal experiments to which we owe our knowledge of the α particle. (B.Sc. Hons.)

(4) Give an outline of the theory of the disintegration of the radioactive materials, and deduce equations showing the amounts of two consecutive products present at any time subsequent to the isolation of the higher product. (B.Sc. Hons.)

Substance.	Half Life.	Rays.
Uranium 1	5×10^9 years	α
Uranium X_1	24·6 days	β
Uranium X_2	1·15 minute	β
Uranium 2	2×10^6 years (?)	α
Uranium Y	25·5 hours	β
Ionium	10^5 years	α
Radium	1730 years	αβ
Radium Emanation	3·85 days	α
Radium A	3·0 minutes	α
Radium B	26·7 minutes	β
Radium C_1	19·5 minutes	α(?)β
Radium C_2	1·4 minute	β
Radium C^1	10^{-6} seconds (?)	α
Radium D	15·83 years	β
Radium E	4·85 days	β
Polonium (Ra. F)	136 days	αβ (?)
Thorium	$1·5 \times 10^{10}$ years	α
Mesothorium 1	5·5 years	——
Mesothorium 2	6·2 hours	β
Radiothorium	2·02 years	α
Thorium X	3·64 days	α
Thorium Emanation	54 seconds	α
Thorium A	0·14 second	α
Thorium B	10·6 hours	β
Thorium C_1	60 minutes	αβ
Thorium D	3·1 minutes	β
Thorium C^1	10^{-11} seconds (?)	α
Actinium	200 years (?)	α (?)
Radioactinium	18·88 days	αβ
[Radioactinium′]	60 hours (?)	α (?)
Actinium X	11·4 days	α
Actinium Emanation	3·9 seconds	α
Actinium A	0·002 second	α
Actinium B	36·1 minutes	β
Actinium C_1	2·15 minutes	αβ (?)
Actinium D	4·71 minutes	β
Actinium C^1	0·001 second	α
Potassium	——	β
Rubidium	——	β

CHAPTER XXV.

ELECTRONIC THEORIES AND THE NEW PHYSICS.

338. Motion of an Electrified Sphere. Mass of an Electron entirely Electrical.—Consider a small sphere O of radius a, electrified and moving in the direction OX (Fig. 515) with velocity v. If this velocity be *not large* we may assume the sphere to carry its Faraday tubes along with it, undisturbed, as shown in the figure. As already indicated, this moving charge is equivalent to a current, and the magnetic field at P due to it is given by the expression:—

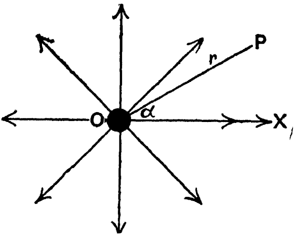


Fig. 515.

$$\text{Magnetic field at } P = \frac{ev \sin \alpha}{r^2},$$

where e is the charge, r the distance of P from O, and α the angle between OP and OX. Now the magnetic energy per unit volume (medium = air) in a field has been proved equal to $H^2/8\pi$: hence the magnetic energy in a small volume dv at P is given by:—

$$\text{Magnetic energy} = \frac{1}{8\pi} \frac{e^2v^2 \sin^2 \alpha}{r^4} . dv$$

and the total magnetic energy in the whole field surrounding the sphere is obtained by imagining the whole field divided up into these small volumes and finding the sum of all these values.

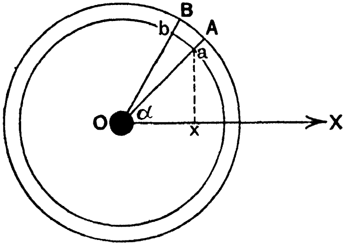


Fig. 516.

Consider now two spheres of radii r and $r + dr$ to be drawn in the field, the charged sphere O being at the centre (Fig. 516). Let, as before, OX be the direction of motion, and let the radius OA make an angle a, and OB an angle $a + da$ with OX. Now imagine OA and OB to revolve about OX. The *area of the belt* on the sphere of radius r which OA and OB will sweep out is

$$2\pi r \sin a \,.\, r da,$$

for the radius of the circular belt swept out is $\overline{ax}$, viz. $r \sin a$, and its width $\overline{ab}$ is $r da$. The *volume of the space* between this belt and the outer sphere is :—

$$2\pi r \sin a \,.\, r da \,.\, dr,$$

and the magnetic energy in this space is, from the preceding :—

$$\frac{1}{8\pi} \cdot \frac{e^2 v^2 \sin^2 a}{r^4} \,.\, 2\pi r \sin a \,.\, r da \,.\, dr,$$

$$\textit{i.e.}\ \frac{e^2v^2}{4r^2} dr \sin^8 a \,.\, da.$$

The total magnetic energy in the entire space between the two spheres is clearly :—

$$\int_0^\pi \frac{e^2v^2}{4r^2} dr \,.\, \sin^3 a \,.\, da$$

$$= \frac{e^2v^2}{4r^2} dr \,.\, 2\int_0^{\frac{\pi}{2}} \sin^8 a \,.\, da$$

$$= \frac{e^2v^2}{4r^2}dr \,.\, 2 \,.\, \frac{2}{3} = \frac{1}{3}\, e^2v^2 \frac{1}{r^2}\, dr.$$

Hence the total magnetic energy in the whole field surrounding the sphere is the integral of this from the surface of the sphere, where $r = a$, to infinity : that is :—

$$\text{Total magnetic energy} = \frac{1}{3} e^2v^2 \int_a^\infty \frac{1}{r^2}\, dr$$

$$= \frac{e^2}{3a} v^2.$$

Now if m be the ordinary mechanical mass of the sphere, its kinetic energy due to its motion is $\frac{1}{2}mv^2$; thus the total energy of the moving electrified sphere is given by :—

$$\text{Energy} = \frac{1}{2}\, mv^2 + \frac{e^2}{3a}v^2$$

$$= \frac{1}{2}\left(m + \frac{2e^2}{3a}\right)v^2,$$

or it is the same as if the ordinary mass m were increased by an amount $\frac{2e^2}{3a}$ due to the charge. This latter is frequently referred to as the "electrical mass," and is denoted by I, *i.e.*

$$I = \frac{2e^2}{3a},$$

or to be more general $2\mu e^2/3a$, where μ is the permeability. Strictly, of course, this mass lies not in the body but in the aether: experimentally, however, it is the same as if it resided in the former.

Up to this point it has been assumed that the velocity is not great and the Faraday tubes undisturbed by the motion. As the velocity increases, however, the tubes tend to set more and more into the equational region YY (Fig. 516a), and as this occurs the mass of aether they drag up with them increases. In other words, the electrical mass is not constant but depends upon the velocity, at any rate, when the latter becomes great, approaching that of light, *i.e.* as previously indicated *the electrical mass increases with the velocity.* Theory shows that, more exactly, the electrical mass I is given by an expression of the form

$$I = \frac{2\mu e^2}{3a}\left(1 + \frac{v^2}{2V^2} + \text{higher powers}\right),$$

where V is the velocity of light, and Dr. Abraham gives the formula:—

$$\frac{\text{Electrical mass at velocity } v}{\text{Electrical mass at low velocity}} = \frac{I}{I_1}$$
$$= \frac{3}{4\beta^2}\left(\frac{1+\beta^2}{2\beta}\log\frac{1+\beta}{1-\beta} - 1\right),$$

where β is the ratio of the velocity of the charged particle to the velocity of light.

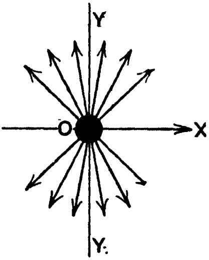


Fig. 516*a*.

Turning from the question of the charged moving sphere to that of an electron, J. J. Thomson conjectured that probably the whole mass of an electron arose in this way. Assuming this, writing the expression for I above in the form—

$$I = m = \frac{2e^2}{3a} \quad \text{or} \quad a = \frac{2}{3}\frac{e}{m}e,$$

and substituting the known values of e/m and e for the electron, we get $a = 1{\cdot}9 \times 10^{-13}$ cm. as the radius of an electron: this is very small compared with that of a gaseous atom, viz. of the order 10^{-8} cm.

Again, working with the β rays from radium, Kaufmann obtained *experimentally* the numbers given in column 4 of the table (p. 471), for the ratios of the mass of the particles to their mass at low velocities. *Assuming the whole mass to be electrical* the numbers given in column 3 were *calculated* for the ratios by Abraham's formula. The close agreement supports the view that **the whole mass of the electron is electrical,** *i.e.* **due to the charge:** in other words, **an electron is a disembodied atom of elec-**

tricity free from association with matter as we know it.

v	β	Ratio by Formula.	Ratio by Exp.
$2{\cdot}88 \times 10^{10}$ cm. per sec.	·963	2·52	2·70
$2{\cdot}80 \times 10^{10}$,, ,,	·933	2·14	2·09
$2{\cdot}49 \times 10^{10}$,, ,,	·830	1·61	1·70
$2{\cdot}20 \times 10^{10}$,, ,,	·732	1·41	1·41

The question then arises, Are there two kinds of mass or is all mass electrical? and J. J. Thomson postulated the latter, viz. that **all mass is electrical**, due to the tubes carrying forward some of the aether. On this view all mass is mass of the aether, all momentum is momentum of the aether, all kinetic energy is kinetic energy of the aether, and further, since the whole mass of a body is all in the aether, the body is therefore everywhere, and every body occupies space occupied by every other body. Mathematics shows, however, that the mass, though everywhere, is intensely localised, for only $\frac{1}{100000}$ of the mass is outside a sphere of the same size as an ordinary atom.

Difficulties in the above may be briefly indicated. It was early suggested that the mass of an atom was due to its electrons, and this would necessitate in round numbers (say) 2,000 in a hydrogen atom, whereas recent work indicates that the number is of the order *one* for a hydrogen atom and only a fraction (1/2) of the atomic weight for other atoms (Art. 339). Of course there is in the atom the "positive sphere" or "positive nucleus" (Art 339), which must be there for the atom to be neutral, but about which we do not know very much. In the case of the hydrogen atom, if the positive charge were on a nucleus 1/2000 of the size of an electron, the mass of the atom might be accounted for, and the theory that mass is all electrical would still hold. The difficulty lies in our ignorance of this positive electricity.

The preceeding explains mass in terms of aether, but the question still remains—*What is the explanation of aether?* To fit in with certain ideas, it should be more fluid than the most perfect gas, but to fit in with others it should be of great density. Again, for many years it was the subject for dispute whether the earth, for example, carried the aether along with it or whether the aether was of such a nature that it was not disturbed by the motion of bodies through it; the latter view was, ultimately, almost universally accepted, but when experiments were carried out on the determination of "velocity relative to the aether" they, practically, one and all failed. Explanations have been given (*e.g.* the Lorentz-Fitzgerald explanation of the Michelson-Morley experiment), but difficulties remain. The recent German attempt to explain away the aether by the "**Principle of Relativity**"—that we never shall know velocity relative to the aether—bristles with difficulties, and is a long way from satisfying as yet. All the knowledge gained in the last decade has, however, been a true development in the right direction, and work in the near future may clear up much that is at present obscure; the key to future progress is the answers to the associated questions—"What is positive electricity?" "How are we to explain aether?"

339. The Structure of the Atom. Van-den-Broek's Hypothesis. Planck's Constant and Quanta.—The periodic law, the study of spectra, radioactivity, etc. indicate that the elements have something in common, and, as electrons, identical in every respect can be ejected from all sorts of matter it is clear that the electron must be a common constituent of all matter.

In a theory of atomic structure due to Professor J. J. Thomson the atom consists of rapidly revolving negative electrons within a sphere of positive electricity, the total positive charge being equal to the negative charge. The force of attraction on an electron will vary directly as its distance from the centre of the sphere, and, treating the problem as a statical one, the system of electrons will be stable if their mutual forces of repulsion balance the attraction of the positive sphere. The three dimensional problem of the arrangement of the electrons has not been worked out in detail, but it has been solved for two dimensions, and it has been shown that when the number of electrons is large they are arranged in several concentric rings.✓ Thus 6 in a ring is unstable, but 5 in a ring and 1 at the centre is stable; 8 in a ring is unstable, but 7 in a ring and 1 at the centre is stable; a stable arrangement of 30 electrons would consist of 15 in an outer ring, 10 in the next ring, and 5 in the central ring, and so on. It has been stated that the atomic weight of an element is proportional to the number of electrons contained in its atom, and the following table gives a stable arrangement of atoms of increasing atomic weight: in the fourth row there are 4 rings of electrons, in the third row 3 rings, and in the second row 2 rings in each atom. The student of chemistry will see many points of resemblance to Mendeleef's periodic classification: in fact, many points in physics and chemistry can be explained on the lines of the preceding atoms, and so also can the facts of radiation if the rotation of the electrons is taken into account, but to go further into details would take us beyond our present province.

1						2	3					4		5		
5 1	6 1	7 1			8 1	8 2	8 3		9 3		10 3	10 4		10 5		11 5
11 5 1	11 6 1	11 7 1		12 7 1	12 8 1	12 8 2	12 8 3	13 8 3	13 9 3		13 10 3	13 10 4	14 10 4	14 10 5	15 10 5	15 11 5
15 11 5 1	15 11 6 1	15 11 7 1	16 11 7 1	16 12 7 1	16 12 8 1	16 12 8 2	16 12 8 3	16 13 8 3	16 13 9 3	17 13 9 3	17 13 10 3	17 13 10 4	17 14 10 4	17 14 10 5	17 15 10 5	17 15 11 5

Later work by Bohr, Rutherford, and others definitely indicated that Thomson's positive sphere must be abandoned and that the main part of an atom is concentrated in a nucleus which has *on the whole* a positive charge (protons). Some at least of the electrons are outside this positive charge, extending to distances comparable with the diameter of the atom and revolving as satellites round the nucleus. The attraction on an electron is, in this case, inversely proportional to the square of the distance from the nucleus. Since the radius of an atom is of the order 10^{-8} cm., whilst that of the electron is of the order 2×10^{-13} cm. (one fifty-thousandth of the atomic radius), and that of the positive nucleus has been estimated to be in most cases probably less than that of the electron, it is evident that the bulk of the atom of matter is "empty space"—a kind of solar system in which the so-called "material lumps" are very very small indeed compared with the total bulk. Hence it is that a helium atom or an electron travelling with great speed can go straight through an atom, straight through, in fact, 10,000 atoms without hitting anything. Hence it is, also, that a helium atom, if only travelling fast enough, can go through 500,000 atoms without coming near enough to the strong minute central nucleus of an atom to suffer appreciable deflection. These facts are shown by Wilson's photographs.

The determination of the actual number of electrons in an atom of any substance is a problem on which much work has been done with conflicting results. Evidence derived from certain experiments gave comparatively small numbers, whilst evidence derived from other experiments gave much larger numbers. Of course, in the former, the number of electrons involved in the phenomena under examination might be only a small part of the total number present, whilst the view already mentioned, that the mass of an atom was the sum of the masses of its electrons, at first gained much favour owing to its simplicity and attractiveness ; on this view for the number N of electrons in an atom we would have

$$N = W\frac{M}{m},$$

where M is the mass of a hydrogen atom, m the mass of an electron, and W the atomic weight.

Modern work, however, does not uphold the theory. From experiments on the deflection of β rays in passing through certain thicknesses of solid matter it is possible to calculate the number of collisions it has made and to find the number of electrons in unit volume, and knowing the number of atoms in unit volume the number of electrons per atom may be deduced. Recent experiments of this nature at the Cavendish Laboratory led to the conclusion that if the positive electrification is distributed over a sphere the size of the atom the number of electrons per atom is equal to three times the atomic weight ($\therefore$ 3 in a hydrogen atom), whereas if the positive electrification is a nucleus at the centre the number of electrons per atom is equal to the atomic weight ($\therefore$ 1 in a hydrogen atom). Certain other experiments gave the number (outside the nucleus) as half the atomic weight.

In still later work by Rutherford the chance that an α particle going through a sheet of gold foil would be deflected through a certain angle was estimated, and then Marsden and Geiger, by means of the scintillations on a zinc sulphide screen, counted what fraction of a certain number of α particles falling on gold foil were deflected through a certain angle. Combining their result with Rutherford's they were able to determine *the number of free positive charges* (each numerically equal to the electronic charge e) *on the nucleus*, and this is, of course, equal to the number of electrons outside the nucleus. The experiments and estimations were repeated with different substances and *in all cases the number of free positive charges was found to be approximately half the atomic weight*, or somewhat less. By a series of quite different experiments—the scattering of X-rays—Barkla estimated *the number of electrons outside the nucleus to be approximately half the atomic weight*, agreeing, as it should do, with the previous experiments on the free positive charges on the nucleus. Finally, Moseley's brilliant work on the wave-lengths of the characteristic X-rays set up when kathode rays fall upon different substances confirmed the above, and added definiteness to it. (Moseley, unfortunately, like many more of our splendid young men, was killed in the trenches by a Turkish bullet at Gallipoli in the summer of 1915.) The final conclusion is that **if the elements in Mendeleeff's series be numbered in order of atomic weight (*i.e.* what is called the "atomic number" N) these numbers, which are approximately half the atomic weight, will give the number of free positive charges on the nucleus and also the number of electrons outside** (*i.e.* if N be the atomic number and e the electronic charge the number of free positive charges in the nucleus is N, the total free charge on the nucleus is $+Ne$, and there are N electrons, total charge $-Ne$, outside). This view is now adopted (Appendix 2).

Thus from the above, hydrogen has a central nucleus with one free positive charge e and one revolving electron outside; helium has two central positive charges and two revolving electrons, lithium has three of each, beryelium four, boron five, carbon six, nitrogen seven, oxygen eight, fluorine nine, neon ten, and so on up to uranium with ninety-two.

In a modified conception of the atom due to Lewis and Langmuir, the electrons outside the nucleus are at rest or merely oscillating. The first two electrons lie on a shell round the nucleus. The next eight take up positions (outside the first two) corresponding to the eight corners of a cube. Thus we have a series of shells containing electrons one outside the other, the successive shells containing respectively 2, 8, 8, 18, 18, 32 electrons. If the outer shell is not complete the atom tends to join with others to complete it, *i.e.* there is chemical union between them. If it is complete, the atom is inert like argon and neon. Further details would, however, take up too much space here, and reference should be made to some modern work on Physical Chemistry.

It has been mentioned that the characteristic radiation of highest frequency which an atom produces—the K radiation—is due to perturbations of the innermost, most rapid ring, the L radiation of lower frequency is from the next ring, and the M radiation from the next ring (the extra high frequency J radiation, recently mentioned will be from a ring within the K ring, very close to the nucleus). The fact that hydrogen, with a single electron in its planetary system, can exhibit these radiations is explained by Bohr's additional views on atomic structure. Bohr's contention is that an electron may rotate round the nucleus in a series of different orbits, and that radiation takes place when the electron jumps from one orbit to the other. The radii of the orbits are as the squares of the natural numbers, viz. 1, 4, 9, 16, etc., the frequencies as the inverse cubes of the natural numbers, viz. 1, $\frac{1}{8}$, $\frac{1}{27}$, $\frac{1}{64}$, etc., and Bohr has shown that the longest wave-length line of the K series is due to jumping from orbit B to orbit A (Fig. 516*b*),

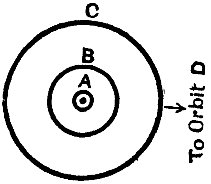


Fig. 516*b*.

whilst the longest wave-length line of the L series is due to jumping from C to B. Further, it has been mentioned that an electron may be expelled by the direct impact of an α or β particle from outside, or by the influence of X-rays, etc.: it can also be expelled by the collapse of particles from one orbit to the next. Thus Sir Oliver Lodge points out that, just as the energy required to throw a planet to infinity is double its energy in its orbit, so the energy of an electron in its orbit is just that "quantum" of energy which must be supplied in addition, in order to bring about an expulsion. Now the kinetic energy is inversely as the distance of the orbit from the nucleus, so that if the K, L, and M orbits have radii 1, 4, and 9, the energies will be as the numbers 1, $\frac{1}{4}$, and $\frac{1}{9}$. Taking, then, the case of an M particle falling to L, the energy acquired will be $\frac{1}{4} - \frac{1}{9}$, *i.e.* $\frac{5}{36}$ of a K unit, so that altogether $\frac{5}{36} + \frac{1}{9}$, *i.e.* $\frac{1}{4}$ of a K unit will be transmitted, and, this being equal to a unit of L energy, will be able to eject the L particle.

The nucleus contains, as has been explained, a number of *free* positive charges equal to the atomic number: that there are also electrons inside the nucleus (and, therefore, an equal number of binding positive charges) is indicated by certain phenomena in radio-activity. To explain these Rutherford supposes that not only the α but also the primary β particles are expelled *from the nucleus*; thus the α particle, being positive, will have its velocity increased in passing through the strong repulsion field, whilst the β particle, being negative, will be impeded in its escape from the nucleus, and must, therefore, possess considerable energy in order to effect its escape. Further, if a β particle escapes from a ring near the surface it constitutes the high speed β ray emitted without the production of γ rays (Radium E and Uranium X are illustrations), while if it escapes from the interior it passes on its way through regions where it gives rise to the production of γ rays.

The statement that the positive charge on the nucleus is equal to the atomic number N (*i.e.* that the charge is $+Ne$) is often referred to as "**Van den Broek's hypothesis.**"

According to Bohr the angular momentum of an electron is constant and given by the expression $h/2\pi$; this is spoken of as the "***h*** **hypothesis,**" h being known as "**Planck's universal constant.**"

Planck's "**Theory of Quanta**" assumes that not only matter but *energy* has an atomistic structure, at least when flung out into space as radiation; it asserts that a simple harmonic aether wave gives energy to matter in quantities of hn ergs at a time, n being the frequency. The actual magnitude of the quanta is very small. The energy of one quanta of radiation of frequency n is hn, and for a frequency of one vibration per second it is only 6×10^{-27} erg.

This is *Planck's constant*; it has proved a useful constant in various investigations, and although its actual nature is not yet defined (it is an energy divided by a frequency, but in the h hypothesis above it is regarded as an angular momentum) it is possible that it may be proved to be one of our fundamental constants.

The **Bohr atom** referred to above may now be dealt with somewhat fuller. Bohr asserts that (a) an atomic system possesses a number of "stationary states" in which there is no emission of energy as radiation, (b) a transition from one of these states to another is accompanied by an emission or absorption of energy; the frequency n of the radiation emitted is given by the expression

$$hn = E_1 - E_2 \quad \dots\dots\dots\dots\dots\dots \quad (1)$$

where E_1 and E_2 denote the energies of the system in the two stationary states, and h is Planck's constant. (c) In the case of an electron rotating round a positive nucleus the possible stationary states are given by the relation

$$T = \tfrac{1}{2}ahn \quad \dots\dots\dots\dots\dots\dots \quad (2)$$

where T is the mean value of the kinetic energy of the system, and a is a whole number; if the electronic orbit is circular it is shown that this is in agreement with the assertion that the angular momentum of the system in the stationary states is an integral multiple of $h/2\pi$. (d) In the case of a system consisting of positive nuclei at rest relative to each other and electrons moving in circular orbits the angular momentum of each electron round the centre of its orbit is $h/2\pi$ when the total energy of the system is a minimum. For further details the reader should consult the original paper.

It has recently been shown that Planck's constant may be connected numerically with the magnetic moment of the "**magneton**" (Art. 282). Consider an electron moving in a circular orbit of radius a with angular velocity ω. Its angular momentum will be $ma^2\omega$; thus $h/2\pi = ma^2\omega$ and therefore $a^2\omega = h/2\pi m$. Further, the magnetic moment of the revolving electron will be $\frac{1}{2}ea^2\omega$, i.e. $eh/4\pi m$; hence

$$h = 4\pi \frac{m}{e} \text{ (magnetic moment).}$$

Now the magnetic moment of the revolving electron is $92{\cdot}7 \times 10^{-2}$, and that of the magneton $18{\cdot}54 \times 10^{-22}$, *i.e.* one fifth of the former; thus

$$h = 20\pi \frac{m}{e} \text{ (moment of the magneton).}$$

An extension of this to magnetic theory and the structure of the atom has recently been made by Peddie. He considers the atom as consisting of concentric spherical shells of electrification rotating round a common axis. Consider a uniform positive sphere of radius r and charge Ne moving with angular velocity β. Outside this imagine there is a ring of n (from 1 to 8) valency electrons also in rotation, and assume the remaining negative electrification to be on a central core having no rotation. The magnetic moment of the rotating sphere will be $\frac{1}{5}Ner^2\beta$. As we have no definite knowledge of the value $r^2\beta$ it is taken as equal to $a^2\omega$ for a ring electron. Thus the magnetic moment of the positive sphere will be $\frac{1}{5}Nea^2\omega = Neh/10\pi m$; but the moment of the magneton is $eh/20\pi m$, so that the magnetic moment of the positive sphere will be equal to $2N$ magnetons Again, the magnetic moment of the ring will be equivalent to $5n$ magnetons (the magneton moment is one-fifth of the electron moment). Hence the resultant magnetic moment for this atomic model will be the difference between the $2N$ magnetons of the core and the $5n$ magnetons of the ring. This view is communicated by Dr. H. S. Allen, and it certainly emphasises that in considering magnetic material and atomic models the magnetic effects of the nucleus will enter into the considerations.

It has been explained that light is an electro-magnetic wave, and it is interesting to note, in passing, the origin of such waves. We have seen that when an electron is suddenly stopped it gives rise to an aether pulse, the electric and magnetic forces being at right angles to each other, and both at right angles to the direction of propagation. If, instead of being stopped, we imagine the electron to execute simple harmonic vibrations to and fro of suitable frequency we have the necessary conditions for the propagation of plane polarised light; if we imagine circular or elliptic vibrations of suitable frequency we have the necessary conditions for the propagation of ordinary light.

In the sections which follow some simple applications of the **electron theory** are dealt with: the subject is a very wide one, so that a very brief treatment only is possible.

340. The Dielectric Constant (Specific Inductive Capacity) and the Index of Refraction.—We have seen that an atom consists of some structure of positive electricity combined with certain corpuscles or electrons, and, on the electron theory, the distinction between a dielectric or insulator and a conductor is that in the former the electrons can be displaced within the atom but cannot be dragged out of it by an external electrical field, whilst in the latter the electrons can be detached from the atom and are quite free to move in the spaces between them.

Consider now a rectangular block of a dielectric placed at right angles to a uniform electrical field (air). Let e be the charge on an electron, x its displacement from its neutral position due to the field, F the electric intensity in the block, and f the restoring force on an electron for unit displacement. The total displacing force on the electron is Fe and the total restoring force on it is fx; hence, as there is equilibrium,

$$Fe = fx, \quad \text{i.e. } x = \frac{Fe}{f} \quad \dots\dots\dots\dots\dots\dots \quad (1)$$

Now let N be the number of electrons per unit volume and A the face area of the block. The displacement of the electrons through a distance x will be equivalent to a negative charge of $NAxe$ at one side and an equal positive charge at the other, *i.e.* will be equivalent to surface densities of Nxe.

The intensity F in the block is due jointly to the external field and the surface densities. The former is $4\pi D$ (Art. 88), where D is the polarisation of the external field ($K = 1$) and the latter is $4\pi\rho$, i.e. $4\pi Nxe$; their directions are clearly opposed, hence

$$F = 4\pi D - 4\pi Nxe = 4\pi D - 4\pi Ne\frac{Fe}{f} \quad \dots\dots \quad (2)$$

$$\therefore\ 4\pi D = F\left(1 + \frac{4\pi Ne^2}{f}\right),$$

i.e.

$$\frac{4\pi D}{F} = 1 + \frac{4\pi Ne^2}{f}.$$

But if K be the dielectric constant of the block, $F = 4\pi D/K$, and if n be the index of refraction $K = n^2$; hence

$$K = n^2 = 1 + \frac{4\pi Ne^2}{f} \quad \dots\dots\dots\dots\dots\dots \quad (3)$$

If the dielectric is a gas the atoms are far apart and therefore f is independent of the density whilst N is proportional to the density; in this case therefore $(K - 1)$ and $(n^2 - 1)$ are proportional to the density. Boltzmann has shown that this is so.

In the preceding we have considered the case of a *steady electrical field* acting on the electrons in the block, and have found expressions for the index of refraction and the dielectric constant To obtain corresponding expressions in the case of *waves of light* falling on a substance certain results from the "theory of vibrations" must be assumed; these the student will find in any good work on Sound.

Let m be the mass of an electron in the substance, then, since it is acted on by a force fx which is proportional to its distance x from a centre of force, it will have a natural period of vibration T_1 given by

$$T_1 = 2\pi\sqrt{\frac{m}{f}}.$$

Now imagine a light wave of period T_2 to fall on the substance. The electron will be subject to a periodic force eF varying harmonically and therefore given by the expression

$$eF_0 \sin 2\pi \frac{t}{T_2}.$$

It will therefore execute "forced vibrations" of period T_2, and the displacement x at any instant will be

$$x = \frac{e}{m} \cdot \frac{F_0}{\frac{f}{m} - \frac{4\pi^2}{T_2^2}} \sin 2\pi \frac{t}{T_2}$$

or, since $f/m = 4\pi^2/T_1^2$,

$$x = \frac{e}{m} \cdot \frac{F_0}{4\pi^2\left(\frac{1}{T_1^2} - \frac{1}{T_2^2}\right)} \sin 2\pi \frac{t}{T_2} \quad \ldots\ldots\ldots \quad (4)$$

Putting now $F_0 \sin 2\pi \frac{t}{T_2}$ in place of F, $D_0 \sin 2\pi \frac{t}{T_2}$ in place of D, and (4) in place of x in (2), viz. $F = 4\pi D - 4\pi Nxe$, we get

$$F_0 \sin 2\pi \frac{t}{T_2}$$

$$= 4\pi D_0 \sin 2\pi \frac{t}{T_2} - \frac{4\pi Ne^2 F_0}{4\pi^2 m\left(\frac{1}{T_1^2} - \frac{1}{T_2^2}\right)} \sin 2\pi \frac{t}{T_2}$$

$$\therefore\ 4\pi D_0 = F_0\left(1 + \frac{Ne^2}{\pi m\left(\frac{1}{T_1^2} - \frac{1}{T_2^2}\right)}\right),$$

$$i.e. \qquad K = \frac{4\pi D_0}{F_0} = 1 + \frac{Ne^2}{\pi m\left(\frac{1}{T_1^2} - \frac{1}{T_2^2}\right)} = n^2 \text{ ......(5)}$$

The difference between the two values of K (and n^2)—the one for steady fields, the other for light waves—should be carefully noted: in the latter if T_2 be great in comparison with T_1 we have

$$K = n^2 = 1 + \frac{Ne^2}{\pi m \frac{1}{T_1^2}} = 1 + \frac{4\pi Ne^2}{f},$$

since $f = 4\pi^2 m/T_1^2$; this is the expression already obtained above for steady fields (see also Art. 314).

341. Electrical Conduction in Metals.—It has been shown that the conduction of electricity in electrolytes and in gases is effected by positively and negatively charged carriers called **ions.** In liquids the ions are free charged atoms or groups of atoms (Art. 203); in gases the negative ion is an electron loaded up by having attached to it one or more neutral atoms (at low pressures the electron throws off its attendant neutral atoms and travels alone), whilst the positive ion is an atom which has lost an electron. The conduction of electricity in solids is also effected by carriers, *but the latter in this case consist solely of free electrons.* In the absence of an electric field these electrons travel promiscuously in all directions, and it is further supposed that they are in temperature equilibrium with the conductor, *i.e.* that their mean kinetic energy is equal to the mean kinetic energy of the molecules of the conductor, and, therefore, that the ordinary laws of the Kinetic Theory apply. On the establishment of an electric field there is, in addition to this movement in all directions, a drift of electrons in the opposite direction to the field (the electrons are *negative*), and this constitutes the current in the conductor.

Now, by the Kinetic Theory,

$$\tfrac{1}{2}mv^2 = aT \text{ .... (1)}$$

where m = mass of an electron, T = absolute temperature, v = mean velocity of the electrons corresponding to temperature equilibrium with the substance, and α = a constant.

Let an electric field of intensity E be applied to the conductor; between two collisions an electron will be subject to a force Ee, and its *acceleration* f parallel to the field will be

$$f = \frac{Ee}{m} \quad\dots\dots\dots\dots\dots\dots\dots\dots \quad (2)$$

If p be the *mean free path*, i.e. the average distance travelled between two collisions, then the time (t) taken to describe this mean free path will be p/v, and the distance travelled (d) parallel to the field in this time will be

$$d = \tfrac{1}{2}ft^2 = \frac{1}{2} \cdot \frac{Ee}{m} \cdot \frac{p^2}{v^2} \quad\dots\dots\dots\dots \quad (3)$$

Again, if V be the average velocity parallel to the field,

$$V = \frac{d}{t} = \frac{dv}{p} = \frac{1}{2} \cdot \frac{Ee}{m} \cdot \frac{p}{v} ;$$

and since $\frac{1}{2}mv^2 = \alpha T$, i.e. $m = 2\alpha T/v^2$,

$$V = \frac{Eepv}{4\alpha T} \quad\dots\dots\dots\dots\dots\dots\dots \quad (4)$$

But if N be the number of electrons in unit volume and I the current density, $I = NeV$, i.e.

$c^2 = e^2$ ×

$$I = \frac{NEc^2pv}{4\alpha T} \quad\dots\dots\dots\dots\dots\dots \quad (5)$$

and, if ρ be the *conductivity*, $I = \rho E$, i.e.

$$\rho = \frac{Ne^2pv}{4\alpha T} \quad\dots\dots\dots\dots\dots\dots\dots \quad (6)$$

Hence (1) Ohm's law is obeyed, for by (5) I is proportional to E. (2) From (6) ρ is inversely proportional to T, *i.e.* specific resistance is proportional to the absolute temperature (this assumes that the product Npv is independent of temperature). (3) Differences in conductivity of different bodies are due to differences in the number of free electrons (N), for on the right hand side of (6) the only factor likely to differ very much in different substances is N.

Although the simple electronic theory of electrical conduction in metals outlined above explains many points and has many points in its favour, there are, nevertheless, serious difficulties to be explained away. Consider, first, the number of free electrons required to account for the high conductivity of most metals. Calculation shows that in the case of silver the number per unit volume should be of the order $2{\cdot}6 \times 10^{24}$, which is about 40 times the number of silver atoms ; this would mean that each atom of silver would have to lose about 40 electrons ; taking the charge on an electron as unity, each atom would have a positive charge of 40 units, and the specific heat of the electrons in unit mass would be considerably greater than the actual specific heat of silver. There are probably reasons for believing that the number of electrons which can be withdrawn from an atom of metal under ordinary conditions is quite small, so that probably the number of free electrons cannot greatly exceed the number of atoms.

Further, other difficulties must be explained away in connection with the variation of resistance with temperature. Kamerlingh Onnes finds that at the temperature of liquid helium the resistance of certain pure metals is less than one-thousand-millionth of its value at 0° C., whilst if it fell in proportion to the absolute temperature it would only be reduced by one-seventieth ; he also finds that an induced current once started in a ring of lead at this temperature lasts almost undiminished for two hours, and takes about four days to fall to half value. These and certain other difficulties are in a measure met by a modified electronic theory of conduction in metals due to J. J. Thomson.

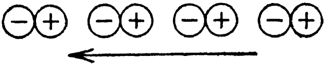


Fig. 517.

Thomson assumes that an atom of the metal contains an electrical doublet, which consists of equal and opposite charges, the negative being the electron. When an electrical field is applied these doublets set themselves in chains parallel to the field (Fig. 517). The doublets will give rise to intense electric forces, and electrons will be dragged out of one atom into the other along the chain. Thus the field results in the chain arrangement, whilst the force which drags out the electrons is the force exerted by the atoms in the neighbourhood. Theory shows that

$$I = \frac{1}{k} \cdot \frac{ET_0dne}{T - T_0},$$

$$\rho = \frac{1}{k} \cdot \frac{T_0dne}{T - T_0},$$

where k = drift per unit electric force, d = distance between the centres of two doublets, n = number of electrons passing along each chain per second; thus ρ becomes infinite (resistance zero) when $T = T_0$. The deviation from this, as shown by the experiments of Onnes and others, is brought out by a more accurate mathematical treatment. Further, the part played by the electric force is the forming of the chains, and theory shows that below a certain temperature the electric force may be removed and the chains will remain, and electrons will still be moved by the forces exerted by the neighbouring atoms, *i.e.* the current will continue to flow, as was found in the lead ring experiment of Onnes. The possibilities of this modified theory of Thomson's merit a more detailed consideration than they have hitherto received, but space forbids their further treatment here.

342. Thermal Conductivity.—The close connection between electrical and thermal conductivity in metals is shown by the Wiedemann and Franz law, which states that at a given temperature

$$\frac{\text{Thermal conductivity}}{\text{Electrical conductivity}} = \text{A constant}$$

and is proportional to the absolute temperature. Consider now a metal rod AB, A being at a higher temperature than B. According to the Kinetic Theory the conduction of heat from A to B was due to molecular collisions; the molecules coming from A possessed greater kinetic energy than those from B, and in a collision the energies tended to be equalised, so that the B molecules gained kinetic energy whilst the A molecules lost kinetic energy, and heat was therefore transferred from A to B. Further, the Kinetic Theory of gases proves that

$$Q = \frac{pvN}{3} \cdot \frac{W_1 - W_2}{t},$$

where Q is the quantity of heat passing per second through a partition of unit area and thickness t, and where W_1 is the mean kinetic energy of a molecule on the hot side and W_2 that on the cold side of the partition.

Now, in the light of modern work there is every reason to believe that the electrons in the metal are the essential agents in this heat transference. Applying, then, the above, we have

$$W_1 = \tfrac{1}{2}mv_1^2 = aT_1 \quad \text{and} \quad W_2 = \tfrac{1}{2}mv_2^2 = aT_2,$$

where T_1 and T_2 are the absolute temperatures ; hence

$$Q = \frac{pvN}{3} \cdot a \cdot \frac{T_1 - T_2}{t},$$

p, v, and N having the same meaning as in Art. 341, and if k be the thermal conductivity

$$Q = k \cdot \frac{T_1 - T_2}{t},$$

$$\therefore\ k = \frac{pvNa}{3},$$

i.e.

$$\frac{\text{Thermal conductivity}}{\text{Electrical conductivity}} = \frac{k}{\rho} = \frac{\dfrac{pvNa}{3}}{\dfrac{Ne^2pv}{4aT}}$$

$$= \left(\frac{a}{e}\right)^2 \cdot \frac{4}{3} \cdot T,$$

that is, the ratio is independent of the metal and is proportional to the absolute temperature; *this is the law of Wiedemann and Franz.* All the quantities are known, and, substituting the values in the equation, we find that the ratio at 18° C. is $6{\cdot}3 \times 10^{10}$ and the temperature coefficient ·366 per cent. Experiments give the following results, which are in close agreement:—

Material.	$\dfrac{\text{Thermal conductivity}}{\text{Electrical conductivity}}$ at 18° C.	Temperature Coefficient of this ratio. Per cent.
Copper, pure... ...	$6{\cdot}65 \times 10^{10}$	0·39
Silver, pure	$6{\cdot}86 \times 10^{10}$	0·37
Gold	$7{\cdot}27 \times 10^{10}$	0·36
Nickel	$6{\cdot}99 \times 10^{10}$	0·39
Zinc (1)	$7{\cdot}05 \times 10^{10}$	0·38
Zinc (2), pure ...	$6{\cdot}72 \times 10^{10}$	0·38
Lead, pure	$7{\cdot}15 \times 10^{10}$	0·40
Tin, pure	$7{\cdot}35 \times 10^{10}$	0·34
Aluminium	$6{\cdot}36 \times 10^{10}$	0·43
Platinum, pure ...	$7{\cdot}53 \times 10^{10}$	0·46
Iron...	$8{\cdot}02 \times 10^{10}$	0·43

These and the results of Art. 341 indicate that in metals the current and at least the greater part of the heat are carried by electrons; in the latter case the atoms, by collision, also play a part.

343. Thermo-Electricity.—Consider two metals A and B in contact, and let the number of electrons per unit volume of A be greater than the number per unit volume of B, so that the pressure of the electrons in A is greater than the pressure of those in B. Electrons will pass from A to B (making B negative and A positive) until the difference of potential produced prevents any more electrons passing from A to B; thus we have a simple explanation of the P.D. arising from the contact of metals.

Again, if an external P.D. be applied to the junction so that a current flows the Peltier effect comes into existence, and if P be the Peltier coefficient P ergs of work will be done in the transference of unit quantity across the junction. If N_1 and N_2 be the concentration of electrons in the two metals (*i.e.* number per unit volume) the charges per unit volume will be N_1e and N_2e, and the volumes corresponding to unit quantity will be $\frac{1}{N_1e}$ and $\frac{1}{N_2e}$ respectively. Further, from the Kinetic Theory, the pressure due to the electrons per unit volume is given by the expression

$$p = \tfrac{2}{3}aNT,$$

and in the case under consideration will therefore be $\tfrac{2}{3}aN_1T$ and $\tfrac{2}{3}aN_2T$ respectively. Hence the work done in transferring unit quantity is given by

$$\begin{aligned} P &= \int_{v_1}^{v_2} p \, . \, dv = \tfrac{2}{3}aT \int_{v_1}^{v_2} N dv \\ &= \tfrac{2}{3}a \, . \, T \, . \, \frac{1}{e} \int_{v_1}^{v_2} \frac{1}{v} dv \quad \left(\because v = \frac{1}{Ne} \right) \\ &= \tfrac{2}{3}a \, . \, T \, . \, \frac{1}{e} \, . \log_e \frac{v_2}{v_1} \\ &= \frac{2}{3} \, . \, \frac{aT}{e} . \log_e \frac{N_1}{N_2} \left(\because \frac{v_2}{v_1} = \frac{\frac{1}{N_2e}}{\frac{1}{N_1e}} = \frac{N_1}{N_2} \right). \end{aligned}$$

This gives the relation between the contact difference of potential, the concentration of the electrons, the electronic charge, and the temperature. From known data the ratio N_1/N_2 can be calculated from the above; for copper and zinc it is about 1·03, for bismuth and antimony it is 4.

The Thomson effect can be partially explained. Consider the end A of a conductor to be at a higher temperature than the end B, and let a current flow from A to B. Since the pressure due to electrons is given by $\tfrac{2}{3}NaT$, the energy of the electrons in the conductor will be increased since they move from parts at a lower to parts at a higher pressure (electrons are *negative*); thus there will, on this account, be a heating effect. At the same time, the electrons will be carried out of the conductor at A, where their kinetic energy is greater, and into B, where their kinetic energy is less, thus leading to a cooling effect. Which of these two effects is greater in any case, so that the sign of the Thomson effect may be predicted, can only be settled by further knowledge.

Connected with the above is the effect produced when one part of a conductor is suddenly heated. This increases the pressure of the electrons in that part, they rush to other parts and the effect of the charges they carry is to set up potential differences; here again, however, further knowledge is required in order to explain observed facts.

344. Conduction in a Magnetic Field. The Hall, Ettinghausen-Nernst, Ettinghausen, Leduc, and Longitudinal Effects.—The **Hall effect** was discovered by Hall in 1879, and may be briefly described as follows. If a current passes between two points, *P* and *Q*, in a thin metal plate the lines of flow diverging from *P* converge to *Q* as shown in the diagram (Fig. 518), and the equipotential lines, everywhere at right angles to the lines of flow, run as indicated by the dotted lines in the figure. If this thin plate carrying a current be placed in a magnetic field with its plane at right angles to the field it is found that the system of lines of flow, and therefore the equipotential lines, suffer distortion. The points *P* and *Q* remaining fixed, the lines appear as if rotated round the direction of the magnetic field as an axis. Looking along the lines of force the system of lines is deformed as if twisted to the right in tellurium, iron, zinc, and antimony, and to the left in bismuth, nickel, and gold. In the former the effect is said to be *positive* and in the latter *negative.*

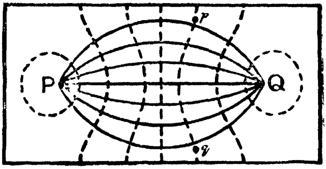


Fig. 518.

Exp. The effect is demonstrated experimentally by noticing the displacement of an equipotential line. If the point *p* be connected to one terminal of a sensitive galvanometer it is easy to find another point *q* at the same potential as *p* by adjusting the position of the point of contact with the other terminal until no current passes through the galvanometer. Let this adjustment be made before placing the plate in the magnetic field. It will then be found that when the plate is placed in the field a current at once passes through the galvanometer, indicating that *p* and *q* are no longer at the same potential. If the equipotential line initially passing through *p* and *q* be supposed to be rotated to the right by the action of the field, the point *p* evidently passes into a region of higher potentialand *q* into one of lower potential than at first, and the current in the galvanometer will be from *p* to *q*. If, however, the equipotential line be rotated to the left the current will be from *q* to *p*. Hence, the direction of the current in the galvanometer indicates the sense of the distortion of the lines of flow and the magnitude of the current indicates the extent of the distortion.

If e denotes the difference of potential thus produced between two points equidistant from P and Q by a field of strength H, in the case of a plate of thickness t carrying a current I it is found experimentally that

$$e = A\,\frac{IH}{t},$$

where A is a constant under given conditions. The Hall effect is very marked in bismuth and tellurium.

The electronic theory supplies, at present, only a partial explanation. In Fig. 519 let I be the direction of the current (i.e. P to Q) and H the direction of the magnetic field. The electrons, being *negative*, will be moving in the direction Q to P, and an application of the rules given in previous pages will show that the magnetic field will result in the electrons being deflected *upwards* so that p is at a lower potential than q, and the sign of the Hall effect is that found in bismuth; this does not account for the Hall effect with reversed sign (*e.g.* antimony). On the other hand, if *positive* carriers could also be imagined moving from P to Q these also would be deflected upwards, tending to make p at a higher potential than q, and both signs could be accounted for according as to which of the two effects predominated. The main objections to the latter idea are (1) the charged carriers must be the same in all metals, and whilst this is the case with electrons the "universal positive particle" has not yet been definitely found, (2) the positive particles found so far are of *atomic* dimensions.

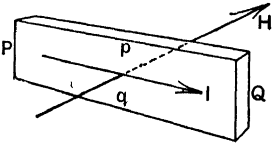


Fig. 519.

Consider again Fig. 519, but instead of a current let heat be flowing from P to Q. The electrons from the end P will have more energy and higher velocities than those from Q and will be more deflected by the magnetic field. Since their direction of deflection is *downwards* the result will be that the face q will be at a lower potential than the face p. Experiment proves that this is so, and it is known as the **Ettinghausen-Nernst effect.** ✓

Again consider Fig. 519 with the current flowing from P to Q. The electrons moving upwards acquire increased energy due to this motion, part of which is communicated to the molecules there, with the result that the upper face becomes at a higher temperature than the lower face. Experiment proves that this is so and it is known as the **Ettinghausen effect.**

Consider again Fig. 519, but instead of a current let heat be flowing from P to Q. For the reasons already given the electrons moving downwards will cause the lower face to be at a higher temperature than the upper face, since they have greater velocities than those moving upwards. Experiment proves that this is so in some metals, but opposite in other metals; it is known as the **Leduc effect.**

It should be noted that, whilst the first and last effects have reversed signs, all metals show the second and third effects as indicated in theory.

It is evident that the *transverse effects* indicated above will result in *longitudinal effects*, i.e. effects parallel to the original current and flow of heat. Thus in the Hall effect (say in bismuth) the path of the electrons becomes curved and it is easy to deduce that the final result is a reduction in the current flowing; this is equivalent to an increase in the apparent resistance of the bismuth, and this has been used for the measurement of magnetic fields. The following briefly summarises the transverse and longitudinal effects; the reader will be able to think out the latter, on the lines already indicated, for himself.

Transverse.	Longitudinal.
Hall effect	Variation in electrical conductivity
Ettinghausen-Nernst effect	Difference in potential
Ettinghausen effect ...	Difference in temperature
Leduc effect	Variation in thermal conductivity

The explanation of the Hall effect on Thomson's theory of conduction in metals would be briefly as follows. Imagine a doublet to rotate about an axis *not at its centre*, so that the two charges move with different velocities. The effect of a magnetic field will be to incline the axis of the doublet to the plane containing the electric and magnetic fields and there will be a flow of electrons at right angles to both, the direction of the flow being determined by that end of the doublet which moves the quicker; thus we have the Hall effect with *either* sign. There are still, however, difficulties with this explanation.

345. Magnetism.—The molecular theory of magnetism has been fully dealt with in Chapter I., but it will be noted that the theory accounts for paramagnetism (and ferromagnetism) only, *not for diamagnetism.* Weber was the first to frame a theory of diamagnetic bodies, and Maxwell gives this theory somewhat as follows. Imagine that the molecule of the substance is a *perfect* conductor of electricity. When placed in the magnetic field due to (say) an inducing magnet, induced currents are formed in each molecule, and these tend to oppose the motion (Lenz's law). These molecular currents act like small magnets whose poles are, therefore, turned *towards* the *like* pole of the inducing magnet, and since they flow in a *perfect* conductor they will continue to do so until they are wiped out by equal and opposite induced currents due to the destruction of the field. These induced currents thus provide the necessary explanation of diamagnetism; up to this point, then, a body is paramagnetic or diamagnetic according as to whether the effects dealt with in Art. 8 are greater or less than the effects due to the induced currents.

We have seen that Ampère postulated currents flowing round perfectly conducting circuits in the molecules, and, further, that in a magnetic field these circuits set themselves so that, for example, their "clockwise" direction was towards the north pole of the inducing magnet. At the same time these perfectly conducting circuits provide the necessary paths for the Maxwell induced currents mentioned above; a body is, therefore, paramagnetic or diamagnetic according as to whether the effect of the original or the induced currents is the greater.

The electronic theory of magnetism is a continuation of the above. Electrons moving in closed orbits within the atoms take the place of the Amperean currents in perfectly conducting circuits, and *the alteration which, as will be seen presently, is produced in these orbits by the inducing field* takes the place of the Maxwell induced currents; *a body is paramagnetic or diamagnetic according as to whether the orientation of the orbits into the necessary direction or the alteration in the orbits due to the magnetic field produces the predominating effect.* ✓

For simplicity we shall assume the electrons moving in circles whose planes are perpendicular to, and axes in the direction of, the inducing field, and shall show that the alteration in the orbits due to the magnetic field leads to the phenomena observed in diamagnetic bodies. In the more general case the motion of the electrons could be resolved into three components, a linear motion in the direction of the field and two equal and opposite circular motions in a plane perpendicular to the field, but the general principle will be equally well illustrated by the simpler treatment.

Consider the electronic orbit and the magnetic field to be as indicated in Fig. 520. Before the magnetic field is put on the centrifugal force is balanced by a force directed towards *O* and proportional to r_1, *i.e.* by a force equal to (say) fr_1, where r_1 is the radius of the orbit; hence

$$\frac{mv^2}{r_1} = fr_1.$$

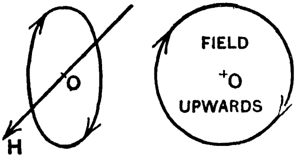


Fig. 520.

When the field is on, the force due to the field is *Hev* and is directed outwards along the radius, and, since the motion of the electron is perpendicular to the forces, v is the same. If r_2 be the radius of the new orbit

$$\frac{mv^2}{r_2} = fr_2 - Hev,$$

$$\therefore \quad \frac{mv^2}{r_2^2} - f = -\frac{Hev}{r_2},$$

i.e.
$$\frac{mv^2}{r_2^2} - \frac{mv^2}{r_1^2} = -\frac{Hev}{r_2}.$$

But, if T_1 be the period in the first case and T_2 that in the second, $T_1 = 2\pi r_1/v$ and $r_1 = T_1 v/2\pi$; so also $r_2 = T_2 v/2\pi$. Substituting and reducing,

$$\frac{1}{T_2^2} - \frac{1}{T_1^2} = -\frac{He}{2\pi m T_2},$$

$$\therefore \quad \frac{(T_1 + T_2)(T_1 - T_2)}{T_1^2 T_2^2} = -\frac{He}{2\pi m T_2},$$

or approximating, by putting $T_1 + T_2 = 2T_1$ and $T_1 T_2 = T_1^2$,

$$\frac{T_1 - T_2}{T_1^2} = -\frac{He}{4\pi m},$$

i.e.
$$T_1 - T_2 = -\frac{He}{4\pi m} \cdot T_1^2.$$

Now, a charge e moving in a circle of area a in periodic time T is equivalent to a current e/T, and the magnetic moment of the orbit is ea/T. Thus the moment in the first case is ea/T_1 or $\pi r_1^2 e/T_1$ and in the second case $\pi r_2^2 e/T_2$, and the change in the moment is $\pi e\left(\frac{r_1^2}{T_1} - \frac{r_2^2}{T_2}\right)$. If there are N such orbits per unit volume, then, since the induced intensity (I) is given by the change in moment per unit volume,

$$I = N\pi e\left(\frac{r_1^2}{T_1} - \frac{r_2^2}{T_2}\right),$$

or, since $r_1/r_2 = T_1/T_2$,

$$I = Ne\pi r_1^2\left(\frac{T_1 - T_2}{T_1^2}\right)$$
$$= -\frac{NHe^2a}{4\pi m},$$

and, if κ be the susceptibility,

$$\kappa = \frac{I}{H} = -\frac{Ne^2a}{4\pi m}.$$

Thus κ is *negative*, as is the case with a diamagnetic body. A more exact treatment (see above) would merely introduce a small factor to the right-hand side.

There is no limiting value to I for diamagnetism, for it is proportional to the field H. On the other hand, a paramagnetic body will be saturated when all the orbits are turned into the proper direction, and this maximum intensity is Nea/T.

Since diamagnetism is due to interatomic actions it is reasonable to expect it independent of temperature; the ferromagnetics of Art. 8, on the other hand, probably involve orientation of atoms and molecules, and may be expected to be affected by temperature (see Art. 8).

In a recent communication by Kotaro Honda the ferromagnetic molecule is nearly spherical, the paramagnetic elongated, and the transformation of a ferromagnetic into a paramagnetic at high temperatures is explained as the result of the gradual flattening of the spherical molecule.

346. Magnetism and Light. (1) The Zeeman Effect.—Zeeman discovered in 1896 that if a source of light producing line-spectra be placed in a magnetic field the lines were resolved into doublets or triplets or even more complex arrangements; this is known as the **Zeeman effect.**

To consider the simplest case: when the source was viewed in a direction *at right angles to that of the field* a triplet was produced, the middle one which occupied the undisturbed position being plane polarised, the electric displacement being parallel to the field and the outer two being plane polarised in a direction at right angles to the middle one. When viewed *along the direction of the field* a doublet was produced, the two being circularly polarised in opposite directions.

The electronic theory affords a satisfactory explanation of the Zeeman effect. Suppose we are looking down on an electron revolving in a counter-clockwise direction in a horizontal plane. If a magnetic field, acting vertically downwards, is produced by a magnet, then (Art. 170) the orbit of the electron will expand. The linear velocity of the electron, however, remains constant, with the result that a smaller number of revolutions will be made per second. By similar reasoning an electron revolving in a clockwise direction will have its orbit diminished in diameter, and will therefore make a greater number of revolutions per second. The orbit of an electron revolving in a vertical plane (*i.e.* a plane parallel to the magnetic field) will not be changed in diameter. ✓

Now the emission of light by (say) a glowing gas is due to the motions of electrons; the number of revolutions per second will determine the number of waves produced per second. For simplicity consider a single electron. All possible motions of this electron can be resolved into a linear motion along the field and two opposite circular motions at right angles to the field. Imagine now the light to be viewed in the direction at right angles to the field. The linear motion along the field will send out vibrations parallel to the field and of the original period, say T_0, so that the spectroscope will reveal a line in the undisturbed position, plane polarised, the vibrations being parallel to the field. The two circular motions at right angles to the field will send out linear vibrations in that direction, since it is their own plane, but as the circular motions are opposite the period of one will be increased to T_1, that of the other decreased to T_2, by the field. Thus the spectroscope will reveal two additional lines, one on each side of the undisturbed line, corresponding to the augmented and diminished frequency respectively; these additional two are plane polarised, the vibrations being at right angles to the field. From Art. 345 it follows that T_1 and T_2 differ from T_0 by an amount given numerically by $\frac{He}{4\pi m} . T_0^2$; hence

$$T_1 - T_2 = + \frac{He}{2\pi m} . T_0^2.$$

Now imagine the light viewed in the direction of the field. The linear vibration along the field gives no waves in that direction. The two opposite circular motions at right angles to the field will result in the emission of opposite circularly polarised light in the direction of the field. The spectroscope will therefore reveal two lines only, one on each side of the original position, and the light in them will be circularly polarised in opposite directions.

The equation above may be expressed in terms of the wave length γ, since $\gamma = VT$, where V is the velocity of light; substituting we get

$$\frac{\gamma_1 - \gamma_2}{\gamma_0^2} = \frac{e}{m} . \frac{H}{2\pi} . \frac{1}{V}.$$

Now γ_1, γ_2, γ_0, and H are quantities which can be measured, and hence e/m can be determined; thus in the case of the triplet with mercury vapour it has been found that $e/m = 1{\cdot}6 \times 10^7$, which is in close agreement with the value found for kathode rays ($1{\cdot}77 \times 10^7$) and further supports the electronic theories given in this and preceding chapters.

347. Magnetism and Light. (2) The Faraday and Kerr Effects.—In 1845 Faraday found that when dense lead glass was placed in a magnetic field it acquired the power of rotating the plane of polarisation of a beam of plane polarised light; this is known as the **Faraday effect.**

Much work has been done on this rotation of the plane of polarisation in a magnetic field by Verdet (1852), Gordon (1877), Becquerel (1877), Rayleigh (1885), and later by Rodger, Watson, and others. It is now known that most substances exhibit this effect, and it is well marked in bodies having a high index of refraction for light; in dealing with metals very thin films must be used, but the effect is very pronounced in the case of an iron film.

The amount of rotation depends upon the material, and is also proportional to the component of the field intensity parallel to the direction of the beam. Hence, for a given substance the rotation is a maximum when the directions of the beam and the magnetic field are parallel, and zero when they are at right angles. The direction of rotation is, however, not reversed by reversing the direction of the beam.

In experimenting, the substance may be placed between the poles of an electromagnet, or more satisfactorily in the interior of a long coil, and a beam of plane polarised light passed through it parallel to the direction of the field. The amount of rotation produced can then be measured by a polarimeter in the usual way. The amount of rotation is proportional to the length of the substance traversed, and as the sense of the rotation is not reversed by reversal of path, it has been found convenient to increase the length of path by multiple reflexions, produced by silvering the ends of the piece of substance, as indicated in Fig. 521. The amount of rotation also depends upon the strength of the field, and results show that the general law of the phenomenon is that the rotation of the plane of polarisation along the path between any two points is directly proportional to the difference of magnetic potential between these two points. That is, if P denotes the *difference* of potential

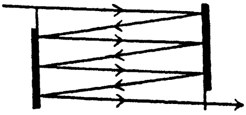


Fig. 521.

between any two points, and δ the observed rotation of the plane of polarisation due to the transmission of a beam of plane polarised light along the line joining the points, then

$$\delta = VP,$$

where V is a constant depending on the material.

This law was enunciated by Verdet and is known as Verdet's Law, and the constant V is known as **Verdet's constant.** The value of V varies with the wave length of the light and is approximately inversely proportional to the square of the wave length.

For all diamagnetic substances the direction of rotation is the same as that of the current which would produce the magnetic field to which the rotation is due. For paramagnetic substances it is in the opposite direction.

For pure carbon bisulphide the value of Verdet's constant at $t°$ C. is given by

$$V_t = V_o (1 - \cdot00104t - \cdot000014t^2),$$

V_o being equal to $0\cdot043'$.

In 1877 Kerr discovered that the plane of polarisation of light was rotated by reflection at the polished surface of a magnet; this is known as the **Kerr effect.**

Kerr's method is shown in Fig. 522. A plane polarised beam from the Nicol prism A is reflected by the mirror (unsilvered) M, and passing through the opening in the soft iron block I falls vertically on the magnet pole N; here it is reflected, and passing upwards is analysed by the Nicol prism B. The nicols are crossed and the field is dark when the magnet is not excited; on exciting the magnet the light appears and is extinguished again on rotating B. Kerr found that if the plane of polarisation is parallel to the plane of incidence the rotation is in the same direction as the magnetising current.

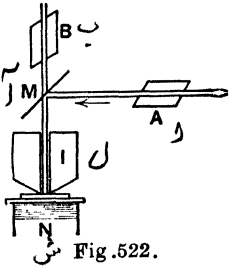


Fig.522.

The student of Optics will readily see that the electronic theory supplies an explanation of the Faraday rotations. The light vibration may be regarded as resolved into two opposite circular vibrations, the relative phases settling the plane of polarisation. When the field is on the circular orbits are altered, the period of one being augmented, the other diminished, and the index of refraction for the two components is therefore not the same. The relative phase on emergence is therefore altered so that they combine to form a plane polarised beam whose plane of polarisation is "rotated." The reader should think this out in detail for himself, for there are many intermediate points, dealt with in this and preceding chapters, which are omitted in this brief summary.

This rotation of the plane of polarisation may be used in measuring a current. Let the current to be measured be passed round a uniformly wound solenoid having n turns per unit length. Then, if the rotation produced by a column of carbon bisulphide, of length l, placed in the interior of the coil with its length parallel to the axis of the coil, be denoted by δ, we have

$$\delta = VP \text{ or, since } P = 4\pi nlI,$$

$$\delta = 4\pi nlIV.$$

That is

$$I = \frac{\delta}{4\pi nlV},$$

and I is determined if V is known and n, l, and δ are observed.

In 1875 Kerr discovered that a dielectric when subjected to electrostatic stress became doubly refracting and converted plane into elliptically polarised light; this also is spoken of as the "Kerr effect."

348. X Ray Diffraction and Crystal Structure.—The absence of regular reflection, refraction, etc., early observed in the case of Röntgen rays, was attributed by Schuster as being due to the shortness of the wave length —a view which modern work certainly upholds.

In some recent experiments by Laue, Friedrich, and Knipping a fine pencil of X rays was passed through a thin crystal slip, thence impressing itself on a photographic plate, and Fig. 523 diagrammatically represents the result in one case—that of a crystal of zinc blende. Round the central spot a number of other spots are symmetrically arranged, the arrangement varying with the structure of the crystal.

The effect is due to the diffraction of the rays by the atoms of the crystal. Thus consider the rays falling on a plane containing atoms: a certain amount of reflection by the atoms ensues. Imagine a second plane of atoms behind the first and parallel to it: the primary, weakened in its passage, is again partially reflected by the atoms, and so on. The reflections from two different planes may on rejoining either reinforce or destroy each other. They will reinforce if the waves exactly fit, crest to crest and trough to trough, and this will be so if the distance "lost" by a reflection at one plane in comparison with a reflection at the preceding plane is an integral number of wave lengths. Mathematically it can be shown that the condition for reinforcement in the case of two parallel planes of atoms is expressed by the relation

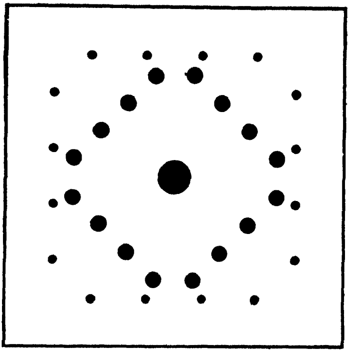


Fig. 523.

$$d = \frac{N\gamma}{2 \sin\left(\frac{\pi}{2} - \theta\right)},$$

where d is the distance between the planes, γ the wave length, θ the angle of incidence, and N an integer; thus reinforcement depends upon the spacing of the planes, the wave length, and the angle of incidence. The explanation of the Laue photograph is, therefore, that the different series of spots are simply reflections under suitable conditions in the different series of planes containing the atoms of the crystal.

Bragg, continuing this work, used the X *ray spectrometer* (Fig. 524). The X rays pass through holes in the lead box *B* and lead plate *P*, and fall on the crystal *C*, which is mounted on a table *T*, capable of rotation; *S* is a slit leading the reflected rays to an ionisation chamber *I*. In experimenting *T* is turned to various positions and the angle θ and the ionisation noted. Clearly, by using the same crystal, the radiation from various X ray bulbs may be examined, or by using the same rays the spacing of the atoms of a crystal in different directions may be examined and crystal structure investigated. Thus, for example, Bragg concludes that in the case, say, of sodium chloride crystals the diffracting centres are in cubical array (Fig. 525), the black dots being sodium atoms and the white circles chlorine atoms. For further details the student should consult the original paper of Professor W. H. Bragg.

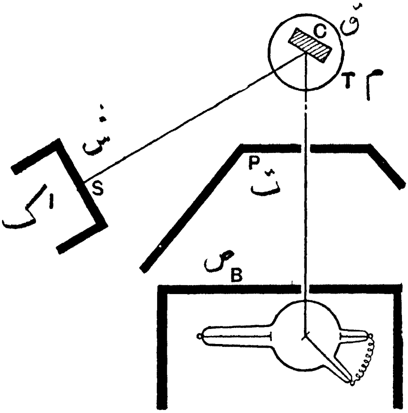


Fig. 524.

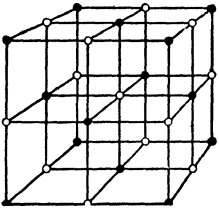


Fig. 525.

Exercises XXIV.

Section C.

(1) Write an essay on the application of the electron theory to the explanation of electric conductivity. (B.E. Hons.)

(2) Explain why there is an apparent increase in mass produced by charging a body with electricity. (B.E. Hons.)

(3) Give a short account of the Zeeman effect and of its explanation on the electronic hypothesis. (B.E. Hons.)

(4) Describe the Hall effect, and discuss the theory which has been advanced to account for it on the hypothesis of electrons. (B.E. Hons.)

(5) An electron of mass 10^{-25} grammes rotates 5×10^{14} times a second round an atom supposed to be at rest; find the force with which the electron must be attracted to the atom in terms of the radius of its orbit. If the atom has a positive charge of 10^{-10} electrostatic units and the electron an equal negative charge, calculate the radius of the orbit, assuming that the attractive force is purely the force between the charges. (B.Sc.)

(6) Give a short account of the effects of transmitting light through a magnetised material, and of reflecting it from the surface of a magnetised body. (B.Sc. Hons.)

APPENDIX.

1. Solid Angles.—The expression employed in (4), page 96, for the solid angle subtended at P (Fig. 91) by the circular shell AB may be readily found as follows :—

Consider a sphere and its *circumscribing cylinder* (*i.e.* let the sphere have a cylinder drawn about it, the base diameter, and height of the cylinder, being therefore each equal to the diameter of the sphere), and imagine a series of planes parallel to the ends of the cylinder, such planes dividing the surface of the sphere and the lateral surface of the cylinder into *zones* ; *the area of any zone of the sphere is equal to the area of the corresponding zone of the cylinder.*

Now consider Fig. 91, and imagine a sphere with centre P and radius PA. The area of the segment cut off by AB will be equal to that of the corresponding zone of the cylinder. The circumference of this zone is $2\pi\overline{AP}$, *i.e.* $2\pi(x^2 + r^2)^{\frac{1}{2}}$, and its width is $(\overline{AP} - x)$, *i.e.* $(x^2 + r^2)^{\frac{1}{2}} - x$; hence its area (and therefore that of the segment cut off by AB) is $2\pi(x^2 + r^2)^{\frac{1}{2}}\{(x^2 + r^2)^{\frac{1}{2}} - x\}$ Thus—

$$\omega = \frac{\text{Area of segment}}{(\text{Radius})^2} = \frac{2\pi(x^2 + r^2)^{\frac{1}{2}}\{(x^2 + r^2)^{\frac{1}{2}} - x\}}{(x^2 + r^2)}$$

$$= \frac{2\pi\{(x^2 + r^2)^{\frac{1}{2}} - x\}}{(x^2 + r^2)^{\frac{1}{2}}} = 2\pi\left(1 - \frac{x}{(x^2 + r^2)^{\frac{1}{2}}}\right)$$

$$= 2\pi\,(1 - \cos\theta).$$

2. The Number of Electrons in an Atom.—Reference has been made in Art. 339 to Rutherford, Marsden and Geiger's work on the determination of the number (N) of free positive charges in the nucleus of an atom, and therefore the number (N) of electrons outside the nucleus. As these results are now accepted, a very brief note on the "mathematics" of the investigation may be of interest to the student. (See *Phil. Mag.* xxi. (669) and xxv. (604)). The treatment below is a modification of Prof. Millkan's.

Consider an α particle rushing straight on to a nucleus, getting to its nearest distance x from the centre of the nucleus, and then being deflected through 180° back again along its line of approach. Evidently the original kinetic energy of the α particle must be equivalent to the work done in moving up to distance x under the repulsion of the nucleus, *i.e.*

$$\tfrac{1}{2}mu^2 = \frac{Ne(2e)}{x} \quad \text{.....................} \quad (1)$$

where $\frac{1}{2}mu^2$ = original kinetic energy of α particle, $+Ne$ = total nucleus charge (e = electron charge), and $2e$ is the known charge on the α particle.

Now consider the α particle rushing not "straight on" to the nucleus A (Fig. 526), but as indicated in the figure where the α particle is deflected through an angle β. In this case if v be the velocity at B and u the velocity originally, our energy expression becomes

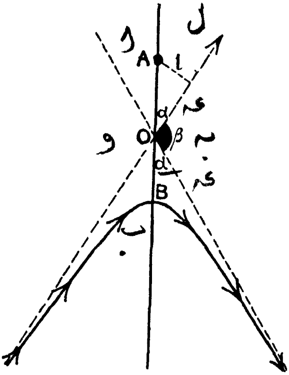


Fig. 526.

$$\tfrac{1}{2}mu^2 - \tfrac{1}{2}mv^2 = \frac{Ne(2e)}{D} \quad \text{..........................} \quad (2)$$

where $D = AB$.

By well known properties of the hyperbola, the eccentricity z is equal to $\frac{1}{\cos\alpha}$ (Fig. 526) and $AO = zOB$; hence:—

$$D = AO + OB = AO\left(1 + \frac{1}{z}\right) = \frac{l}{\sin\alpha}(1 + \cos\alpha) = l\left(\frac{1 + \cos\alpha}{\sin\alpha}\right)$$

$$i.e\ D = l\cot\frac{\alpha}{2} \quad \text{.................................} \quad (3)$$

and from elementary mechanics (conservation of momentum) :—

$$Dv = lu \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Now from (4) $l^2 = D^2 \frac{v^2}{u^2}$, and from (1) and (2) $\frac{v^2}{u^2} = 1 - \frac{x}{D}$, hence :—

$$l^2 = D(D - x) = l \cot\frac{a}{2}\left(l \cot\frac{a}{2} - x\right),$$

$$\therefore\ x = 2l \cot a,$$

$$= 2l \cot\frac{\pi - \beta}{2},$$

$$i.e.\quad l = \frac{x}{2} \cot \frac{\beta}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \cancel{(4)}\ (5)$$

where β = the deflection of the a particle = $(\pi - 2a)$.

Further, if there are n molecules in a cube of 1 cm. side the probability that another molecule shot through the cube will "hit" one of the contained molecules is known to be $\pi n d^2$ where d is the diameter of a molecule. In the same way if there are n atoms per c. cm. in a piece of metal foil of thickness t the probability P that a small particle shot into the foil will come within a distance l of a nucleus is given by :—

$$P = \pi n t l^2,$$

and this is the fraction f of any given number of small particles shot into it which will come within this distance l. The fraction which will come within the distances l and $l + dl$ is

$$dP = 2\pi n t l dl.$$

Now from (4)

$$\frac{dl}{d\beta} = -\frac{x}{2} \cdot \frac{1}{2} \operatorname{cosec}^2 \frac{\beta}{2}$$

and substituting for dl and l in the preceding :—

$$dP = 2\pi n t \frac{x}{2} \cot \frac{\beta}{2}\left(-\frac{x}{2} \cdot \frac{1}{2} \operatorname{cosec}^2 \frac{\beta}{2}\right) d\beta$$

$$= -\frac{\pi n t}{4} x^2 \cot \frac{\beta}{2} \operatorname{cosec}^2 \frac{\beta}{2}\, d\beta$$

Hence the fraction f deflected between the limits β_1 and β_2 will be obtained from the above by integrating between these limits: the integration gives :—

$$f = \frac{\pi n t}{4} x^2 \left(\cot^2 \frac{\beta_1}{2} - \cot^2 \frac{\beta_2}{2}\right) \ldots\ldots\ldots\ldots\ldots \cancel{(5)}\ 6$$

Marsden and Geiger determined, as previously indicated, this fraction for the known angles β_1 and β_2 and then, knowing n and t, x was found. Substituting the value of x in (1) and knowing the energy $\frac{1}{2}mu^2$ and the value of e, the required value N was found. The number N of free positive charges in the nucleus (and therefore the number of electrons outside) was found to be as mentioned in Art. 339, equal to half the atomic weight or somewhat less, and x was of the order 10^{-12} cm. for gold and smaller for lighter elements. N must of course be a whole number and is taken now to be the **atomic number** which ($H = 1$) is roughly, for other elements, half the atomic weight, really somewhat less (see Art. 339).

3. Atomic Weights, Valencies, and Electro-chemical Equivalents. (International Atomic Weights 1912; 0 = 16.)

Element.	Atomic Weight (O = 16).	Valency.
Aluminium ... ... (Al)	27·1	3
Antimony ... ... (Sb)	120·2	3
Bismuth... ... ... (Bi)	208·0	3
Bromine . ... (Br)	79·92	1
Cadmium ... ... (Cd)	112·40	2
Calcium ... ... (Ca)	40·07	2
Chlorine... ... ... (Cl)	35·46	1
Copper ... . (Cu)	63·57	1 or 2
Gold ... ... (Au)	197·2	3
Hydrogen ... ... (H)	1·008	1
Iodine ... ... (I)	126·92	1
Iron... ... ... (Fe)	55·84	2 or 3
Lead ... ... (Pb)	207·10	2
Mercury... ... ... (Hg)	200·6	1 or 2
Oxygen ... ... (O)	16·0	2
Platinum ... ... (Pt)	195·2	4
Potassium ... ... (K)	39·10	1
Silver ... ... (Ag)	107 88	1
Sodium ... ... (Na)	23·00	1
Tin ... ... (Sn)	119·0	2 or 4
Zinc... ... ... (Zn)	65·37	2

Electro-chemical equivalents (grammes per coulomb) :—
Cu= ·0003293; Ag = ·0011183; Zn = ·0003387; H = ·00001044.

Note.—Chemical equivalent = Atomic weight ÷ Valency.

4. Specific Resistances, Conductivities, and Temperature Coefficients.

Substance.	Sp. Res. at 18° C. (ohms per cm. cube).	Conductivity at 18°.	Temperature Coefficient.
Silver	$1{\cdot}65 \times 10^{-6}$	$6{\cdot}06 \times 10^{5}$	·0040
Copper (drawn)	$1{\cdot}78 \times 10^{-6}$	$5{\cdot}62 \times 10$	·0043
Zinc	$6{\cdot}1 \times 10^{-6}$	$1{\cdot}6 \times 10^{5}$	·0037
Iron	$1{\cdot}4 \times 10^{-5}$	$7{\cdot}1 \times 10^{4}$	·0062
Lead	$2{\cdot}1 \times 10^{-5}$	$4{\cdot}8 \times 10^{4}$	·0043
Mercury	$9{\cdot}41 \times 10^{-5}$	$1{\cdot}06 \times 10^{4}$	·0009
Platinum	$1{\cdot}16 \times 10^{-5}$	$8{\cdot}6 \times 10^{4}$	·0037
Aluminium ...	$3{\cdot}0 \times 10^{-6}$	$3{\cdot}3 \times 10^{5}$	·0038

Manganin (Cu = 84, Ni = 4, Mn = 12) :—Sp. Res. = $4{\cdot}76 \times 10^{-5}$ at 0° C. ; Temp. Coeff. = ·000018.

Platinoid (Cu = 62, Ni = 15, Zn = 22) :—Sp. Res. = $3{\cdot}44 \times 10^{-5}$ at 18° C. ; Temp Coeff. = ·00021.

Resista (Fe = 80, Ni = 15, Mn = 5) :—Sp. Res. = 12×10^{-5} ; Temp. Coeff. = ·00109.

German Silver (Cu = 4, Zn = 1, Ni = 2) :—Sp. Res. = 3×10^{-5} ; Temp. Coeff. = ·0004.

5. Data for Reference.

Ratio e/m for electron	$1{\cdot}772 \times 10^{7}$ e.m. units per grm. $5\ 32 \times 10^{17}$ e.s. units per grm.
Electronic charge (e)	$1{\cdot}55 \times 10^{-20}$ e.m. units. $4{\cdot}65 \times 10^{-10}$ e.s. units.
Electronic mass (m)	$8{\cdot}9 \times 10^{-28}$ grm.
Radius of electron	$1{\cdot}87 \times 10^{-13}$ cm.
Ratio e/m for H ion in electrolysis	$\cdot 96 \times 10^{4}$ e.m. units per grm. $2{\cdot}88 \times 10^{14}$ e.s. units per grm.
Charge on monovalent ion	e as above.
Mass of H atom	$1{\cdot}65 \times 10^{-24}$ grm.
Radius of H atom	$1{\cdot}2 \times 10^{-8}$ cm.
Ratio of mass of electron to mass of H ion	1 : 1850 (taking e/m as above)
Number of molecules in 1 c.c. of gas at 0° C. and 760 mm.	$2{\cdot}9 \times 10^{19}$
Velocity of light	$2{\cdot}998 \times 10^{10}$ cm. per sec.
"v"—ratio of units—Rosa and Dorsey (1907)	$2{\cdot}997 \times 10^{10}$ cm. per sec.
"Magneton" moment	$18{\cdot}54 \times 10^{-22}$
Planck's universal constant (h)	6×10^{-27}

6. Books to Read.

(*a*) *Molecular Physics*, by J. A. Crowther.
Modern Electrical Theory, by N. R. Campbell.

(*b*) *Manual of Radiotelegraphy and Radiotelephony*, by J. A. Fleming.

(*c*) *Conduction of Electricity through Gases*, by Sir J. J. Thomson.
Rays of Positive Electricity, by Sir J. J. Thomson.

(*d*) *Radio-active Substances and their Radiations*, by Sir E. Rutherford.

(*e*) *Corpuscular Theory of Matter*, by Sir J. J. Thomson.
Electron Theory of Matter, by O. W. Richardson.
Relativity and the Electron Theory, by E. Cunningham.

(*f*) *A Treatise on the Theory of Solution*, by W. C. D. Whetham.

(*g*) *Electrical Engineering—Continuous Current*, by W. T. Maccall.
Alternating Currents, by C. G. Lamb.

ANSWERS.

Exercises XI.

B.—(1) ·25 volt. (2) $\frac{2}{7}$ of 35, *i.e.* 10 ft. from copper terminal.

(3) $I = \cdot\dot{3}$ ampere P.D. $= 1\cdot\dot{3}$ volts. (4) 4 : 3.

(5) One wrongly connected.

(6) $+ \cdot 25$; $- 1\cdot 45$; $- \cdot 95$; $- 25$.

C.—(1) 20 cells. (2) 3 rows, 12 cells per row.

(3) ·62 amp.; ·31 amp.; $H_1 : H_2 = 2 : 1$.

(4) 275 ohms; 60 watts in battery; 40 watts in leads; 1100 watts in lamps.

(6) 2·28 amperes; 2·86 amperes.

(7) Resistance = that of a piece of the same wire of length $(2 - \sqrt{2})$ times the length of one side of the square.

Exercises XII.

B.—(1) 111 ohms; 107·73 ohms; 3555·27 ohms.

(2) 6000 ohms; 4. (3) 70 divisions; 2. (4) 39 : 20.

C.—(2) 5·03. (5) $\pi a^2 iH$.

(8) $\frac{172}{81}$ C.G.S. units. (9) $\frac{\theta_A}{\theta_B} = \frac{1}{n^3}$.

(10) $$2\pi nc\,(1 - \cos\theta) = 2\pi nc\left(1 - \frac{x}{(x^2 + r^2)^{\frac{1}{2}}}\right);$$

$$2\pi nc\,.\,\frac{r^2}{(x^2 + r^2)^{\frac{3}{2}}}\,.\,m;\ x = \frac{r}{2};\quad \cdot 266 \text{ C.G.S. unit.}$$

(11) 0042.

Exercises XIII.

B.—(1) 10 : 1; 1 : 1; 10 : 1.
(3) Res. of A = 15 ohms; Res. of B = 6 ohms;
Heat in A : Heat in B = 5 : 8.
(4) 6 ohms. (5) 29° C.

C.—(1) ·62 ampere; ·31 ampere; 2 : 1.
(2) Heat in Case 1 = 38·1 calories per second.
Heat in Case 2 = 8·48 calories per second.
(3) 114·3. (4) 1·508 volts.
(5) ·3402 grm.; 142·3 calories; 69·7 calories; 46·5 calories.
(6) 10 J absolute units. (7) ·9 volt; ·01 ohm. (8) 10·49.

Exercises XIV.

B.—(1) ·001 grm. per coulomb.
(2) Cu = 6560·6 grm., Hg = 20747 grm., H = 103·8 grm.,
NaOH = 4153·5 grm., Cu = 3280·3 grm.

C.—(4) ·505 ampere. (5) 4 : 1; 16 : 1.
(6) ·1588 grm.; 30964 calories.
(8) Current in A = 1·03; Current in B = ·345.

Exercises XV.

C.—(5) 750.

Exercises XVII.

B.—(1) 12·57 × 10^{-5} volts.
(2) 250 dynes; perpendicular to field and conductor.

C.—(2) ·0000141372 volt. (3) 1·28 × 10^{-3} volts.
(5) L = ·05 henry; M = ·018 henry. Reduced.

Exercises XVIII.

C.—(3) $\frac{1125 \times 10^5}{\pi^2}$ dynes.

Exercises XIX.

C.—(1) 14·85 for (*a*) and 19·09 for (*b*) nearly. (2) 4 m.f.

Exercises XX.

C.—(8) ·134 H.P.; 78·8 per cent.

Exercises XXIV.

C.—(5) If R be the radius the force is $\frac{2\pi R}{10^{10}}$ dynes.

$$\text{The radius } R = \frac{1}{\sqrt{2\pi 10^{10}}} \text{ cm.}$$

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